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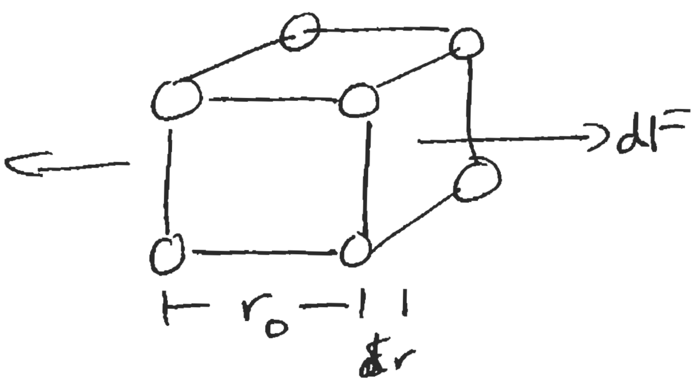
SOLUTIONS

$$U = -\frac{A}{r^m} + \frac{B}{r^n} \qquad (0)$$

long range electrostatic attraction

Short range repulsion inner electron orbitals, nuclei.

Cubic unit cell



Young's modulus

$$= \frac{dF}{A} \cdot \frac{dr_0}{dr} = \frac{dF\, r_0}{dr\; r_0^2}$$

$$= \frac{1}{r_0}\frac{dF}{dr} = \frac{1}{r_0}\left.\frac{d^2U}{dr^2}\right|_{r=r_0} \qquad (1)$$

$$\frac{dU}{dr} = mA\,r^{(-m-1)} - nB\,r^{(-n-1)}, \quad \cancel{\frac{d^2U}{dr^2}}$$

at $r = r_0$ $\quad \frac{dU}{dr} = 0$

$$\Rightarrow B = \frac{m}{n} A\, r_0^{\,n-m} \qquad \text{Substitute into (0)}$$

$$\Rightarrow U = -A r^{-m} + \frac{m}{n} A\, r_0^{\,n-m}\, r^{-n} \qquad (2)$$

$$\therefore U(r_0) = -A r_0^{-m} + \frac{m}{n} A\, r_0^{(n-m-n)} = A r_0^{-m}\left(\frac{m}{n} - 1\right) = A r_0^{-m}\left(\frac{m-n}{n}\right)$$

From definition in question

$$-KT_m = A r_0^{-m}\left(\frac{m-n}{n}\right)$$

$$\therefore\ A = \frac{n}{(m-n)} K T_m r_0^{m}, \qquad B = -\frac{m}{m-n} K T_m \cancel{r_0^{m}} \cdot r_0^{\cancel{n-m}} = \frac{m}{m-n} K T_m r_0^{n}$$

$$U = +\frac{n}{m-n} K T_m \frac{r_0^{m}}{r^{m}} - \frac{m}{m-n} K T_m \frac{r_0^{n}}{r^{n}}$$

$$\therefore\ \frac{dU}{dr} = -\frac{mn}{m-n} K T_m r_0^{m} r^{(-m-1)} + \frac{nm}{m-n} K T_m r_0^{n} r^{(-n-1)}$$

$$\frac{dF}{dr} = +\frac{(m+1)mn}{m-n} K T_m r_0^{m} r^{-(m+2)} - \frac{(n+1)nm}{m-n} K T_m r_0^{n} r^{-(n+2)}$$

for $r = r_0$,

$$\frac{1}{r_0}\frac{dF}{dr} = +\frac{(m+1)mn}{m-n} K T_m \cancel{r_0^{m}} \cdot \cancel{r_0^{-m-2}} \cdot r_0^{-1} - \frac{(n+1)nm}{m-n} K T_m \cancel{r_0^{n}} \cancel{r_0^{-n-2}} r_0^{-1}$$

