

PART 0

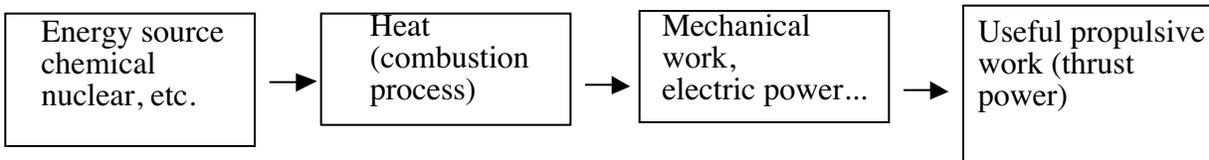
**PRELUDE: REVIEW OF "UNIFIED ENGINEERING
THERMODYNAMICS"**

PART 0 - PRELUDE: REVIEW OF “UNIFIED ENGINEERING THERMODYNAMICS”

[IAW pp 2-22, 32-41 (see IAW for detailed SB&VW references); VN Chapter 1]

0.1 What it's All About

The focus of thermodynamics in 16.050 is on the production of work, often in the form of kinetic energy (for example in the exhaust of a jet engine) or shaft power, from different sources of heat. For the most part the heat will be the result of combustion processes, but this is not always the case. The course content can be viewed in terms of a “propulsion chain” as shown below, where we see a progression from an energy source to useful propulsive work (thrust power of a jet engine). In terms of the different blocks, the thermodynamics in Unified Engineering and in this course are mainly about how to progress from the second block to the third, but there is some examination of the processes represented by the other arrows as well. The course content, objectives, and lecture outline are described in detail in Handout #1.



0.2 Definitions and Fundamental Ideas of Thermodynamics

As with all sciences, thermodynamics is concerned with the mathematical modeling of the real world. In order that the mathematical deductions are consistent, we need some precise definitions of the basic concepts.

A **continuum** is a smoothed-out model of matter, neglecting the fact that real substances are composed of discrete molecules. **Classical thermodynamics** is concerned only with continua. If we wish to describe the properties of matter at a molecular level, we must use the techniques of **statistical mechanics** and **kinetic theory**.

A **closed system** is a fixed quantity of matter around which we can draw a boundary. Everything outside the boundary is the **surroundings**. Matter cannot cross the boundary of a closed system and hence the principle of the conservation of mass is automatically satisfied whenever we employ a closed system analysis.

The **thermodynamic state** of a system is defined by the value of certain **properties** of that system. For fluid systems, typical properties are pressure, volume and temperature. More complex systems may require the specification of more unusual properties. As an example, the state of an electric battery requires the specification of the amount of electric charge it contains.

Properties may be **extensive** or **intensive**. Extensive properties are additive. Thus, if the system is divided into a number of sub-systems, the value of the property for the whole system is equal to the sum of the values for the parts. **Volume** is an extensive property. Intensive properties do not depend on the quantity of matter present. **Temperature** and **pressure are intensive properties**.

Specific properties are extensive properties per unit mass and are denoted by lower case letters. For example:

$$\text{specific volume} = V/m = v.$$

Specific properties are intensive because they do not depend on the mass of the system,

A **simple system** is a system having uniform properties throughout. In general, however, properties can vary from point to point in a system. We can usually analyze a general system by sub-dividing it (either conceptually or in practice) into a number of simple systems in each of which the properties are assumed to be uniform.

If the state of a system changes, then it is undergoing a **process**. The succession of states through which the system passes defines the **path** of the process. If, at the end of the process, the properties have returned to their original values, the system has undergone a **cyclic process**. Note that although the system has returned to its original state, the state of the surroundings may have changed.

Muddy points

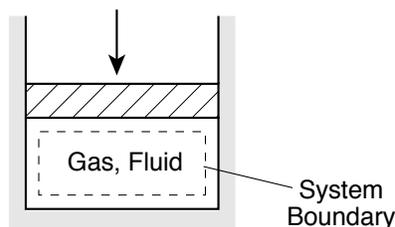
Specific properties (MP 0.1)

What is the difference between extensive and intensive properties? (MP 0.2)

0.3 Review of Thermodynamic Concepts

The following is a brief discussion of some of the concepts introduced in Unified Engineering, which we will need in 16.050. Several of these will be further amplified in the lectures and in other handouts. If you need additional information or examples concerning these topics, they are described clearly and in-depth in the Unified Notes of Professor Waitz, where detailed references to the relevant sections of the text (SB&VW) are given. They are also covered, although in a less detailed manner, in Chapters 1 and 2 of the book by Van Ness.

