

## 1.C Applications of the Second Law

[VN-Chapter 6; VWB&S-8.1, 8.2, 8.5, 8.6, 8.7, 8.8, 9.6]

### 1.C.1 Limitations on the Work that Can be Supplied by a Heat Engine

The second law enables us to make powerful and general statements concerning the maximum work that can be derived from any heat engine which operates in a cycle. To illustrate these ideas, we use a Carnot cycle which is shown schematically at the right. The engine operates between two heat reservoirs, exchanging heat

$Q_H$  with the high temperature reservoir at  $T_H$  and  $Q_L$  with the reservoir at  $T_L$ . The entropy changes of the two reservoirs are:

$$\Delta S_H = \frac{Q_H}{T_H}; \quad Q_H < 0$$

$$\Delta S_L = \frac{Q_L}{T_L}; \quad Q_L > 0$$

The same heat exchanges apply to the system, but with opposite signs; the heat received from the high temperature source is positive, and conversely. Denoting the heat transferred to the engines by subscript "e",

$$Q_{H_e} = -Q_H; \quad Q_{L_e} = -Q_L.$$

The total entropy change during any operation of the engine is,

$$\Delta S^{total} = \underbrace{\Delta S_H}_{\text{Reservoir at } T_H} + \underbrace{\Delta S_L}_{\text{Reservoir at } T_L} + \underbrace{\Delta S_e}_{\text{Engine}}$$

For a cyclic process, the third of these ( $\Delta S_e$ ) is zero, and thus (remembering that  $Q_H < 0$ ),

$$\Delta S^{total} = \Delta S_H + \Delta S_L = \frac{Q_H}{T_H} + \frac{Q_L}{T_L} \quad (\text{C.1.1})$$

For the engine we can write the first law as

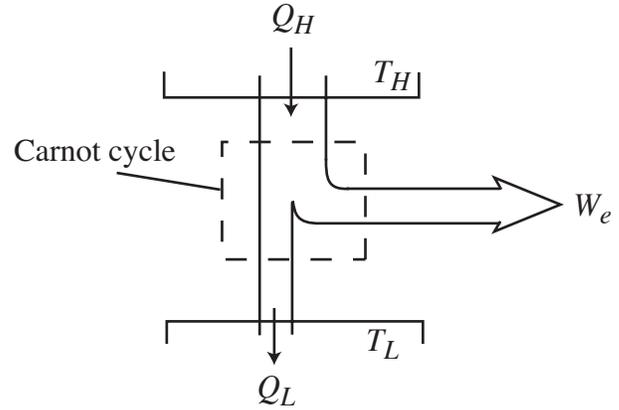
$$\Delta U_e = 0 \text{ (cyclic process)} = Q_{H_e} + Q_{L_e} - W_e.$$

Or,

$$\begin{aligned} W_e &= Q_{H_e} + Q_{L_e} \\ &= -Q_H - Q_L. \end{aligned}$$

Hence, using (C.1.1)

$$W_e = -Q_H - T_L \Delta S^{total} + Q_H \left( \frac{T_L}{T_H} \right)$$



$$= (-Q_H) \left[ 1 - \left( \frac{T_L}{T_H} \right) \right] - T_L \Delta S^{total}.$$

The work of the engine can be expressed in terms of the heat received by the engine as

$$W_e = (Q_{H_e}) \left[ 1 - \left( \frac{T_L}{T_H} \right) \right] - T_L \Delta S^{total}.$$

The upper limit of work that can be done occurs during a reversible cycle, for which the total entropy change ( $\Delta S^{total}$ ) is zero. In this situation:

$$\text{Maximum work for an engine working between } T_H \text{ and } T_L: W_e = (Q_{H_e}) \left[ 1 - \left( \frac{T_L}{T_H} \right) \right]$$

Also, for a reversible cycle of the engine,

$$\frac{Q_H}{T_H} + \frac{Q_L}{T_L} = 0.$$

These constraints apply to all reversible heat engines operating between fixed temperatures. The thermal efficiency of the engine is

$$\begin{aligned} \eta &= \frac{\text{Work done}}{\text{Heat received}} = \frac{W}{Q_{H_e}} \\ &= 1 - \frac{T_L}{T_H} = \eta_{Carnot}. \end{aligned}$$

The Carnot efficiency is thus the maximum efficiency that can occur in an engine working between two given temperatures.

We can approach this last point in another way. The engine work is given by

$$\begin{aligned} W_e &= -Q_H - T_L \Delta S^{total} + Q_H (T_L / T_H) \\ \text{or, } T_L \Delta S^{total} &= -Q_H + Q_H (T_L / T_H) - W_e \end{aligned}$$

The total entropy change can be written in terms of the Carnot cycle efficiency and the ratio of the work done to the heat absorbed by the engine. The latter is the efficiency of any cycle we can devise:

$$\Delta S^{total} = \frac{Q_{H_e}}{T_L} \left[ 1 - \frac{T_L}{T_H} - \frac{W_e}{Q_{H_e}} \right] = \frac{Q_{H_e}}{T_L} \left[ \eta_{Carnot} - \eta_{\text{Any other cycle}} \right].$$

The second law says that the total entropy change is equal to or greater than zero. This means that the Carnot cycle efficiency is equal to or greater than the efficiency for any other cycle, with the equality only occurring if  $\Delta S^{total} = 0$ .

### Muddy points

So, do we lose the capability to do work when we have an irreversible process and entropy increases? (MP 1C.1)

Why do we study cycles starting with the Carnot cycle? Is it because it is easier to work with? (MP 1C.2)

### 1.C.2 The Thermodynamic Temperature Scale

The considerations of Carnot cycles in this section have not mentioned the working medium. They are thus not limited to an ideal gas and hold for Carnot cycles with any medium. Because we derived the Carnot efficiency with an ideal gas as a medium, the temperature definition used in the ideal gas equation is not essential to the thermodynamic arguments. More specifically, we can define a thermodynamic temperature scale that is independent of the working medium. To see this, consider the situation shown below in Figure C-1, which has three reversible cycles. There is a high temperature heat reservoir at  $T_3$  and a low temperature heat reservoir at  $T_1$ . For any two temperatures  $T_1, T_2$ , the ratio of the magnitudes of the heat absorbed and rejected in a Carnot cycle has the same value for all systems.

