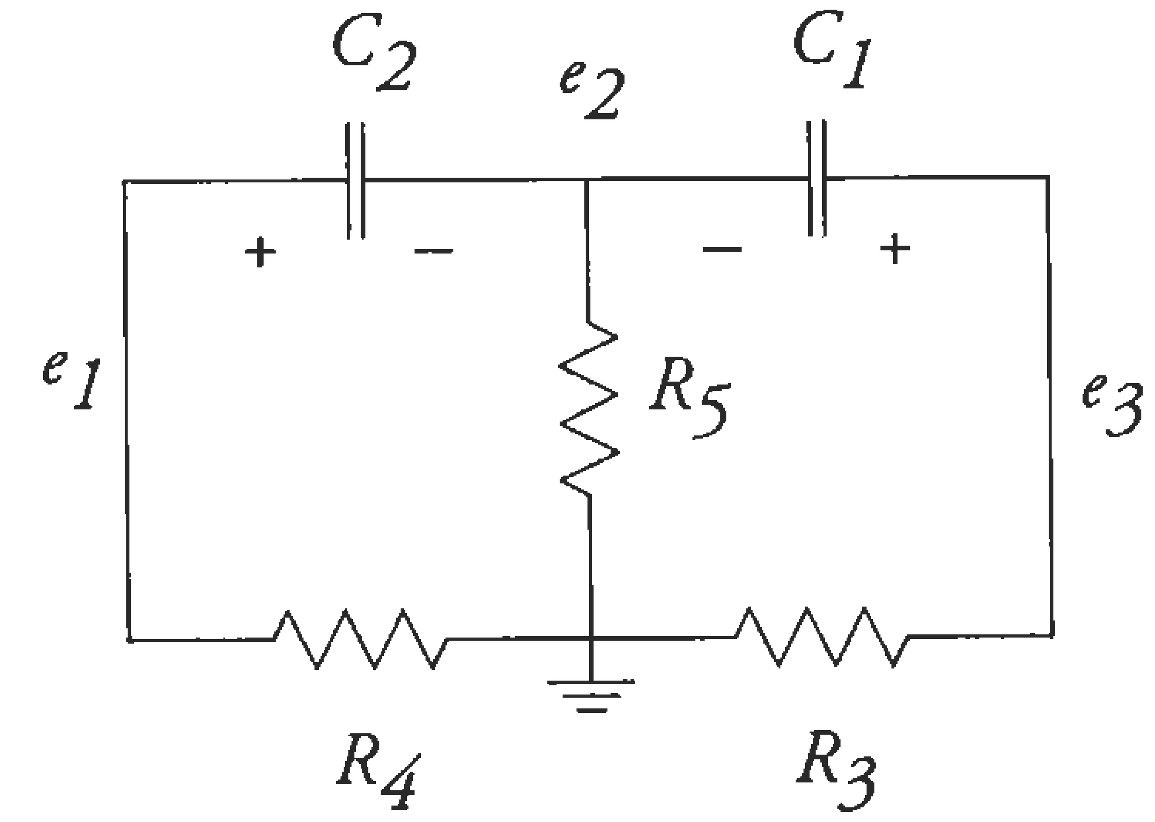

The node equations are:

$$e_1: \left(C_2\frac{d}{dt} + G_4\right)e_1 - C_2\frac{d}{dt}e_2 = 0$$

$$-C_2\frac{d}{dt}e_1 + \left(C_1\frac{d}{dt} + C_2\frac{d}{dt} + G_5\right)e_2 - C_1\frac{d}{dt}e_3 = 0$$