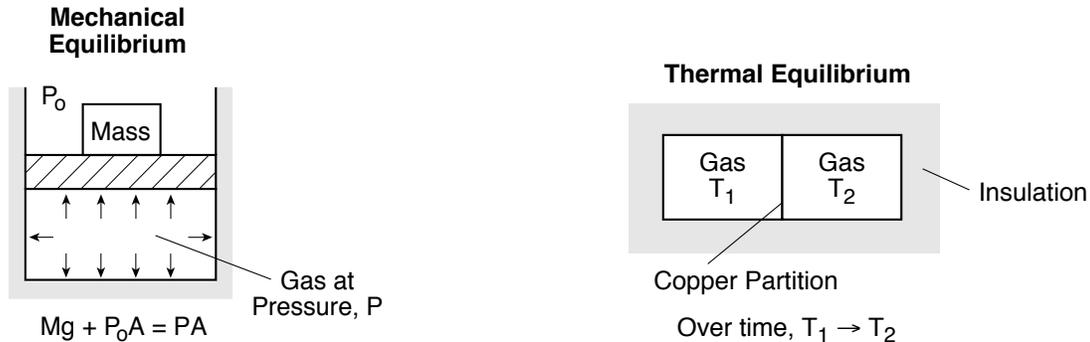
- 1) Thermodynamics can be regarded as a generalization of an enormous body of empirical evidence. It is extremely general, and there are no hypotheses made concerning the structure and type of matter that we deal with.
- 2) Thermodynamic system:
A quantity of matter of fixed identity. Work or heat (see below) can be transferred across the system boundary, but mass cannot.



- 3) Thermodynamic properties:
For engineering purposes, we want "averaged" information, i.e., macroscopic not microscopic (molecular) description. (Knowing the position and velocity of each of 10^{20+} molecules that we meet in typical engineering applications is generally not useful.)
- 4) State of a system:
The thermodynamic state is defined by specifying values of a (small) set of measured properties which are sufficient to determine all the remaining properties.

5) Equilibrium:

The state of a system in which properties have definite (unchanged) values as long as external conditions are unchanged is called an equilibrium state. Properties (P , pressure, T , temperature, ρ , density) describe states only when the system is in equilibrium.



6) Equations of state:

For a simple compressible substance (e.g., air, water) we need to know two properties to set the state. Thus:

$$P = P(v, T), \text{ or } v = v(P, T), \text{ or } T = T(P, v)$$

where v is the volume per unit mass, $1/\rho$.

Any of these is equivalent to an equation $f(P, v, T) = 0$ which is known as an equation of state. The equation of state for an ideal gas, which is a very good approximation to real gases at conditions that are typically of interest for aerospace applications is:

$$P\bar{v} = RT,$$

where \bar{v} is the volume per mol of gas and R is the "Universal Gas Constant", 8.31 kJ/kmol-K.

A form of this equation which is more useful in fluid flow problems is obtained if we divide by the molecular weight, M :

$$Pv = RT, \text{ or } P = \rho RT$$

where R is R/M , which has a different value for different gases. For air at room conditions, R is 0.287 kJ/kg-K.

7) Quasi-equilibrium processes:

A system in thermodynamic equilibrium satisfies:

- a) mechanical equilibrium (no unbalanced forces)
- b) thermal equilibrium (no temperature differences)
- c) chemical equilibrium.

For a finite, unbalanced force, the system can pass through non-equilibrium states. We wish to describe processes using thermodynamic coordinates, so we cannot treat situations in which such imbalances exist. An extremely useful idealization, however, is that only "infinitesimal" unbalanced forces exist, so that the process can be viewed as taking place in a series of "quasi-equilibrium" states. (The term *quasi* can be taken to mean "as if"; you will see it used in a number of contexts such as quasi-one-dimensional, quasi-steady, etc.) For this to be true the process must be slow in relation to the time needed for the system to come to equilibrium internally. For a gas

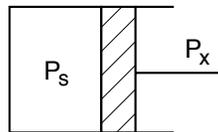
at conditions of interest to us, a given molecule can undergo roughly 10^{10} molecular collisions per second, so that, if ten collisions are needed to come to equilibrium, the equilibration time is on the order of 10^{-9} seconds. This is generally much shorter than the time scales associated with the bulk properties of the flow (say the time needed for a fluid particle to move some significant fraction of the length of the device of interest). Over a large range of parameters, therefore, it is a very good approximation to view the thermodynamic processes as consisting of such a succession of equilibrium states.

8) Reversible process

For a simple compressible substance,

$$\text{Work} = \int P dV.$$

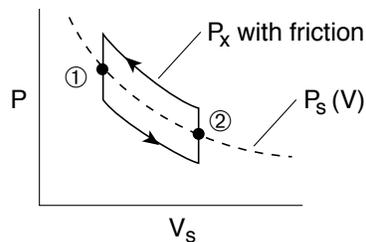
If we look at a simple system, for example a cylinder of gas and a piston, we see that there can be two pressures, P_s , the system pressure and P_x , the external pressure.



The work done by the system on the environment is

$$\text{Work} = \int P_x dV.$$

This can only be related to the system properties if $P_x \approx P_s$. For this to occur, there cannot be any friction, and the process must also be slow enough so that pressure differences due to accelerations are not significant.



$$\text{①} \rightarrow \text{②} \rightarrow \text{①}$$

$$\int P_x dV \neq 0 \quad \text{but} \quad \int P_s dV = 0$$

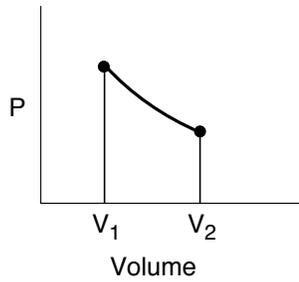
Work during an irreversible process $\neq \int P_s dV$

Under these conditions, we say that the process is reversible. The conditions for reversibility are that:

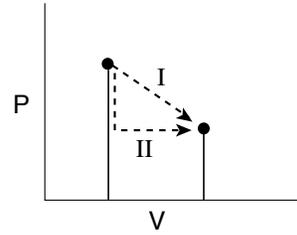
- If the process is reversed, the system and the surroundings will be returned to the original states.
- To reverse the process we need to apply only an infinitesimal dP . A reversible process can be altered in direction by infinitesimal changes in the external conditions (see Van Ness, Chapter 2).

