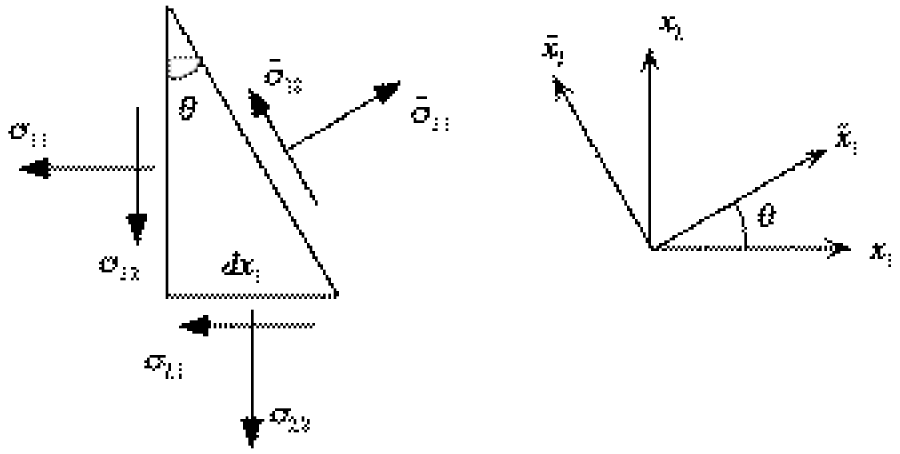
Problem M12 (Materials and Structures)

The figure below shows a triangular element in a two dimensional plane. The element is defined by an angle θ and the length of the opposite side dx_1. The element is of uniform thickness, dx_3. The element is acted on by a state of stress in the plane, σ_{11}, σ_{22} and σ_{12}. Stresses $\tilde{\sigma}_{11}$ and $\tilde{\sigma}_{12}$ y corresponding to a rotated axis system, $\tilde{x}_{\theta}$. By considering equilibrium of the forces acting on the triangular element drawn below show that:

$$\tilde{\sigma}_{11} = \cos^2\theta\,\sigma_{11} + \sin^2\theta\,\sigma_{22} + 2\cos\theta\sin\theta\,\sigma_{12} \qquad (1)$$

and

$$\tilde{\sigma}_{12} = -\cos\theta\sin\theta\,\sigma_{11} + \cos\theta\sin\theta\,\sigma_{22} + \left(\cos^2\theta - \sin^2\theta\right)\sigma_{12} \qquad (2)$$



Find the values of θ that produce the maximum and minimum values of $\tilde{\sigma}_{11}$, what are the corresponding values of $\tilde{\sigma}_{12}$?. Do not try to distinguish between the maximum and minimum values.

Note: This is a "plane stress" problem, i.e. stresses only act in the plane of the drawing ($\sigma_{33}=\sigma_{13}=\sigma_{23}=0$). This problem is at the heart of transforming stress, take the time to make sure that you understand the procedure.