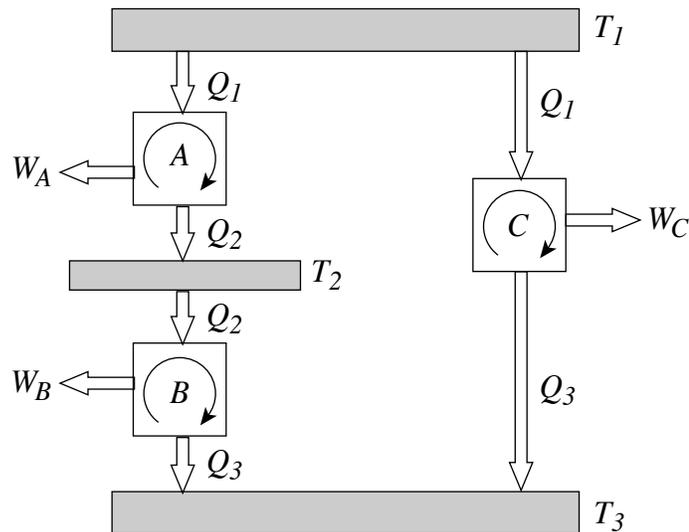


Figure C-1: Arrangement of heat engines to demonstrate the thermodynamic temperature scale

We choose the cycles so  $Q_1$  is the same for A and C. Also  $Q_3$  is the same for B and C. For a Carnot cycle

$$\eta = 1 + \frac{Q_L}{Q_H} = F(T_L, T_H); \eta \text{ is only a function of temperature.}$$

Also

$$\begin{aligned} Q_1/Q_2 &= F(T_1, T_2) \\ Q_2/Q_3 &= F(T_2, T_3) \\ Q_1/Q_3 &= F(T_1, T_3). \end{aligned}$$

But

$$\frac{Q_1}{Q_3} = \frac{Q_1}{Q_2} \frac{Q_2}{Q_3}.$$

Hence

$$\underbrace{F(T_1, T_3)}_{\substack{\text{Not a function} \\ \text{of } T_2}} = \underbrace{F(T_1, T_2) \times F(T_2, T_3)}_{\text{Cannot be a function of } T_2}.$$

We thus conclude that  $F(T_1, T_2)$  has the form  $f(T_1)/f(T_2)$ , and similarly  $F(T_2, T_3) = f(T_2)/f(T_3)$ . The ratio of the heat exchanged is therefore

$$\frac{Q_1}{Q_3} = F(T_1, T_3) = \frac{f(T_1)}{f(T_3)}.$$

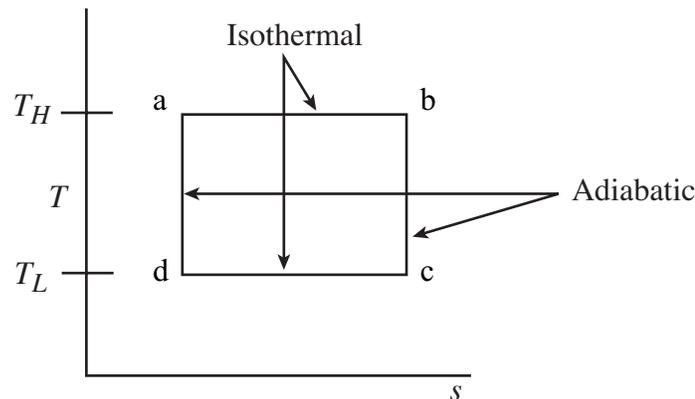
In general,

$$\frac{Q_H}{Q_L} = \frac{f(T_H)}{f(T_L)},$$

so that the ratio of the heat exchanged is a function of the temperature. We could choose any function that is monotonic, and one choice is the simplest:  $f(T) = T$ . This is the thermodynamic scale of temperature,  $Q_H/Q_L = T_H/T_L$ . The temperature defined in this manner is the same as that for the ideal gas; the thermodynamic temperature scale and the ideal gas scale are equivalent

### 1.C.3 Representation of Thermodynamic Processes in $T$ - $s$ coordinates.

It is often useful to plot the thermodynamic state transitions and the cycles in terms of temperature (or enthalpy) and entropy,  $T, S$ , rather than  $P, V$ . The maximum temperature is often the constraint on the process and the enthalpy changes show the work done or heat received directly, so that plotting in terms of these variables provides insight into the process. A Carnot cycle is shown below in these coordinates, in which it is a rectangle, with two horizontal, constant temperature legs. The other two legs are reversible and adiabatic, hence isentropic ( $dS = dQ_{rev}/T = 0$ ), and therefore vertical in  $T$ - $s$  coordinates.



Carnot cycle in  $T, s$  coordinates

If the cycle is traversed clockwise, the heat added is

$$\text{Heat added: } Q_H = \int_a^b T dS = T_H (S_b - S_a) = T_H \Delta S.$$

The heat rejected (from  $c$  to  $d$ ) has magnitude  $|Q_L| = T_L \Delta S$ .

The work done by the cycle can be found using the first law for a reversible process:

$$\begin{aligned} dU &= dQ - dW. \\ &= TdS - dW \quad (\text{This form is only true for a reversible process}). \end{aligned}$$

We can integrate this last expression around the closed path traced out by the cycle:

$$\oint dU = \oint TdS - \oint dW$$

However  $dU$  is an exact differential and its integral around a closed contour is zero:

$$0 = \oint TdS - \oint dW.$$

The work done by the cycle, which is represented by the term  $\oint dW$ , is equal to  $\oint Tds$ , the area enclosed by the closed contour in the  $T$ - $S$  plane. This area represents the difference between the heat absorbed ( $\int TdS$  at the high temperature) and the heat rejected ( $\int TdS$  at the low temperature). Finding the work done through evaluation of  $\oint TdS$  is an alternative to computation of the work in a reversible cycle from  $\oint PdV$ . Finally, although we have carried out the discussion in terms of the entropy,  $S$ , all of the arguments carry over to the specific entropy,  $s$ ; the work of the reversible cycle per unit mass is given by  $\oint Tds$ .