9) Work:

For simple compressible substances in reversible processes, the work done by the system on the environment is $\int P dV$. This can be represented as the area under a curve in a Pressure-volume diagram:



Work is area under curve of P(V)



Work depends on the path

$$W_{1-2}]_I \neq W_{1-2}]_{II}$$

- Work is path dependent;
- Properties only depend on states;
- Work is not a property, not a state variable;
- When we say W_{1-2} , the work between states 1 and 2, we need to specify the path;
- For *irreversible* (non-reversible) processes, we cannot use $\int PdV$; either the work must be given or it must be found by another method.

Muddy points

How do we know when work is done? (MP 0.3)

10) Heat

Heat is energy transferred due to temperature differences.

- Heat transfer can alter system states;
- Bodies don't "contain" heat; heat is identified as it comes across system boundaries;
- The amount of heat needed to go from one state to another is path dependent;
- Heat and work are different modes of energy transfer;
- Adiabatic processes* are ones in which no heat is transferred.

11) First Law of Thermodynamics

For a system,

$$\Delta E = Q - W$$

E is the energy of the system,

Q is the heat input *to* the system, and

W is the work done *by* the system.

$$E = U \text{ (thermal energy)} + E_{kinetic} + E_{potential} + \dots$$

If changes in kinetic and potential energy are not important,

$$\Delta U = Q - W$$

- U arises from molecular motion.
- U is a function of state, and thus ΔU is a function of state (as is ΔE).
- Q and W are not functions of state.

Comparing (b) and (c) we have the striking result that:

- ΔU is independent of path even though Q and W are not!**

Muddy points

What are the conventions for work and heat in the first law? (MP 0.4)

When does E->U? (MP 0.5)

12) Enthalpy:

A useful thermodynamic property, especially for flow processes, is the enthalpy. Enthalpy is usually denoted by H , or h for enthalpy per unit mass, and is defined by:

$$H = U + PV.$$

In terms of the specific quantities, the enthalpy per unit mass is

$$h = u + Pv = u + P/\rho.$$

13) Specific heats - relation between temperature change and heat input

For a change in state between two temperatures, the "specific heat" is:

$$\text{Specific heat} = Q/(T_{\text{final}} - T_{\text{initial}})$$

We must, however, specify the process, i.e., the path, for the heat transfer. Two useful processes are constant pressure and constant volume. The specific heat at constant pressure is denoted as C_p and that at constant volume as C_v , or c_p and c_v per unit mass.

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p \quad \text{and} \quad c_v = \left(\frac{\partial u}{\partial T} \right)_v$$

For an ideal gas

$$dh = c_p dT \quad \text{and} \quad du = c_v dT.$$

The ratio of specific heats, c_p/c_v is denoted by γ . This ratio is 1.4 for air at room conditions.

The specific heats c_v and c_p have a basic definition as derivatives of the energy and enthalpy. Suppose we view the internal energy per unit mass, u , as being fixed by specification of T , the temperature and v , the specific volume, i.e., the volume per unit mass. (For a simple compressible substance, these two variables specify the state of the system.) Thus,

$$u = u(T, v).$$

The difference in energy between any two states separated by small temperature and specific volume differences, dT and dv is

$$du = \left(\frac{\partial u}{\partial T} \right)_v dT + \left(\frac{\partial u}{\partial v} \right)_T dv$$

The derivative $(\partial u/\partial T)_v$ represents the slope of a line of constant v on a u - T plane. The derivative is also a function of state, i. e., a thermodynamic property, and is called the specific heat at constant volume, c_v .

The name specific heat is perhaps unfortunate in that only for special circumstances is the derivative related to energy transfer as heat. If a process is carried out slowly at constant volume, no work will be done and any energy increase will be due only to energy transfer as heat. For such a

process, c_v does represent the energy increase per unit of temperature (per unit of mass) and consequently has been called the "specific heat at constant volume". However, it is more useful to think of c_v in terms of its definition as a certain partial derivative, which is a thermodynamic property, rather than a quantity related to energy transfer as heat in the special constant volume process.

The enthalpy is also a function of state. For a simple compressible substance we can regard the enthalpy as a function of T and P , that is view the temperature and pressure as the two variables that define the state. Thus,

$$h = h(T,P).$$

Taking the differential,

$$dh = \left(\frac{\partial h}{\partial T}\right)_P dT + \left(\frac{\partial h}{\partial P}\right)_T dP$$

The derivative $\left(\frac{\partial h}{\partial T}\right)_P$ is called the specific heat at constant pressure, denoted by c_p .

The derivatives c_v and c_p constitute two of the most important thermodynamic derivative functions. Values of these properties have been experimentally determined as a function of the thermodynamic state for an enormous number of simple compressible substances.