$$-C_1\frac{d}{dt}e_2 + \left(C_1\frac{d}{dt} + G_3\right)e_3 = 0$$

Plugging in component values,

$$\left(2\frac{d}{dt} + 1\right)e_1 - 2\frac{d}{dt}e_2$$

$$-2\frac{d}{dt}e_1 + \left(3\frac{d}{dt} + 1\right)e_2 - \frac{d}{dt}e_3 = 0$$

$$-\frac{d}{dt}e_2 + \left(\frac{d}{dt} + 0.5\right)e_3 = 0$$

To find the solution, assume

$$e_1(t) = E_1 e^{st}$$

$$e_2(t) = E_2 e^{st}$$

Then

$$(2s+1)E_1 - 2sE_2 = 0$$

$$-2sE_1 + (3s+1)E_2 - sE_3 = 0$$

$$-sE_2 + (s+0.5)E_3 = 0$$

In matrix form,

$$\begin{bmatrix} 2s+1 & -2s & 0 \\ -2s & 3s+1 & -s \\ 0 & -s & s+0.5 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = 0$$

$$= M(s)\,\underline{E}$$

For there to be a nontrivial solution,

$$\det(M(s)) = 0$$
$$= 5s^2 + 3.5s + 0.5$$

This equation can be solved by using the quadratic formula, or a polynomial solver. The roots are

$$s_1 = -0.2\ \text{sec}^{-1}$$
$$s_2 = -0.5\ \text{sec}^{-1}$$

Solve for $\underline{E}$ in each case:

$\underline{s_1 = -0.2}$ $\Rightarrow M(s) = \begin{bmatrix} 0.6 & +0.4 & 0 \\ +0.4 & 0.4 & +0.2 \\ 0 & +0.2 & 0.3 \end{bmatrix}$ *

Normally, would solve by row reduction. Because of the zeros in M, can solve as follows: Set $E_3 = 1$. From last row of M,

$$+0.2E_2 + 0.3E_3 = 0$$ *

$$\Rightarrow E_2 = -1.5$$ *

From row 1 of M,

$$0.6\,E_1 + 0.4\,E_2 = 0 \qquad *$$

$$\Rightarrow E_1 = 1$$

So

$$\underline{E}_1 = \begin{bmatrix} 1 \\ -1.5 \\ 1 \end{bmatrix} \qquad *$$

(Of course, any multiple of this is also a solution.)

$\underline{\underline{S_2 = -0.5}}$ $\Rightarrow M(s) = \begin{bmatrix} 0 & +1 & 0 \\ +1 & -0.5 & +0.5 \\ 0 & +0.5 & 0 \end{bmatrix}$ *

From row 1 (or row 3),

$$+E_2 = 0 \Rightarrow E_2 = 0 \qquad *$$

Arbitrarily choose $E_3 = 1$. Then from row 2,

$$+E_1 - 0.5\,E_2 + 0.5\,E_3 = 0 \qquad *$$

$$\Rightarrow E_1 = -0.5$$

Therefore,

$$\underline{E}_2 = \begin{bmatrix} -0.5 \\ 0 \\ 1 \end{bmatrix} \qquad \text{(or any multiple)}$$

<u>Total solution</u>

The total solution is given by

$$\begin{pmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{pmatrix} = a\,\underline{E}_1 e^{s_1 t} + b\,\underline{E}_2 e^{s_2 t}$$

From the circuit,

$$v_1(t) = e_3(t) - e_2(t)$$

$$v_2(t) = e_1(t) - e_2(t)$$

To match the initial conditions,

$$v_1(0) = 10\ \text{V} = a(1+1.5)e^0 + b(1-0)e^0 \quad *$$

$$= 2.5a + b \quad *$$

$$v_2(0) = 0\ \text{V} = a(1+1.5)e^0 + b(-0.5-0)e^0 \quad *$$

$$= 2.5a - 0.5b \quad *$$

Therefore,

$$\left.\begin{array}{l} 2.5a + b = 10 \\ 2.5a - 0.5b = 0 \end{array}\right\} \Longrightarrow \begin{array}{l} a = 1.333 \quad * \\ b = 6.667 \end{array}$$

The final solution is then

$$\boxed{\begin{array}{l} v_1(t) = \left(+3.333\,e^{-0.2t} + 6.667\,e^{-0.5t}\right)\ \text{V} \\ v_2(t) = \left(3.333\,e^{-0.2t} - 3.333\,e^{-0.5t}\right)\ \text{V} \end{array}}$$

N.B.: Corrected lines are marked with an asterisk.