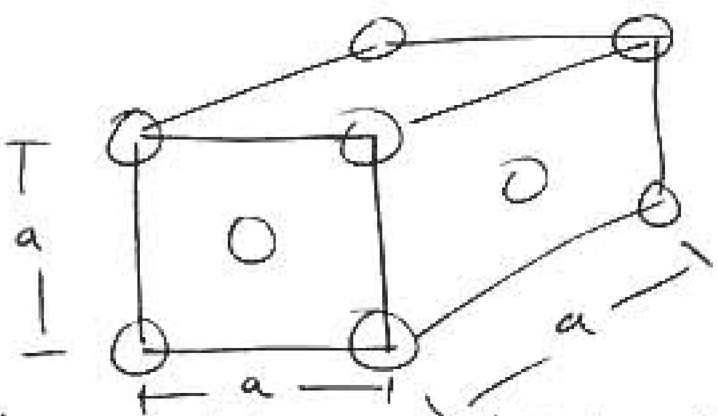
$$= \frac{mnKT_m}{\cancel{(m-n)}\, r_0^{3}} \Big(\underbrace{+(m+1) - (n+1)}_{=\ \cancel{m-n}} \Big), \quad r_0^{3} = \Omega$$

$$\therefore\ E = \frac{1}{r_0}\frac{dF}{dr}\bigg|_{r=r_0} = \frac{mnKT_m}{\Omega} \quad !$$

The purpose of this question is to demonstrate intrinsic link between moduli & T_m. Diamond, SiC have high E high T_m, Polymers have low E, low T_m.

M29 FCC Packing density

- close packed directions on face diagonals

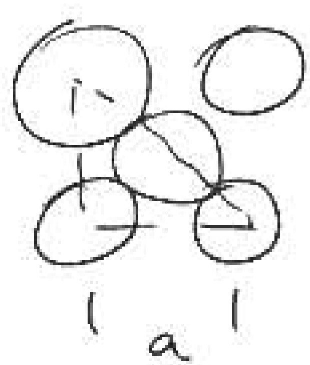


atomic radius r, atomic volume $\frac{4}{3}\pi r^3$

number of atoms / cube $= \underbrace{(8 \times \frac{1}{8})}_{\text{corners}} + \underbrace{6 \times (\frac{1}{2})}_{\text{faces}} = 4$

$\therefore \; 4 \times \frac{4}{3}\pi r^3$ / cube $= \frac{16}{3}\pi r^3$ of 'solid'

side of cube =



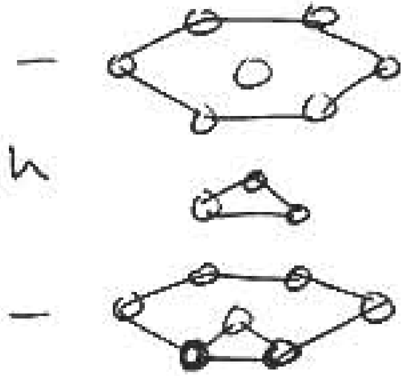
$$2a^2 = (4r)^2 = 16r^2$$
$$a = \sqrt{8}\, r$$

$\therefore$ packing density

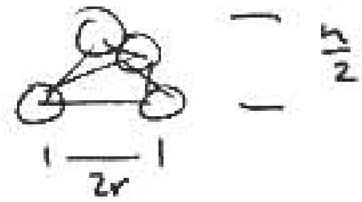
$$= \frac{16\pi \cancel{r^3}}{3} \Big/ (\sqrt{8})^3 \cancel{r^3} = \frac{16\pi}{3(\sqrt{8})^3} =$$

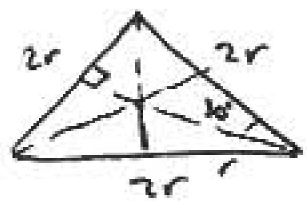
$$= 0.740$$

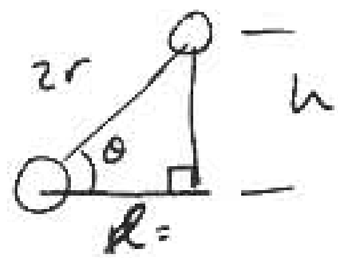
M24 HCP unit cell



consider tetrahedron







$$l \cos 30^\circ = r$$

$$l = \frac{2r}{\sqrt{3}}$$

$$\cos\theta = \frac{2r}{\sqrt{3}} \cdot \frac{1}{2r} = \frac{1}{\sqrt{3}}$$

$$h^2 = 4r^2 - \frac{4r^2}{3} = \frac{8}{3}r^2$$

area of triangle =

$$2r \times 2r \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}\, r^2$$

$$h = \sqrt{\frac{8}{3}}\, r$$

$\therefore$ volume of hexagonal unit cell $= 6 \times 2\sqrt{3}\, r^2 \times \sqrt{\frac{8}{3}}\, r$