14) Ideal Gases

The equation of state for an ideal gas is

$$PV = NRT,$$

where N is the number of moles of gas in the volume V . Ideal gas behavior furnishes an extremely good approximation to the behavior of real gases for a wide variety of aerospace applications. It should be remembered, however, that describing a substance as an ideal gas constitutes a model of the actual physical situation, and the limits of model validity must always be kept in mind.

One of the other important features of an ideal gas is that its internal energy depends only upon its temperature. (For now, this can be regarded as another aspect of the model of actual systems that the perfect gas represents, but it can be shown that this is a consequence of the form of the equation of state.) Since u depends only on T ,

$$du = c_v(T)dT$$

In the above equation we have indicated that c_v can depend on T .

Like the internal energy, the enthalpy is also only dependent on T for an ideal gas. (If u is a function of T , then, using the perfect gas equation of state, $u + Pv$ is also.) Therefore,

$$dh = c_p(T)dT.$$

Further, $dh = du + d(Pv) = c_v dT + R dT$. Hence, for an ideal gas,

$$c_v = c_p - R.$$

In general, for other substances, u and h depend on pressure as well as on temperature. In this respect, the ideal gas is a very special model.

The specific heats do not vary greatly over wide ranges in temperature, as shown in VWB&S Figure 5.11. It is thus often useful to treat them as constant. If so

$$u_2 - u_1 = c_v(T_2 - T_1)$$

$$h_2 - h_1 = c_p(T_2 - T_1)$$

These equations are useful in calculating internal energy or enthalpy differences, but it should be remembered that they hold only for an ideal gas with constant specific heats.

In summary, the specific heats are thermodynamic properties and can be used even if the processes are not constant pressure or constant volume. The simple relations between changes in energy (or enthalpy) and temperature are a consequence of the behavior of an ideal gas, specifically the dependence of the energy and enthalpy on temperature only, and are not true for more complex substances.

Adapted from "Engineering Thermodynamics", Reynolds, W. C and Perkins, H. C, McGraw-Hill Publishers

15) Specific Heats of an Ideal Gas

1. All ideal gases:

- (a) The specific heat at constant volume (c_v for a unit mass or C_V for one kmol) is a function of T only.
- (b) The specific heat at constant pressure (c_p for a unit mass or C_P for one kmol) is a function of T only.
- (c) A relation that connects the specific heats c_p , c_v , and the gas constant is

$$c_p - c_v = R$$

where the units depend on the mass considered. For a unit mass of gas, e. g., a kilogram, c_p and c_v would be the specific heats for one kilogram of gas and R is as defined above. For one kmol of gas, the expression takes the form:

$$C_P - C_V = R,$$

where C_P and C_V have been used to denote the specific heats for one kmol of gas and R is the universal gas constant.

- (d) The specific heat ratio, γ , = c_p/c_v (or C_P/C_V), is a function of T only and is greater than unity.

2. Monatomic gases, such as He, Ne, Ar, and most metallic vapors:

- (a) c_v (or C_V) is constant over a wide temperature range and is very nearly equal to $(3/2)R$ [or $(3/2)R$, for one kmol].
- (b) c_p (or C_P) is constant over a wide temperature range and is very nearly equal to $(5/2)R$ [or $(5/2)R$, for one kmol].
- (c) γ is constant over a wide temperature range and is very nearly equal to $5/3$ [$\gamma = 1.67$].

3. So-called permanent diatomic gases, namely H₂, O₂, N₂, Air, NO, and CO:

- (a) c_v (or C_V) is nearly constant at ordinary temperatures, being approximately $(5/2)R$ [$(5/2)R$, for one kmol], and increases slowly at higher temperatures.
- (b) c_p (or C_P) is nearly constant at ordinary temperatures, being approximately $(7/2)R$ [$(7/2)R$, for one kmol], and increases slowly at higher temperatures.
- (c) γ is constant over a temperature range of roughly 150 to 600K and is very nearly equal to 7/5 [$\gamma = 1.4$]. It decreases with temperature above this.

4. Polyatomic gases and gases that are chemically active, such as CO₂, NH₃, CH₄, and Freons:

The specific heats, c_v and c_p , and γ vary with the temperature, the variation being different for each gas. The general trend is that heavy molecular weight gases (i.e., more complex gas molecules than those listed in 2 or 3), have values of γ closer to unity than diatomic gases, which, as can be seen above, are closer to unity than monatomic gases. For example, values of γ below 1.2 are typical of Freons which have molecular weights of over one hundred.

Adapted from Zemansky, M. W. and Dittman, R. H., "Heat and Thermodynamics", Sixth Edition, McGraw-Hill book company, 1981

16) Reversible adiabatic processes for an ideal gas

From the first law, with $Q = 0$, $du = c_v dT$, and $Work = Pdv$

$$du + Pdv = 0 \tag{i}$$

Also, using the definition of enthalpy

$$dh = \underline{du} + \underline{Pdv} + vdP. \tag{ii}$$

The underlined terms are zero for an adiabatic process. Re-writing (i) and (ii),

$$\begin{aligned} \gamma c_v dT &= -\gamma Pdv \\ c_p dT &= vdP. \end{aligned}$$

Combining the above two equations we obtain

$$-\gamma Pdv = vdP \quad \text{or} \quad -\gamma dv/v = dP/P \tag{iii}$$

Equation (iii) can be integrated between states 1 and 2 to give

$$\begin{aligned} \gamma \ln(v_2/v_1) &= \ln(P_2/P_1), \text{ or, equivalently,} \\ (P_2 v_2^\gamma) / (P_1 v_1^\gamma) &= 1 \end{aligned}$$