### **Muddy points**

How does one interpret h-s diagrams? (MP 1C.3)

Is it always OK to "switch" T-s and h-s diagram? (MP 1C.4)

What is the best way to become comfortable with T-s diagrams? (MP 1C.5)

What is a reversible adiabat physically? (MP 1C.6)

### **1.C.4 Brayton Cycle in T-s Coordinates**

The Brayton cycle has two reversible adiabatic (i.e., isentropic) legs and two reversible, constant pressure heat exchange legs. The former are vertical, but we need to define the shape of the latter. For an ideal gas, changes in specific enthalpy are related to changes in temperature by  $dh = c_p dT$ , so the shape of the cycle in an  $h$ - $s$  plane is the same as in a  $T$ - $s$  plane, with a scale factor of  $c_p$  between the two. This suggests that a place to start is with the combined first and second law, which relates changes in enthalpy, entropy, and pressure:

$$dh = Tds + \frac{dp}{\rho}.$$

On constant pressure curves  $dp=0$  and  $dh = Tds$ . The quantity desired is the derivative of temperature,  $T$ , with respect to entropy,  $s$ , at constant pressure:  $(\partial T / \partial s)_p$ . From the combined first and second law, and the relation between  $dh$  and  $dT$ , this is

$$\left(\frac{\partial T}{\partial s}\right)_p = \frac{T}{c_p} \quad (\text{C.4.1})$$

The derivative is the slope of the constant pressure legs of the Brayton cycle on a  $T$ - $s$  plane. For a given ideal gas (specific  $c_p$ ) the slope is positive and increases as  $T$ .

We can also plot the Brayton cycle in an  $h$ - $s$  plane. This has advantages because changes in enthalpy directly show the work of the compressor and turbine and the heat added and rejected. The slope of the constant pressure legs in the  $h$ - $s$  plane is  $(\partial h / \partial s)_p = T$ .

Note that the similarity in the shapes of the cycles in  $T$ - $s$  and  $h$ - $s$  planes is true for ideal gases only. As we will see when we examine two-phase cycles, the shapes look quite different in these two planes when the medium is not an ideal gas.

Plotting the cycle in  $T$ - $s$  coordinates also allows another way to address the evaluation of the Brayton cycle efficiency which gives insight into the relations between Carnot cycle efficiency and efficiency of other cycles. As shown in Figure C-2, we can break up the Brayton cycle into many small Carnot cycles. The " $i^{\text{th}}$ " Carnot cycle has an efficiency of  $\eta_{c_i} = \left[1 - \left(T_{low_i} / T_{high_i}\right)\right]$ , where the indicated lower temperature is the heat rejection temperature for that elementary cycle and the higher temperature is the heat absorption temperature for that cycle. The upper and lower curves of the Brayton cycle, however, have constant pressure. All of the elementary Carnot cycles therefore have the same pressure ratio:

$$\frac{P(T_{high})}{P(T_{low})} = PR = \text{constant (the same for all the cycles)}.$$

From the isentropic relations for an ideal gas, we know that pressure ratio,  $PR$ , and temperature ratio,  $TR$ , are related by :  $PR^{(\gamma-1)/\gamma} = TR$ .

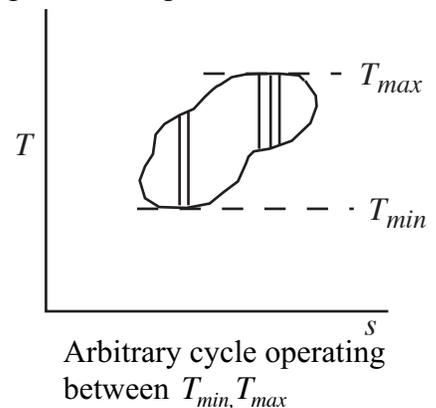
Figure C-2 available from:  
 Kerrebrock, *Aircraft Engines and Gas Turbines*, 2nd Ed. MIT Press. Figure 1.3, p.8.

Figure C-2: Ideal Brayton cycle as composed of many elementary Carnot cycles [Kerrebrock]

The temperature ratios  $(T_{low_i} / T_{high_i})$  of any elementary cycle “i” are therefore the same and each of the elementary cycles has the same thermal efficiency. We only need to find the temperature ratio across any one of the cycles to find what the efficiency is. We know that the temperature ratio of the first elementary cycle is the ratio of compressor exit temperature to engine entry (atmospheric for an aircraft engine) temperature,  $T_2 / T_0$  in Figure C-2. If the efficiency of all the elementary cycles has this value, the efficiency of the overall Brayton cycle (which is composed of the elementary cycles) must also have this value. Thus, as previously,

$$\eta_{Brayton} = 1 - \left( \frac{T_{inlet}}{T_{compressor\ exit}} \right).$$

A benefit of this view of efficiency is that it allows us a way to comment on the efficiency of any thermodynamic cycle. Consider the cycle shown on the right, which operates between some maximum and minimum temperatures. We can break it up into small Carnot cycles and evaluate the efficiency of each. It can be seen that the efficiency of any of the small cycles drawn will be less than the efficiency of a Carnot cycle between  $T_{max}$  and  $T_{min}$ . This graphical argument shows that the efficiency of any other thermodynamic cycle operating between these maximum and minimum temperatures has an efficiency less than that of a Carnot cycle.



**Muddy points**

If there is an ideal efficiency for all cycles, is there a maximum work or maximum power for all cycles? (MP 1C.7)

### 1.C.5 Irreversibility, Entropy Changes, and “Lost Work”

Consider a system in contact with a heat reservoir during a reversible process. If there is heat  $Q$  absorbed by the reservoir at temperature  $T$ , the change in entropy of the reservoir is  $\Delta S = Q/T$ . In general, reversible processes are accompanied by heat exchanges that occur at different temperatures. To analyze these, we can visualize a sequence of heat reservoirs at different temperatures so that during any infinitesimal portion of the cycle there will not be any heat transferred over a finite temperature difference.

During any infinitesimal portion, heat  $dQ_{rev}$  will be transferred between the system and one of the reservoirs which is at  $T$ . If  $dQ_{rev}$  is absorbed by the system, the entropy change of the system is

$$dS^{system} = \frac{dQ_{rev}}{T}.$$

The entropy change of the reservoir is

$$dS^{reservoir} = -\frac{dQ_{rev}}{T}.$$

The total entropy change of system plus surroundings is

$$dS^{total} = dS^{system} + dS^{reservoir} = 0.$$

This is also true if there is a quantity of heat rejected by the system.

