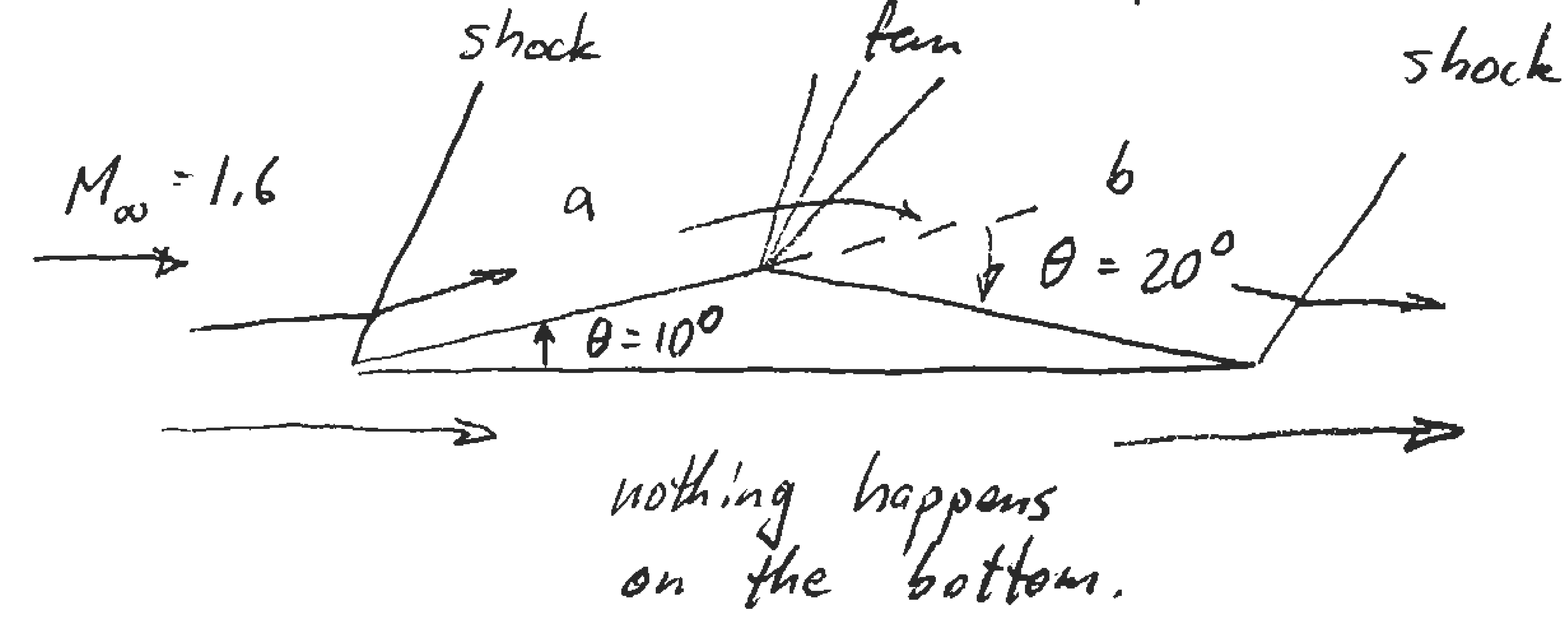
UE Fluids | F17 + F18 Solution | Spring '04

Flow sketch



a) Oblique Shock. $M_1 = 1.6,\ \theta = 10° \rightarrow \beta = 51°$ (chart, Anderson p. 513)

$$M_{n_1} = M_1 \sin 51° = 1.6 \cdot 0.777 = 1.243$$

$$p_2/p_1 = f(M_{n_1}) = 1.636 \quad \text{(Anderson eq. 9.16, or Appendix B table.)}$$

Since $p_1 = p_\infty \rightarrow \quad p_2 = \boxed{1.636\, p_\infty = p_a}$

$$M_{n_2} = f(M_{n_1}) = 0.817 \quad \text{(eq. 9.14)}$$

$$\boxed{M_2 = M_a = \frac{M_{n_2}}{\sin(51° - 10°)} = 1.245}$$

b) Expansion fan. $M_1 = M_a = 1.245$, $\nu(M_1) = 4.7°$

$$\nu(M_2) = \nu(M_1) + 20° = 24.7° \longrightarrow M_2 = 1.94$$

$$p_2 = p_{o_2}\left[1 + \frac{\gamma-1}{2}M_2^2\right]^{\frac{-\gamma}{\gamma-1}}$$

$$p_{o_2} = p_{o_1} = p_{oa} \quad \text{(behind oblique shock).}$$

$$p_{oa}/p_{o\infty} = f(M_{n_1}) = 0.987$$

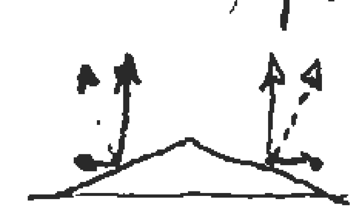
$$p_{oa} = 0.987\, p_{o\infty} = 0.987\, p_\infty\left[1 + \frac{\gamma-1}{2}M_\infty^2\right]^{\frac{\gamma}{\gamma-1}} = 4.195\, p_\infty$$

$$\therefore\ p_2 = 4.195\, p_\infty\left[1 + \frac{\gamma-1}{2}M_2^2\right]^{\frac{-\gamma}{\gamma-1}} = \boxed{0.588\, p_\infty = p_b}$$