For an *ideal gas* undergoing a *reversible, adiabatic process*, the relation between pressure and volume is thus:

$$\begin{aligned} Pv^\gamma &= \text{constant, or} \\ P &= \text{constant} \times \rho^\gamma. \end{aligned}$$

17) Examples of flow problems and the use of enthalpy

a) Adiabatic, steady, throttling of a gas (flow through a valve or other restriction)

Figure 0-1 shows the configuration of interest. We wish to know the relation between properties upstream of the valve, denoted by "1" and those downstream, denoted by "2".

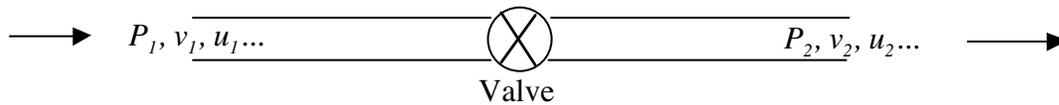


Figure 0-1: Adiabatic flow through a valve, a generic throttling process

To analyze this situation, we can define the system (choosing the appropriate system is often a critical element in effective problem solving) as a unit mass of gas in the following two states. Initially the gas is upstream of the valve and just through the valve as indicated. In the final state the gas is downstream of the valve plus just through the valve. The figures on the left show the actual configuration just described. In terms of the system behavior, however, we could replace the fluid external to the system by pistons which exert the same pressure that the external fluid exerts, as indicated schematically on the right side of Figure 0-2 below.

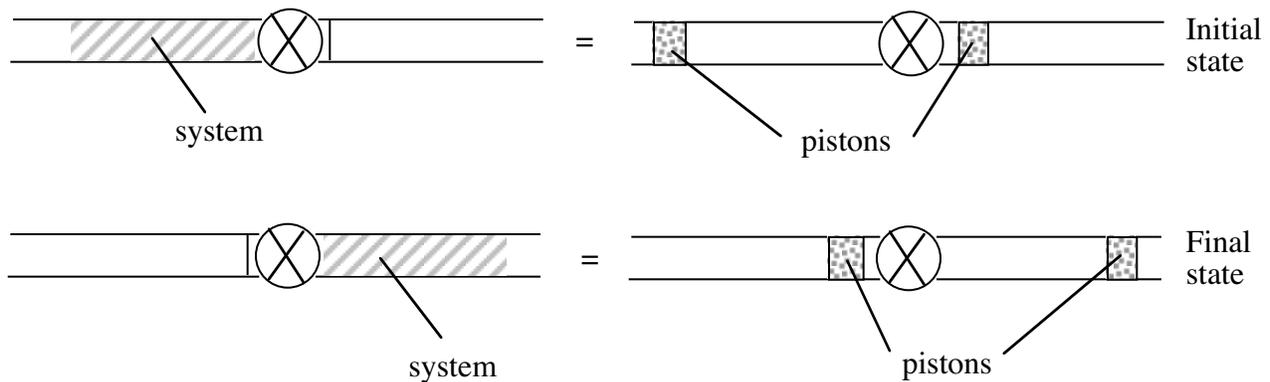


Figure 0-2: Equivalence of actual system and piston model

The process is adiabatic, with changes in potential energy and kinetic energy assumed to be negligible. The first law for the system is therefore

$$\Delta U = -W$$

The work done by the system is

$$W = P_2 V_2 - P_1 V_1$$

Use of the first law leads to

$$U_2 + P_2 V_2 = U_1 + P_1 V_1$$

In words, the initial and final states of the system have the same value of the quantity $U+PV$. For the case examined, since we are dealing with a unit mass, the initial final states of the system have the same value of $u+Pv$.

Muddy points

When is enthalpy the same in initial and final states? (MP 0.6)

b) Another example of a flow process, this time for an unsteady flow, is the transient process of filling a tank, initially evacuated, from a surrounding atmosphere, which is at a pressure P_0 and a temperature T_0 . The configuration is shown in Figure 0-3.

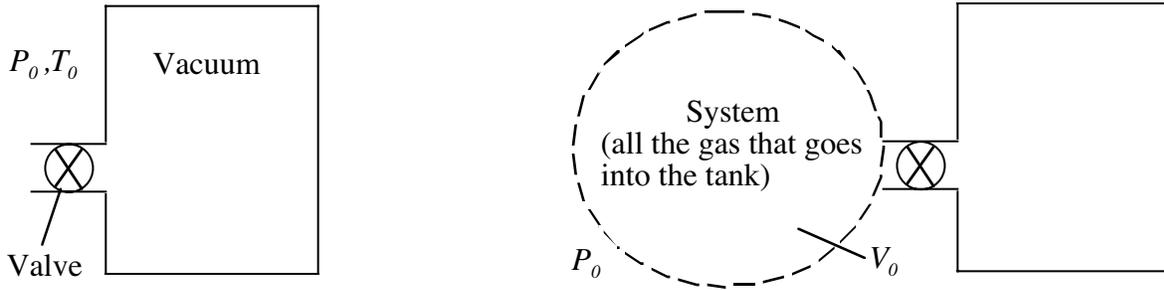


Figure 0-3: A transient problem—filling of a tank from the atmosphere

At a given time, the valve at the tank inlet is opened and the outside air rushes in. The inflow stops when the pressure inside is equal to the pressure outside. The tank is insulated, so there is no heat transfer to the atmosphere. What is the final temperature of the gas in the tank?