The conclusion is that for a reversible process, no change occurs in the total entropy produced, i.e., the entropy of the system plus the entropy of the surroundings:  $\Delta S^{total} = 0$ .

We now carry out the same type of analysis for an irreversible process, which takes the system between the same specified states as in the reversible process. This is shown schematically at the right, with I and R denoting the irreversible and reversible processes.

In the irreversible process, the system receives heat  $dQ$  and does work  $dW$ .

The change in internal energy for the irreversible process is

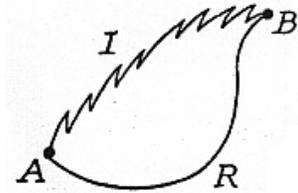
$$dU = dQ - dW \quad (\text{Always true - first law}).$$

For the reversible process

$$dU = TdS - dW_{rev}.$$

Because the state change is the same in the two processes (we specified that it was), the change in internal energy is the same. Equating the changes in internal energy in the above two expressions yields

$$dQ_{actual} - dW_{actual} = TdS - dW_{rev}.$$



Irreversible and reversible state changes

The subscript “actual” refers to the actual process (which is irreversible). The entropy change associated with the state change is

$$dS = \frac{dQ_{actual}}{T} + \frac{1}{T} [dW_{rev} - dW_{actual}]. \quad (C.5.1)$$

If the process is not reversible, we obtain less work (see IAW notes) than in a reversible process,  $dW_{actual} < dW_{rev}$ , so that *for the irreversible process*,

$$dS > \frac{dQ_{actual}}{T}. \quad (C.5.2)$$

*There is no equality between the entropy change  $dS$  and the quantity  $dQ/T$  for an irreversible process. The equality is only applicable for a reversible process.*

The change in entropy for any process that leads to a transformation between an initial state “a” and a final state “b” is therefore

$$\Delta S = S_b - S_a \geq \int_a^b \frac{dQ_{actual}}{T}$$

where  $dQ_{actual}$  is the heat exchanged in the actual process. The equality only applies to a reversible process.

The difference  $dW_{rev} - dW_{actual}$  represents work we could have obtained, but did not. It is referred to as *lost work* and denoted by  $W_{lost}$ . In terms of this quantity we can write,

$$dS = \frac{dQ_{actual}}{T} + \frac{dW_{lost}}{T}. \quad (C.5.3)$$

The content of Equation (C.5.3) is that the entropy of a system can be altered in two ways: (i) through heat exchange and (ii) through irreversibilities. The lost work ( $dW_{lost}$  in Equation C.5.3) is always greater than zero, so the only way to decrease the entropy of a system is through heat transfer.

To apply the second law we consider the total entropy change (system plus surroundings). If the surroundings are a reservoir at temperature T, with which the system exchanges heat,

$$dS^{reservoir} (= dS^{surroundings}) = -\frac{dQ_{actual}}{T}.$$

The total entropy change is

$$dS^{total} = dS^{system} + dS^{surroundings} = \left( \frac{dQ_{actual}}{T} + \frac{dW_{lost}}{T} \right) - \frac{dQ_{actual}}{T}$$

$$dS^{total} = \frac{dW_{lost}}{T} \geq 0.$$

The quantity ( $dW_{lost}/T$ ) is the entropy generated due to irreversibility.

Yet another way to state the distinction we are making is

$$dS^{system} = dS_{\substack{\text{from} \\ \text{heat} \\ \text{transfer}}} + dS_{\substack{\text{generated due to} \\ \text{irreversible} \\ \text{processes}}} = dS_{\text{heat transfer}} + dS_{Gen}. \quad (C.5.4)$$

The lost work is also called dissipation and noted  $d\phi$ . Using this notation, the infinitesimal entropy change of the system becomes:

$$\begin{aligned} dS^{system} &= dS_{\text{heat transfer}} + \frac{d\phi}{T} \\ \text{or } TdS^{system} &= dQ_r + d\phi \end{aligned} \quad |$$

Equation (C.5.4) can also be written as a rate equation,

$$\frac{dS}{dt} = \dot{S} = \dot{S}_{\text{heat transfer}} + \dot{S}_{Gen}. \quad (C.5.5)$$

Either of equation (C.5.4) or (C.5.5) can be interpreted to mean that the entropy of the system,  $S$ , is affected by two factors: the flow of heat  $Q$  and the appearance of additional entropy, denoted by  $dS_{Gen}$ , due to irreversibility<sup>1</sup>. This additional entropy is zero when the process is reversible and always positive when the process is irreversible. Thus, one can say that the system develops *sources* which create entropy during an irreversible process. The second law asserts that *sinks* of entropy are impossible in nature, which is a more graphic way of saying that  $dS_{Gen}$  and  $\dot{S}_{Gen}$  are positive definite, or zero, for reversible processes.

The term  $\dot{S}_{\text{heat transfer}} \left( = \frac{1}{T} \frac{dQ}{dt}, \text{ or } \frac{\dot{Q}}{T} \right)$ , which is associated with heat transfer to the system, can be interpreted as a flux of entropy. The boundary is crossed by heat and the ratio of this heat flux to temperature can be defined as a flux of entropy. There are no restrictions on the sign of this quantity, and we can say that this flux either contributes towards, or drains away, the system's entropy. During a reversible process, only this flux can affect the entropy of the system. This terminology suggests that we interpret entropy as a kind of weightless fluid, whose quantity is conserved (like that of matter) during a reversible process. During an irreversible process, however, this fluid is not conserved; it cannot disappear, but rather is created by sources throughout the system. While this interpretation should not be taken too literally, it provides an easy mode of expression and is in the same category of concepts such as those associated with the phrases "flux of

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<sup>1</sup> This and the following paragraph are excerpted with minor modifications from *A Course in Thermodynamics, Volume I*, by J. Kestin, Hemisphere Press (1979)

energy" or "sources of heat". In fluid mechanics, for example, this graphic language is very effective and there should be no objections to copying it in thermodynamics.