Note: If we neglect the oblique shock's loss: $p_{ob} = p_{oa} = p_{o\infty} \rightarrow p_2 = 0.596\, p_\infty$ (not quite correct)

c) $L' = L'_a + L'_b = (p_\infty - p_a)\frac{c}{2} + (p_\infty - p_b)\frac{c}{2} = (1 - 1.636)p_\infty\frac{c}{2} + (1 - 0.588)p_\infty\frac{c}{2}$

$$\boxed{L' = -0.112\, p_\infty c}$$



$$D' = D'_a + D'_b = -L'_a \tan 10° + L'_b \tan 10° = \left[-(1 - 1.636) + (1 - 0.588)\right]\tan 10°\, p_\infty \frac{c}{2}$$

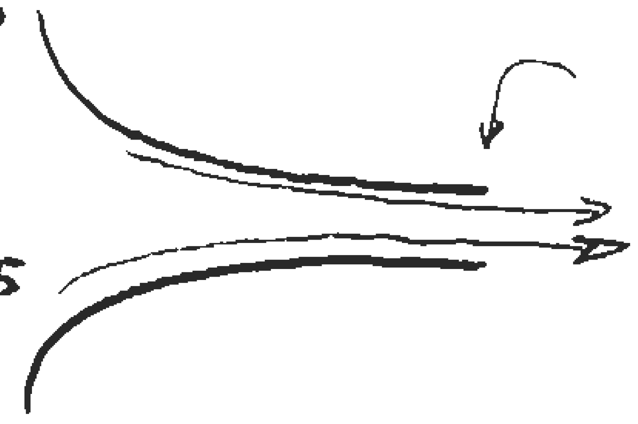
$$\boxed{D' = 0.0924\, p_\infty c}$$

Using $\frac{1}{2}\rho_\infty V_\infty^2 = \frac{\gamma}{2}p_\infty M_\infty^2 = 1.792\, p_\infty \rightarrow$

$$\boxed{\begin{aligned} c_l &= L'/\tfrac{1}{2}\rho_\infty V_\infty^2 = -0.0625 \\ c_d &= D'/\tfrac{1}{2}\rho_\infty V_\infty^2 = 0.0516 \end{aligned}}$$

Flow out of tank through air hose:

$T_0 = 300K°$



smallest $A = A^*$ at sonic conditions

$P_0 = 3.45 \times 10^5$ Pa

Required $\dot{m} = 0.01$ kg/s $= \rho^* a^* A^*$

$$\rho^* = \rho_0 \left[1 + \frac{\gamma-1}{2}\right]^{-\frac{1}{\gamma-1}} = 0.634\, \rho_0$$