This time we take the system to be all the gas that enters the tank. The initial state has the system completely outside the tank, and the final state has the system completely inside the tank. The kinetic energy initially and in the final state is negligible, as is the change in potential energy so the first law again takes the form,

$$\Delta U = -W$$

Work is done on the system, of magnitude $P_0 V_0$, where V_0 is the initial volume of the system, so

$$\Delta U = P_0 V_0$$

In terms of quantities per unit mass ($\Delta U = m \Delta u$, $V_0 = m v_0$, where m is the mass of the system),

$$\Delta u = u_{final} - u_i = P_0 v_0$$

The final value of the internal energy is

$$\begin{aligned} u_{final} &= u_i + P_0 v_0 \\ &= h_i = h_0. \end{aligned}$$

For a perfect gas with constant specific heats,

$$\begin{aligned} u &= c_v T; \quad h = c_p T, \\ c_v T_{final} &= c_p T_0, \end{aligned}$$

$$T_{final} = \frac{c_p}{c_v} T_0 = \gamma T_0 .$$

The final temperature is thus roughly 200°F hotter than the outside air!

It may be helpful to recap what we used to solve this problem. There were basically four steps:

- 1 Definition of the system
- 2 Use of the first law
- 3 Equating the work to a “ PdV ” term
- 4 Assuming the fluid to be a perfect gas with constant specific heats.

A message that can be taken from both of these examples (as well as from a large number of other more complex situations, is that the quantity $h = u + Pv$ occurs naturally in problems of fluid flow. Because the combination appears so frequently, it is not only defined but also tabulated as a function of temperature and pressure for a number of working fluids.

Muddy points

In the filling of a tank, why (physically) is the final temperature in the tank higher than the initial temperature? (MP 0.7)

18) Control volume form of the system laws (Waitz pp 32-34, VWB&S, 6.1, 6.2)

The thermodynamic laws (as well as Newton’s laws) are for a system, a specific quantity of matter. More often, in propulsion and power problems, we are interested in what happens in a fixed volume, for example a rocket motor or a jet engine through which mass is flowing. For this reason, the control volume form of the system laws is of great importance. A schematic of the difference is shown below. Rather than focus on a particle of mass which moves through the engine, it is more convenient to focus on the volume occupied by the engine. This requires us to use the control volume form of the thermodynamic laws.

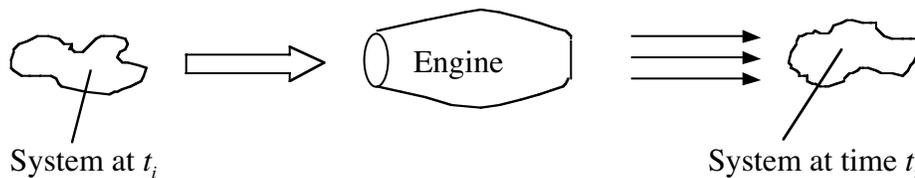


Figure 0-4: Control volume and system for flow through a propulsion device

The first of these is conservation of mass. For the control volume shown, the rate of change of mass inside the volume is given by the difference between the mass flow rate in and the mass flow rate out. For a single flow coming in and a single flow coming out this is

$$\frac{dm_{CV}}{dt} = \dot{m}_{in} - \dot{m}_{out} .$$

If the mass inside the control volume changes with time it is because some mass is added or some is taken out.

The first law of thermodynamics can be written as a rate equation:

$$\frac{dE}{dt} = \dot{Q} - \dot{W} .$$

To derive the first law as a rate equation for a control volume we proceed as with the mass conservation equation. The physical idea is that any rate of change of energy in the control

volume must be caused by the rates of energy flow into or out of the volume. The heat transfer and the work are already included and the only other contribution must be associated with the mass flow in and out, which carries energy with it. The figure below shows a schematic of this idea.