***Muddy points***

Do we ever see an absolute variable for entropy? So far, we have worked with deltas only (MP 1C.8)

I am confused as to  $dS = \frac{dQ_{rev}}{T}$  as opposed to  $dS \geq \frac{dQ_{rev}}{T}$ . (MP 1C.9)

For irreversible processes, how can we calculate  $dS$  if not equal to  $\frac{dQ}{T}$  (MP 1C.10)

**1.C.6 Entropy and Unavailable Energy (Lost Work by Another Name)**

Consider a system consisting of a heat reservoir at  $T_2$  in surroundings (the atmosphere) at  $T_0$ . The surroundings are equivalent to a second reservoir at  $T_0$ . For an amount of heat,  $Q$ , transferred from the reservoir, the maximum work we could derive is  $Q$  times the thermal efficiency of a Carnot cycle operated between these two temperatures:

$$\text{Maximum work we could obtain} = W_{max} = Q(1 - T_0 / T_2). \quad (C.6.1)$$

Only part of the heat transferred can be turned into work, in other words only part of the heat energy is *available* to be used as work.

Suppose we transferred the same amount of heat from the reservoir directly to another reservoir at a temperature  $T_1 < T_2$ . The maximum work available from the quantity of heat,  $Q$ , before the transfer to the reservoir at  $T_1$  is,

$$W_{max}_{T_2, T_0} = Q(1 - T_0 / T_2); \quad [\text{Maximum work between } T_2, T_0].$$

The maximum amount of work available after the transfer to the reservoir at  $T_1$  is,

$$W_{max}_{T_1, T_0} = Q(1 - T_0 / T_1); \quad [\text{Maximum work between } T_1, T_0].$$

There is an amount of energy that could have been converted to work prior to the irreversible heat transfer process of magnitude  $E'$ ,

$$E' = Q \left[ \left( 1 - \frac{T_0}{T_2} \right) - \left( 1 - \frac{T_0}{T_1} \right) \right] = Q \left[ \frac{T_0}{T_1} - \frac{T_0}{T_2} \right],$$

or

$$E' = T_0 \left[ \frac{Q}{T_1} - \frac{Q}{T_2} \right].$$

However,  $Q/T_1$  is the entropy gain of the reservoir at  $T_1$  and  $(-Q/T_2)$  is the entropy decrease of the reservoir at  $T_2$ . The amount of energy,  $E'$ , that could have been converted to work (but now cannot be) can therefore be written in terms of entropy changes and the temperature of the surroundings as

$$E' = T_0 \left( \Delta S_{\text{reservoir at } T_1} + \Delta S_{\text{reservoir at } T_2} \right) \\ = T_0 \Delta S_{\text{irreversible heat transfer process}}$$

$E'$  = “Lost work”, or energy which is no longer available as work.

The situation just described is a special case of an important principle concerning entropy changes, irreversibility and the loss of capability to do work. We thus now develop it in a more general fashion, considering an arbitrary system undergoing an irreversible state change, which transfers heat to the surroundings (for example the atmosphere), which can be assumed to be at constant temperature,  $T_0$ . The change in internal energy of the system during the state change is  $\Delta U = Q - W$ . The change in entropy of the surroundings is (with  $Q$  the heat transfer to the system)

$$\Delta S^{\text{surroundings}} = -\frac{Q}{T_0}.$$

Now consider restoring the system to the initial state by a reversible process. To do this we need to do work,  $W_{rev}$  on the system and extract from the system a quantity of heat  $Q_{rev}$ . (We did this, for example, in “undoing” the free expansion process.) The change in internal energy is (with the quantities  $Q_{rev}$  and  $W_{rev}$  both regarded, in this example, as positive for work done by the surroundings and heat given to the surroundings)<sup>2</sup>

$$\Delta U_{rev} = -Q_{rev} + W_{rev}.$$

In this reversible process, the entropy of the surroundings is changed by

$$\Delta S^{\text{surroundings}} = \frac{Q_{rev}}{T}.$$

For the combined changes (the irreversible state change and the reversible state change back to the initial state), the energy change is zero because the energy is a function of state,

$$\Delta U_{rev} + \Delta U = 0 = Q - W + (-Q_{rev} + W_{rev}).$$

Thus,

$$Q_{rev} - Q = W_{rev} - W.$$

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<sup>2</sup> In the above equation, and in the arguments that follow, the quantities  $Q_{rev}$  and  $W_{rev}$  are both regarded as positive for work done by the surroundings and heat given to the surroundings. Although this is not in accord with the convention we have been using, it seems to me, after writing the notes in both ways, that doing this gives easier access to the ideas. I would be interested in your comments on whether this perception is correct.

For the system, the overall entropy change for the combined process is zero, because the entropy is a function of state,

$$\Delta S_{\text{system;combined process}} = \Delta S_{\text{irreversible process}} + \Delta S_{\text{reversible process}} = 0.$$

The total entropy change is thus only reflected in the entropy change of the surroundings:

$$\Delta S^{\text{total}} = \Delta S_{\text{surroundings}}.$$

The surroundings can be considered a constant temperature heat reservoir and their entropy change is given by

$$\Delta S^{\text{total}} = \frac{(Q_{\text{rev}} - Q)}{T_0}.$$

We also know that the total entropy change, for system plus surroundings is,

$$\Delta S^{\text{total}} = \left[ \Delta S_{\text{irreversible process}} + \Delta S_{\text{reversible process}} \right]_{\text{system+surroundings}} \quad \begin{array}{c} \nearrow \\ 0 \end{array}$$

The total entropy change is associated only with the irreversible process and is related to the work in the two processes by

$$\Delta S^{\text{total}} = \frac{(W_{\text{rev}} - W)}{T_0}.$$

The quantity  $W_{\text{rev}} - W$  represents the *extra work* required to restore the system to the original state. If the process were reversible, we would not have needed any *extra work* to do this. It represents a quantity of work that is now unavailable because of the irreversibility. The quantity  $W_{\text{rev}}$  can also be interpreted as the work that the system would have done if the original process were reversible. From either of these perspectives we can identify  $(W_{\text{rev}} - W)$  as the quantity we denoted previously as  $E'$ , representing lost work. The lost work in any irreversible process can therefore be related to the total entropy change (system plus surroundings) and the temperature of the surroundings by

$$\text{Lost work} = W_{\text{rev}} - W = T_0 \Delta S^{\text{total}}.$$

To summarize the results of the above arguments for processes where heat can be exchanged with the surroundings at  $T_0$ :

- 1)  $W_{\text{rev}} - W$  represents the difference between work we actually obtained and work that would be done during a reversible state change. It is the extra work that would be needed to restore the system to its initial state.
- 2) For a reversible process,  $W_{\text{rev}} = W$ ;  $\Delta S^{\text{total}} = 0$

- 3) For an irreversible process,  $W_{rev} > W$  ;  $\Delta S^{total} > 0$   
 4)  $(W_{rev} - W) = E' = T_0 \Delta S^{total}$  is the energy that becomes unavailable for work during an irreversible process.