$$= 12\sqrt{8}\, r^3$$

Number of atoms/cell $= 2 \times \frac{1}{2} + 12 \times \frac{1}{6} + 3 \times 1 = 6$

top & bottom faces; edge atoms; centre atoms

$\therefore$ packing density $= \dfrac{6 \times \frac{4}{3}\pi r^3}{12\sqrt{8}\, r^3} = \dfrac{2\pi}{3\sqrt{8}} = 0.74$

b) Density $= \frac{\text{Mass}}{\text{volume}}$

at unit cell level $= \frac{\text{\#atoms/cell} \cancel{\times \text{volume of}}}{\text{volume of cell}}$

i) Nickel. FCC. Mass/atom $= \frac{58.69}{6.023\times10^{23}}$

volume of cell $= \frac{\frac{58.69}{6.023\times10^{23}}\times 4}{8.90\times\cancel{10^6}} = a^3.$

$a^3 = 4.38\times10^{-26}$, $a = 3.52\times10^{-\cancel{8}10}$

$= 0.3.59$ nm $\Leftarrow$

$a = \sqrt{8}\, r \Rightarrow r = 1.24\times10^{-9}$ m $\Leftarrow$.

ii) For Magnesium, HCP Mass/atom $= \frac{24.31}{6.023\times10^{23}}$

volume of cell $= \frac{\frac{24.31\times 6}{\cancel{1.74\times10^3}\, 6.023\times10^{23}}}{1.74\times\cancel{10^6}} = 6\times2\sqrt{8}\, r^3$

$r^3 = 4.1\times10^{-27}$ $r = 1.60\times10^{-\cancel{8}10}$ m $\Leftarrow$

Problem M25

In addition to chapters 4-7 of Ashby and Jones Engineering Materials. You may also find the chapters on polymers in Ashby and Jones, Engineering Materials 2, helpful (this is a green covered book, available in the Aero-Astro library).

a) Define the term *polymer*; list three engineering polymers. A polymer is a large molecule made up of smaller repeating units (mers). Typically polymers have carbon "backbones" with side groups consisting of other organic atoms (C, H, O, N). Engineering polymers: polyethylene, polystyrene, epoxy

b) Define a *thermoplastic* and a *thermoset*. A thermoplastic softens dramatically with increasing temperature. A thermoset does not. Thermoplastics consist of long polymer chains with no covalent cross-links between the chains. The chains are bonded together by Van der Waals bonds. Thermosets have covalent crosslinks between the chains.

c) Distinguish between a cross-linked and a non-cross-linked polymer. See above. Thermoplastics are non-cross-linked and Thermosets have covalent cross-links

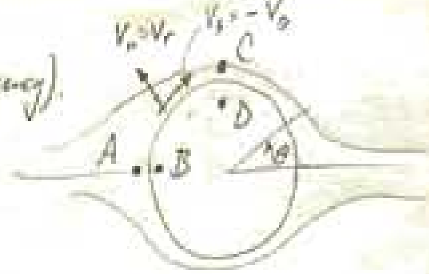
d) What is the glass transition temperature?. The glass transition temperature is the temperature at which the Van der Waals bonds melt. It is the temperature at which the elastic properties drop dramatically in thermoplastics.

e) Explain the change in moduli of polymers at the glass transition temperature. The Van der Waals bonds "melt" at this temperature, i.e. the thermal vibration exceeds the ability of the bonds to hold the molecules together. Thermoplastics rely on Van der Waals bonds for their elastic response at low temperatures. If these bonds are removed,then the polymer behaves viscoelastically, with the elastic component coming from entanglements between the polymer chains.

f) What is the range of temperature in which T_G lies for most engineering polymers?. 100-500 K.

g) How would you increase the modulus of a polymer? Introduce covalent cross-links.

Increase degree of crystallinity. Increase alignment of polymer chains.