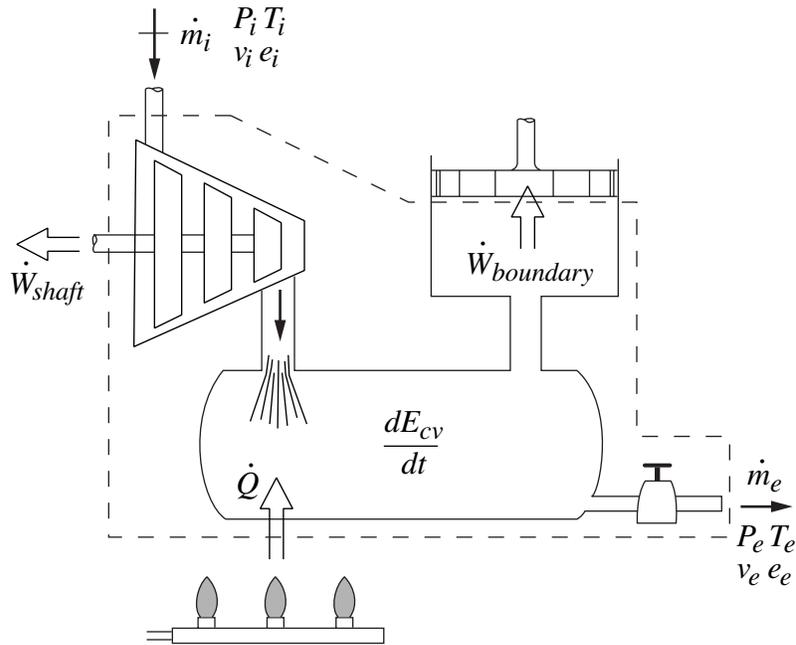


Figure 0-5: Schematic diagram illustrating terms in the energy equation for a control volume

The fluid that enters or leaves has an amount of energy per unit mass given by

$$e = u + c^2 / 2 + gz ,$$

where c is the fluid velocity. In addition, whenever fluid enters or leaves a control volume there is a work term associated with the entry or exit. We saw this in example 16a, and the present derivation is essentially an application of the ideas presented there. Flow exiting at station “e” must push back the surrounding fluid, doing work on it. Flow entering the volume at station “i” is pushed on by, and receives work from the surrounding air. The rate of flow work at exit is given by the product of the pressure times the exit area times the rate at which the external flow is “pushed back”. The last of these, however, is equal to the volume per unit mass times the rate of mass flow. Put another way, in a time dt , the work done on the surroundings by the flow at the exit station is

$$dW_{flow} = Pvdm_e .$$

The net rate of flow work is

$$\dot{W}_{flow} = P_e v_e \dot{m}_e - P_i v_i \dot{m}_i .$$

Including all possible energy flows (heat, shaft work, shear work, piston work etc.), the first law can then be written as:

$$\frac{d}{dt} \sum E_{CV} = \sum \dot{Q} + \sum \dot{W}_{shaft} + \sum \dot{W}_{shear} + \sum \dot{W}_{piston} + \sum \dot{W}_{flow} + \sum \dot{m}(u + \frac{1}{2}c^2 + gz)$$

where Σ includes the sign associated with the energy flow. If heat is added or work is done **on** the system then the sign is positive, if work or heat are extracted **from** the system then the sign is negative. NOTE: this is consistent with $\Delta E = Q - W$, where W is the work done **by** the system **on** the environment, thus work is flowing out of the system.

We can then collect the specific energy term e included in E_{cv} and the specific flow term Pv to make the enthalpy appear:

$$\text{Total energy associated with mass flow: } e + Pv = u + c^2 / 2 + gz + Pv = h + c^2 / 2 + gz = h_t ,$$

where h_t is the stagnation enthalpy (IAW, p.36).

Thus, the first law can be written as:

$$\frac{d}{dt} \sum E_{CV} = \sum \dot{Q} + \sum \dot{W}_{shaft} + \sum \dot{W}_{shear} + \sum \dot{W}_{piston} + \sum \dot{m}(h + \frac{1}{2}c^2 + gz) .$$

For most of the applications done in this course, there will be no shear work and no piston work. Hence, the first law for a control volume will be most often used as:

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W}_{shaft} + \dot{m}_i \left(h_i + c_i^2 / 2 + gz_i \right) - \dot{m}_e \left(h_e + c_e^2 / 2 + gz_e \right) .$$

The rate of work term is the sum of the shaft work and the flow work. In writing the control volume form of the equation we have assumed only one entering and one leaving stream, but this could be generalized to any number of inlet and exit streams.

Muddy points

What distinguishes shaft work from other works? (MP 0.8)

For problems of interest in aerospace applications the velocities are high and the term that is associated with changes in the elevation is small. From now on, we will neglect this term unless explicitly stated. The control volume form of the first law is thus

$$\begin{aligned} \frac{dE_{CV}}{dt} &= \dot{Q}_{CV} - \dot{W}_{shaft} + \dot{m}_i \left(h_i + c_i^2 / 2 \right) - \dot{m}_e \left(h_e + c_e^2 / 2 \right) \\ &= \dot{Q}_{CV} - \dot{W}_{shaft} + \dot{m}_i h_{ti} - \dot{m}_e h_{te} . \end{aligned}$$

For steady flow ($d/dt = 0$) the inlet and exit mass flow rates are the same and the control volume form of the first law becomes the ‘‘Steady Flow Energy Equation’’ (SFEE)

$$\text{Steady Flow Energy Equation: } \dot{m}(h_{te} - h_{ti}) = \dot{Q}_{CV} - \dot{W}_{shaft} .$$

The steady flow energy equation finds much use in the analysis of power and propulsion devices and other fluid machinery. Note the prominent role of enthalpy.

Using what we have just learned we can attack the tank filling problem solved in (16b) from an alternate point of view using the control volume form of the first law. In this problem the shaft work is zero, and the heat transfer, kinetic energy changes, and potential energy changes are neglected. In addition there is no exit mass flow.