### Muddy points

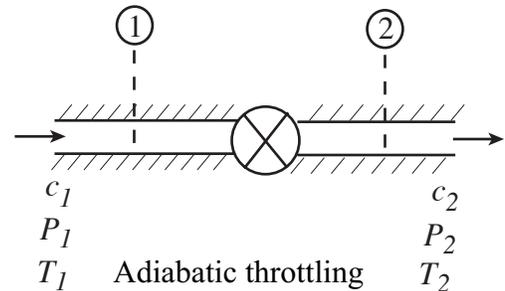
Is  $\Delta S$  path dependent? (MP 1C.11)

Are  $Q_{rev}$  and  $W_{rev}$  the  $Q$  and  $W$  going from the final state back to the initial state? (MP 1C.12)

### 1.C.7 Examples of Lost Work in Engineering Processes

#### a) Lost work in Adiabatic Throttling: Entropy and Stagnation Pressure Changes

A process we have encountered before is adiabatic throttling of a gas, by a valve or other device as shown in the figure at the right. The velocity is denoted by  $c$ . There is no shaft work and no heat transfer and the flow is steady. Under these conditions we can use the first law for a control volume (the Steady Flow Energy Equation) to make a statement about the conditions upstream and downstream of the valve:



$$h_1 + c_1^2 / 2 = h_2 + c_2^2 / 2 = h_t,$$

where  $h_t$  is the stagnation enthalpy, corresponding to a (possibly fictitious) state with zero velocity.

The stagnation enthalpy is the same at stations 1 and 2 if  $Q=W=0$ , even if the flow processes are not reversible.

For an ideal gas with constant specific heats,  $h = c_p T$  and  $h_t = c_p T_t$ . The relation between the static and stagnation temperatures is:

$$\frac{T_t}{T} = 1 + \frac{c^2}{2c_p T} = 1 + \frac{(\gamma - 1)c^2}{2\underbrace{\gamma RT}_{a^2}} = 1 + \frac{(\gamma - 1)c^2}{2a^2},$$

$$\frac{T_t}{T} = 1 + \left(\frac{\gamma - 1}{2}\right) M^2,$$

where  $a$  is the speed of sound and  $M$  is the Mach number,  $M = c/a$ . In deriving this result, use has only been made of the first law, the equation of state, the speed of sound, and the definition of the Mach number. Nothing has yet been specified about whether the process of stagnating the fluid is reversible or irreversible.

When we define the stagnation pressure, however, we do it with respect to isentropic deceleration to the zero velocity state. For an isentropic process

$$\frac{P_2}{P_1} = \left( \frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)}$$

The relation between static and stagnation pressures is

$$\frac{P_t}{P} = \left( \frac{T_t}{T} \right)^{\gamma/(\gamma-1)}$$

The stagnation state is defined by  $P_t, T_t$ . In addition,  $s_{stagnation\ state} = s_{static\ state}$ . The static and stagnation states are shown below in T-s coordinates.

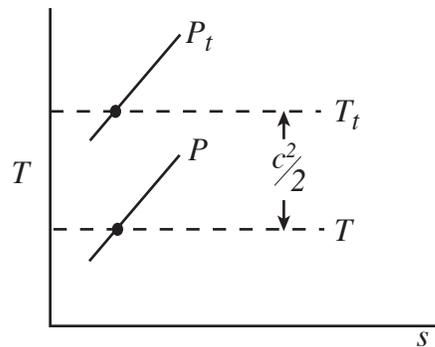


Figure C-1: Static and stagnation pressures and temperatures

Stagnation pressure is a key variable in propulsion and power systems. To see why, we examine the relation between stagnation pressure, stagnation temperature, and entropy. The form of the combined first and second law that uses enthalpy is

$$Tds = dh - \frac{1}{\rho} dP. \quad (C.7.1)$$

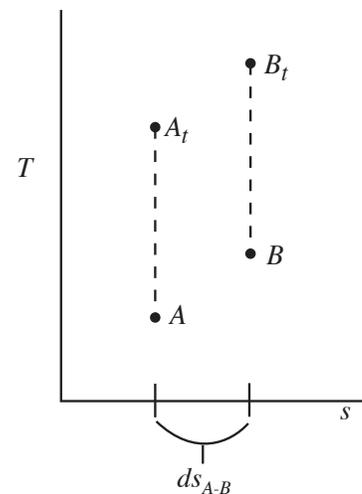
This holds for small changes between any thermodynamic states and we can apply it to a situation in which we consider differences between stagnation states, say one state having properties  $(T_t, P_t)$

and the other having properties  $(T_t + dT_t, P_t + dP_t)$  (see at right). The corresponding static states are

also indicated. Because the entropy is the same at static and stagnation conditions,  $ds$  needs no subscript. Writing (1.C.8) in terms of stagnation

$$\text{conditions yields } ds = \frac{c_p dT_t}{T_t} - \frac{1}{\rho_t T_t} dP_t = \frac{c_p dT_t}{T_t} - \frac{R}{P_t} dP_t.$$

Both sides of the above are perfect differentials and can be integrated as



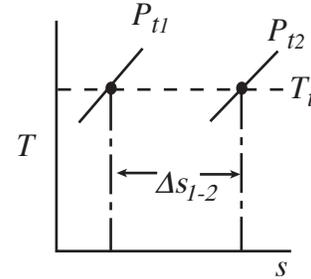
Stagnation and static states

$$\frac{\Delta s}{R} = \frac{\gamma}{\gamma - 1} \ln\left(\frac{T_{t2}}{T_{t1}}\right) - \ln\left(\frac{P_{t2}}{P_{t1}}\right).$$

For a process with  $Q = W = 0$ , the stagnation enthalpy, and hence the stagnation temperature, is constant. In this situation, the stagnation pressure is related directly to the entropy as,

$$\frac{\Delta s}{R} = -\ln\left(\frac{P_{t2}}{P_{t1}}\right). \quad (C.7.2)$$

The figure on the right shows this relation on a  $T$ - $s$  diagram. We have seen that the entropy is related to the loss, or irreversibility. The stagnation pressure plays the role of an indicator of loss if the stagnation temperature is constant. The utility is that it is the stagnation pressure (and temperature) which are directly measured, not the entropy. The throttling process is a representation of flow through inlets, nozzles, stationary turbomachinery blades, and the use of stagnation pressure as a measure of loss is a practice that has widespread application.