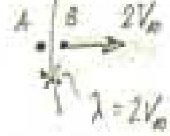
On all points on cylinder $V_n = 0$ (flow tangency).

a) At point A: $\lambda = -2V_\infty \cos(180°) = 2V_\infty$

$$\Delta V_n = V_{n_B} - \cancel{V_{n_A}}^{\,0} = \lambda$$

or $V_{n_B} = \lambda = 2V_\infty$

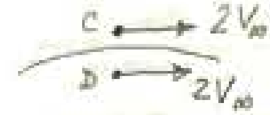
$V_{s_B} = 0$ by symmetry (no vertical velocity)



b) At point C: $\lambda = -2V_\infty \cos(90°) = 0$

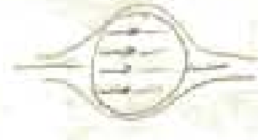
$$\Delta V_n = V_{n_D} - \cancel{V_{n_C}}^{\,0} = 0 \Rightarrow V_{n_D} = 0$$

Also $\Delta V_s = V_{s_D} - V_{s_C} = \gamma = 0 \Rightarrow V_{s_D} = V_{s_C} = 2V_\infty \quad (= -V_\theta)$

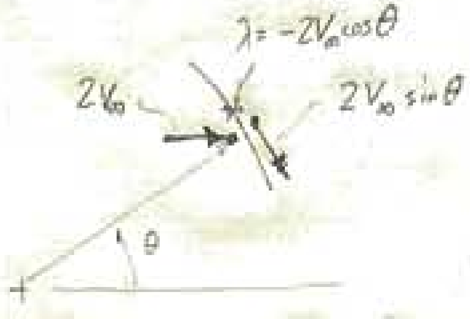


c) Velocities at both B and D are $2V_\infty$ in x-direction.

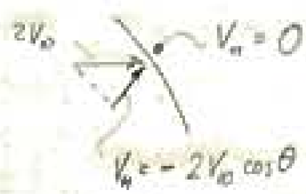
Interior velocity appears to be equal to $2V_\infty$ everywhere.



d) Examine some other general θ location:



Look at normal components



Check: $V_{n_{inside}} - V_{n_{outside}} \stackrel{?}{=} \lambda$

$$-2V_\infty \cos\theta - 0 \stackrel{\checkmark}{=} -2V_\infty \cos\theta$$

Source sheet model is consistent with flow about cylinder

Interior flow is $2V_\infty$ in x direction

a) $L' = \frac{1}{2}\rho V_\infty^2 c\, c_\ell \qquad D' = \frac{1}{2}\rho V_\infty^2 c\, c_d$

$$L = \int_{-b/2}^{b/2} L'\,dy = \int_{-b/2}^{b/2} \underbrace{\tfrac{1}{2}\rho V_\infty^2 c\, c_\ell}_{\text{all constant}}\,dy = \tfrac{1}{2}\rho V_\infty^2 c\, c_\ell \cdot b = \tfrac{1}{2}\rho V^2 S c_\ell$$

$$C_L = \frac{L}{\frac{1}{2}\rho V_\infty^2 S} = \frac{\frac{1}{2}\rho V_\infty^2 S c_\ell}{\frac{1}{2}\rho V_\infty^2 S} \Rightarrow \boxed{C_L = c_\ell}$$

likewise $\boxed{C_D = c_d}$

b) In level flight, $L = mg$ = constant

$$mg = \tfrac{1}{2}\rho V^2 S C_L$$

$$\rightarrow V(C_L) = \sqrt{\frac{mg}{S}\frac{2}{\rho C_L}} = \left(\frac{mg}{S}\frac{2}{\rho}\right)^{1/2}\frac{1}{C_L^{1/2}}$$

also $D = \frac{1}{2}\rho V^2 S C_D$

$$P = DV = \tfrac{1}{2}\rho V^3 S C_D = \tfrac{1}{2}\rho V^3 S\left[0.01 + 0.015\, C_L^3\right]$$

$$P(C_L) = \frac{1}{2}\rho S\left(\frac{mg}{S}\frac{2}{\rho}\right)^{3/2} \times \left[0.01\frac{1}{C_L^{3/2}} + 0.015\, C_L^{3/2}\right]$$

Ignoring constants: $\bar{V}(C_L) = \dfrac{1}{C_L^{1/2}}$, $\bar{P}(C_L) = \dfrac{0.01}{C_L^{3/2}} + 0.015\, C_L^{3/2}$

Can plot $\bar{P}(C_L)$ verses $\bar{V}(C_L)$ with $C_L = 0.1 .. 1.2$

Or note that $\bar{P}(\bar{V}) = 0.01\bar{V}^3 + \dfrac{0.015}{\bar{V}^3}$, plot $\bar{P}(\bar{V})$