but $h_0 = c_p T_0 = 1004\ J/kg°K \cdot 300° = 301200\ m^2/s^2$

$$a_0 = \sqrt{(\gamma-1) h_0} = 347.1\ m/s$$

$$\rho_0 = \gamma P_0 / (\gamma-1) h_0 = 4.01\ kg/m^3$$

So $\rho^* = 0.634\, \rho_0 = 2.542\ kg/m^3$

$$a^* = a_0 \left[1 + \frac{\gamma-1}{2}\right]^{-\frac{1}{2}} = 316.8\ m/s$$

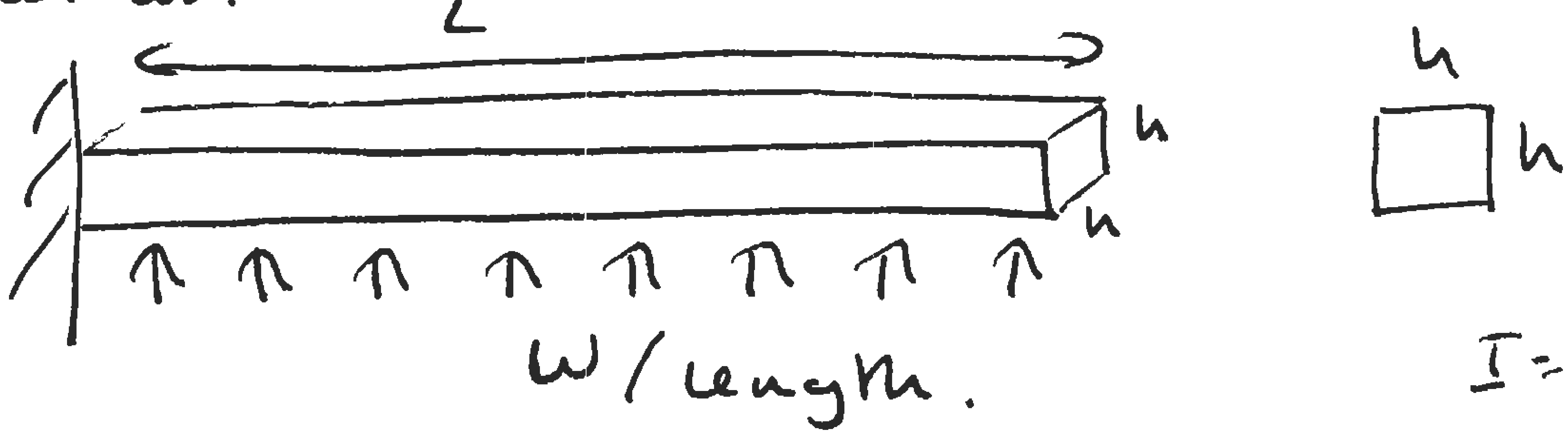
$$A^* = \frac{\dot{m}}{\rho^* a^*} = \frac{0.01}{2.542 \cdot 316.8} = 1.24 \times 10^{-5}\ m^2 = 0.124\ cm^2$$

$r = 2$ mm

diameter $= 4$ mm $= 0.156$ in

M19 Wing, designed for bending loads

Model as:



$I = \frac{1}{12}h^4$

Max moment at root = $\frac{wL^2}{2}$

Max stress = $\frac{Mz}{I} = \sigma_{max} = \frac{wL^2}{\cancel{2}}\cdot\frac{\cancel{h}}{\cancel{2}}\cdot\frac{\cancel{12}\,3}{h^{\cancel{4}\,3}}$

$= \frac{3wL^2}{h^3}$ cannot exceed σ_y

Mass $= \rho A L = \rho h^2 L$ $\qquad h = \sqrt{\frac{m}{\rho L}}$

substitute for h

$$\sigma_y = 3wL^2 \left(\frac{\rho L}{m}\right)^{3/2}$$

$\therefore$ minimum mass $m = (3wL^2)^{2/3}\,\frac{\rho L}{\sigma_y^{2/3}}$

$\therefore$ choose material with min $\rho/\sigma_y^{2/3}$

or max $\sigma_y^{2/3}/\rho$.

For turbine disk $\sigma_{max} < \sigma_y$

$$\therefore \quad \frac{3}{8} \rho V^2 = \sigma_y$$

choose material with max σ_y/ρ

	ρ	σ_y	σ_y/ρ	$\sigma_y^{2/3}/\rho$
6) Al 2024	2800	345	0.12	0.018
Al 7075	2800	495	0.18	0.022
Ti 6-4	4510	910	0.20	0.021
Ph 17-7 Ph.	8000	1435	0.18	0.016
steel	7800	260	0.03	0.005
			↓	↓
			Ti	Al 7075

Critical crack size for

Al 7075 $\quad a_c = \frac{1}{\pi}\left(\frac{K_c}{0.4\sigma_y}\right)^2 = \frac{1}{\pi}\left(\frac{24\times10^6}{0.4\times495\times10^6}\right)^2$

$a_c = 4.7$ mm. - detectable

Ti 6-4 $\quad a_c = \frac{1}{\pi}\left(\frac{50\times10^6}{0.9\times910\times10^6}\right)^2 = 1.2$ mm

small

d) Critical crack size for Ti 6-4 is small, harder to detect, more difficult to implement a damage tolerant design approach.

M20

$$\frac{da}{dN} = 2.7\times10^{-12}(\Delta K)^{5.0}$$

$$= 2.7\times10^{-12}\left(Y\Delta\sigma\sqrt{\pi a}\right)^{5.0}$$

$$= 2.7\times10^{-12}\, Y^5\, \Delta\sigma^5\, \pi^{2.5}\, a^{2.5}$$

$$a_{crit} = \frac{1}{\pi}\left(\frac{K_c}{\sigma_{max}}\right)^2 \qquad K_c = 44\ MPa\sqrt{m}$$

$$\sigma_{max} = \sigma_{mean} + 100\ MPa = 250\ MPa$$

$$a_{crit} = \frac{1}{\pi}\left(\frac{44}{250}\right)^2 = 9.9\ mm$$

$$N_f = \int_{a_0}^{a_f} \frac{da}{2.7\times10^{-12}\, Y^5\, \Delta\sigma^5\, \pi^{2.5}\, a^{2.5}}$$

$$N_f = \frac{1}{2.7\times10^{-12}(1.2)^5(200)^5\pi^{2.5}}\left[\frac{a^{-1.5}}{1.5}\right]_{9.9\times10^{-3}}^{3.0\times10^{-3}}$$

$$N_f = 89.88\ \text{cycles}$$

Inspect at more frequent intervals $\Rightarrow$ Maybe every 45 cycles