Eq. (C.7.2) can be put in several useful approximate forms. First, we note that for aerospace applications we are (hopefully!) concerned with low loss devices, so that the stagnation pressure change is small compared to the inlet level of stagnation pressure  $\Delta P_t / P_{t1} = (P_{t1} - P_{t2}) / P_{t1} \ll 1$ .

Expanding the logarithm [using  $\ln(1-x) \approx -x + \dots$ ],

$$\ln\left(\frac{P_{t2}}{P_{t1}}\right) = \ln\left(1 - \frac{\Delta P_t}{P_{t1}}\right) \approx -\frac{\Delta P_t}{P_{t1}},$$

or,

$$\frac{\Delta s}{R} \approx \frac{\Delta P_t}{P_{t1}}.$$

Another useful form is obtained by dividing both sides by  $c^2/2$  and taking the limiting forms of the expression for stagnation pressure in the limit of low Mach number ( $M \ll 1$ ). Doing this, we find:

$$\frac{T \Delta s}{(c^2/2)} \approx \frac{\Delta P_t}{(\rho c^2/2)}$$

The quantity on the right can be interpreted as the change in the “Bernoulli constant” for incompressible (low Mach number) flow. The quantity on the left is a non-dimensional entropy change parameter, with the term  $T \Delta s$  now representing the loss of mechanical energy associated with the change in stagnation pressure.

To summarize:

- 1) for many applications the stagnation temperature is constant and the change in stagnation pressure is a direct measure of the entropy increase
- 2) stagnation pressure is the quantity that is actually measured so that linking it to entropy (which is not measured) is useful

- 3) we can regard the throttling process as a “free expansion” at constant temperature  $T_{t_1}$  from the initial stagnation pressure to the final stagnation pressure. We thus know that, for the process, the work we need to do to bring the gas back to the initial state is  $T_t \Delta s$ , which is the “lost work” per unit mass.

### Muddy points

Why do we find stagnation enthalpy if the velocity never equals zero in the flow?

(MP 1C.13)

Why does  $T_t$  remain constant for throttling? (MP 1C.14)

### b) Adiabatic Efficiency of a Propulsion System Component (Turbine)

A schematic of a turbine and the accompanying thermodynamic diagram are given in Figure C-2. There is a pressure and temperature drop through the turbine and it produces work.

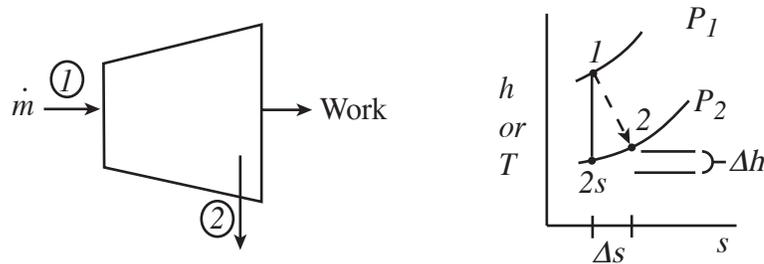


Figure C-2: Schematic of turbine and associated thermodynamic representation in  $h$ - $s$  coordinates

There is no heat transfer so the expressions that describe the overall shaft work and the shaft work per unit mass are:

$$\begin{aligned} \dot{m}(h_{t_2} - h_{t_1}) &= \dot{W}_{shaft} \\ (h_{t_2} - h_{t_1}) &= w_{shaft} \end{aligned} \quad (C.7.3)$$

If the difference in the kinetic energy at inlet and outlet can be neglected, Equation (C.7.3) reduces to

$$(h_2 - h_1) = w_{shaft}.$$

The *adiabatic efficiency* of the turbine is defined as

$$\eta_{ad} = \left[ \frac{\text{actual work}}{\text{ideal work } (\Delta s = 0)} \right]_{\text{For a given pressure ratio}}$$

The performance of the turbine can be represented in an  $h$ - $s$  plane (similar to a  $T$ - $s$  plane for an ideal gas) as shown in Figure C-2. From the figure the adiabatic efficiency is

$$\eta_{ad} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{h_1 - h_{2s} - (h_2 - h_{2s})}{h_1 - h_{2s}}$$

The adiabatic efficiency can therefore be written as

$$\eta_{ad} = 1 - \left( \frac{\Delta h}{\text{Ideal work}} \right).$$

The non-dimensional term ( $\Delta h/\text{Ideal work}$ ) represents the departure from isentropic (reversible) processes and hence a loss. The quantity  $\Delta h$  is the enthalpy difference for two states along a constant pressure line (see diagram). From the combined first and second laws, for a constant pressure process, small changes in enthalpy are related to the entropy change by  $Tds = dh.$ , or approximately,

$$T_2 \Delta s = \Delta h.$$

The adiabatic efficiency can thus be approximated as

$$\eta_{ad} = 1 - \frac{T_2 \Delta s}{h_1 - h_{2s}} = 1 - \left( \frac{\text{Lost work}}{\text{Ideal work}} \right).$$

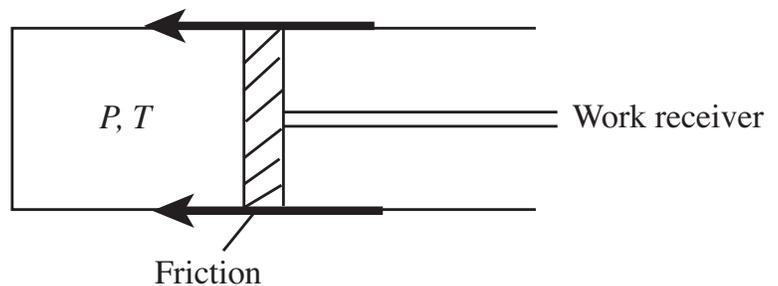
The quantity  $T\Delta s$  represents a useful figure of merit for fluid machinery inefficiency due to irreversibility.