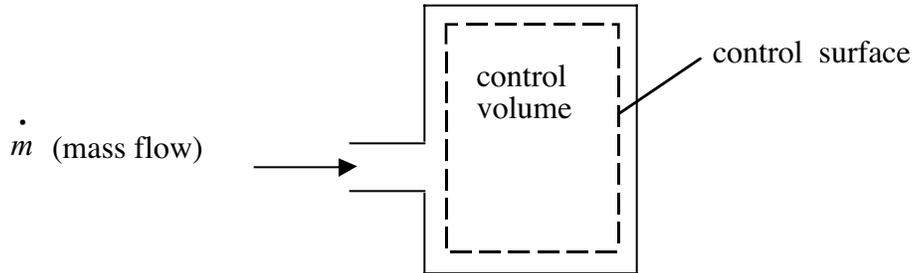


Figure 0-6: A control volume approach to the tank filling problem

The control volume form of the first law is therefore

$$\frac{dU}{dt} = \dot{m}_i h_i .$$

The equation of mass conservation is

$$\frac{dm}{dt} = \dot{m}_i .$$

Combining we have

$$\frac{dU}{dt} = \frac{dm}{dt} h_i .$$

Integrating from the initial time to the final time (the incoming enthalpy is constant) and using $U = mu$ gives the result $u_{final} = h_i = h_0$ as before.

Muddy points

Definition of a control volume (MP 0.9)

What is the difference between enthalpy and stagnation enthalpy? (MP 0.10)

Muddiest Points on Part 0

0.1 Specific properties

Energy, volume, enthalpy are all extensive properties. Their value depends not only on the temperature and pressure but also on “how much”, i.e., what the mass of the system is. The internal energy of two kilograms of air is twice as much as the internal energy of one kilogram of air. It is very often useful to work in terms of properties that do not depend on the mass of the system, and for this purpose we use the specific volume, specific energy, specific enthalpy, etc., which are the values of volume, energy, and enthalpy for a unit mass (kilogram) of the substance. For a system of mass m , the relations between the two quantities are:

$$V = mv ; U = mu ; H = mh$$

0.2 What is the difference between extensive and intensive properties?

Intensive properties are properties that do not depend on the quantity of matter. For example, pressure and temperature are intensive properties. Energy, volume and enthalpy are all extensive properties. Their value depends on the mass of the system. For example, the enthalpy of a certain mass of a gas is doubled if the mass is doubled; the enthalpy of a system that consists of several parts is equal to the sum of the enthalpies of the parts.

0.3 How do we know when work is done?

A rigorous test for whether work is done or not is whether a weight could have been raised in the process under consideration. I will hand out some additional material to supplement the notes on this point, which seems simple, but can be quite subtle to unravel in some situations.

0.4 What are the conventions for work and heat in the first law?

Heat is positive if it is given to the system. Work is positive if it is done by the system.

0.5 When does $E \rightarrow U$?

We deal with changes in energy. When the changes in the other types of energy (kinetic, potential, strain, etc) can be neglected compared to the changes in thermal energy, then it is a good approximation to use ΔU as representing the total energy change.

0.6 When is enthalpy the same in initial and final states?

Initial and final stagnation enthalpy is the same if the flow is steady and if there is no net shaft work plus heat transfer. If the change in kinetic energy is negligible, the initial and final enthalpy is the same. The “tank problem” is unsteady so the initial and final

enthalpies are not the same. See the discussion of steady flow energy equation in notes [(17) in Section 0].

0.7 In the filling of a tank, why (physically) is the final temperature in the tank higher than the initial temperature?

Work is done on the system, which in this problem is the mass of gas that is pushed into the tank.

0.8 What distinguishes shaft work from other works?

The term shaft work arises in using a control volume approach. As we have defined it, “shaft work” is all *work over and above work associated with the “flow work” (the work done by pressure forces)*. Generally this means work done by rotating machinery, which is carried by a shaft from the control volume to the outside world. There could also be work over and above the pressure force work done by shear stresses at the boundaries of the control volume, but this is seldom important if the control boundary is normal to the flow direction.

If we consider a *system* (a mass of fixed identity, say a blob of gas) flowing through some device, neglecting the effects of raising or lowering the blob the only mode of work would be the work to compress the blob. This would be true even if the blob were flowing through a turbine or compressor. (In doing this we are focusing on the *same* material as it undergoes the unsteady compression or expansion processes in the device, rather than looking at a control volume, through which mass passes.)

The question about shaft work and non shaft work has been asked several times. I am not sure how best to answer, but it appears that the difficulty people are having might be associated with being able to know when one can say that shaft work occurs. There are several features of a process that produces (or absorbs) shaft work. First of all, the view taken of the process is one of control volume, rather than control mass (see the discussion of control volumes in section 0 or in IAW). Second, there need to be a shaft or equivalent device (a moving belt, a row of blades) that can be identified as the work carrier. Third, the shaft work is work over and above the flow work that is done by (or received by) the streams that exit and enter the control volume.

0.9 Definition of a control volume.

A control volume is an enclosure that separates a quantity of matter from the surroundings or environment. The enclosure does not necessarily have to consist of a solid boundary like the walls of a vessel. It is only necessary that the enclosure forms a closed surface and that its properties are defined everywhere. An enclosure may transmit heat or be a heat insulator. It may be deformable and thus capable of transmitting work to the system. It may also be capable of transmitting mass.