### **Muddy points**

How do you tell the difference between shaft work/power and flow work in a turbine, both conceptually and mathematically? (MP 1C.15)

#### **c) Isothermal Expansion with Friction**

In a more general look at the isothermal expansion, we now drop the restriction to frictionless processes. As seen in the diagram at the right, work is done to overcome friction. If the kinetic energy of the piston is negligible, a balance of forces tells us that



Isothermal expansion with friction

$$W_{\text{system on piston}} = W_{\text{done by friction}} + W_{\text{received}}.$$

During the expansion, the piston and the walls of the container will heat up because of the friction. The heat will be (eventually) transferred to the atmosphere; all frictional work ends up as heat transferred to the surrounding atmosphere.

$$W_{\text{friction}} = Q_{\text{friction}}$$

The amount of heat transferred to the atmosphere due to the frictional work *only* is thus,

$$Q_{friction} = \underbrace{W_{system}}_{\text{Work produced on piston}} - \underbrace{W_{received}}_{\text{Work received}}$$

The entropy change of the atmosphere (considered as a heat reservoir) due to the frictional work is

$$(\Delta S_{atm})_{\text{due to frictional work only}} = \frac{Q_{friction}}{T_{atm}} = \frac{W_{system} - W_{received}}{T_{atm}}$$

The difference between the work that the system did (the work we could have received if there were no friction) and the work that we actually received can be put in a (by now familiar) form as

$$W_{system} - W_{received} = T_{atm} \Delta S_{atm} = \text{Lost or unavailable work}$$

### **Muddy points**

Is the entropy change in the equations two lines above the total entropy change? If so, why does it say  $\Delta S_{atm}$ ? (MP 1C.16)

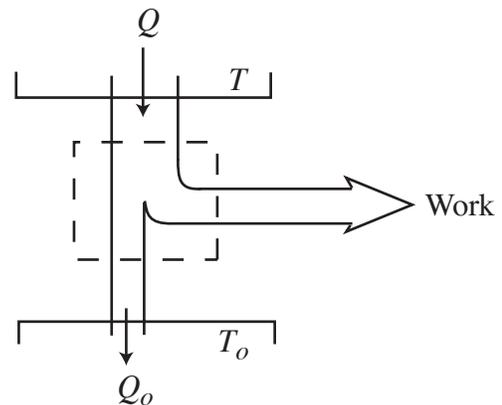
### **d) Entropy Generation, Irreversibility, and Cycle Efficiency**

As another example, we show the links between entropy changes and cycle efficiency for an irreversible cycle. The conditions are:

- i) A source of heat at temperature,  $T$
  - ii) A sink of heat (rejection of heat) at  $T_0$
  - iii) An engine operating in a cycle irreversibly
- During the cycle the engine extracts heat  $Q$ , rejects heat  $Q_0$  and produces work,  $W$ :

$$W = Q - Q_0.$$

$$\Delta S = \Delta S_{engine} + \Delta S_{surroundings}.$$



The engine operates in a cycle and the entropy change for the complete cycle is zero. Therefore,

$$\Delta S = 0 + \underbrace{\Delta S_{heat, source} + \Delta S_{heat, sink}}_{\Delta S_{surroundings}}$$

The total entropy change is,

$$\Delta S^{total} = \Delta S_{heat, source} + \Delta S_{heat, sink} = \frac{-Q}{T} + \frac{Q_0}{T_0}.$$

Suppose we had an ideal reversible engine working between these same two temperatures, which extracted the same amount of heat,  $Q$ , from the high temperature reservoir, and rejected heat of magnitude  $Q_{0rev}$  to the low temperature reservoir. The work done by this reversible engine is

$$W_{rev} = Q - Q_{0rev}.$$

For the reversible engine the total entropy change over a cycle is

$$\Delta S^{total} = \Delta S_{heat\ source} + \Delta S_{heat\ sink} = \frac{-Q}{T} + \frac{Q_{0rev}}{T_0} = 0.$$

Combining the expressions for work and for the entropy changes,

$$Q_0 = Q_{0rev} + W_{rev} - W$$

The entropy change for the irreversible cycle can therefore be written as

$$\Delta S^{total} = \underbrace{\frac{-Q}{T} + \frac{Q_{0rev}}{T_0}}_{=0} + \frac{W_{rev} - W}{T_0}.$$

The difference in work that the two cycles produce is equal to the entropy that is generated during the cycle:

$$T_0 \Delta S^{total} = W_{rev} - W.$$

The second law states that the total entropy generated is greater than zero for an irreversible process, so that the reversible work is greater than the actual work of the irreversible cycle.

An “engine effectiveness”,  $E_{engine}$ , can be defined as the ratio of the actual work obtained divided by the work that *would have been* delivered by a reversible engine operating between the two temperatures  $T$  and  $T_0$ .

$$E_{engine} = \frac{\eta_{engine}}{\eta_{reversible\ engine}} = \frac{W}{W_{rev}} = \frac{\text{Actual work obtained}}{\text{Work that would be delivered by a reversible cycle between } T, T_0}$$

$$E_{engine} = \frac{W_{rev} - T_0 \Delta S^{total}}{W_{rev}} = 1 - \frac{T_0 \Delta S^{total}}{W_{rev}}$$

The departure from a reversible process is directly reflected in the entropy change and the decrease in engine effectiveness.

### Muddy points

Why does  $\Delta S^{\text{irrev}} = \Delta S^{\text{total}}$  in this example? (MP 1C.17)

In discussing the terms "closed system" and "isolated system", can you assume that you are discussing a cycle or not? (MP 1C.18)

Does a cycle process have to have  $\Delta S = 0$ ? (MP 1C.19)

In a real heat engine, with friction and losses, why is  $\Delta S$  still 0 if  $TdS = dQ + d\phi$ ? (MP 1C.20)

### e) Propulsive Power and Entropy Flux

The final example in this section combines a number of ideas presented in this subject and in Unified in the development of a relation between entropy generation and power needed to propel a vehicle. Figure C-3 shows an aerodynamic shape (airfoil) moving through the atmosphere at a constant velocity. A coordinate system fixed to the vehicle has been adopted so that we see the airfoil as fixed and the air far away from the airfoil moving at a velocity  $c_0$ . Streamlines of the flow have been sketched, as has the velocity distribution at station "0" far upstream and station "d" far downstream. The airfoil has a wake, which mixes with the surrounding air and grows in the

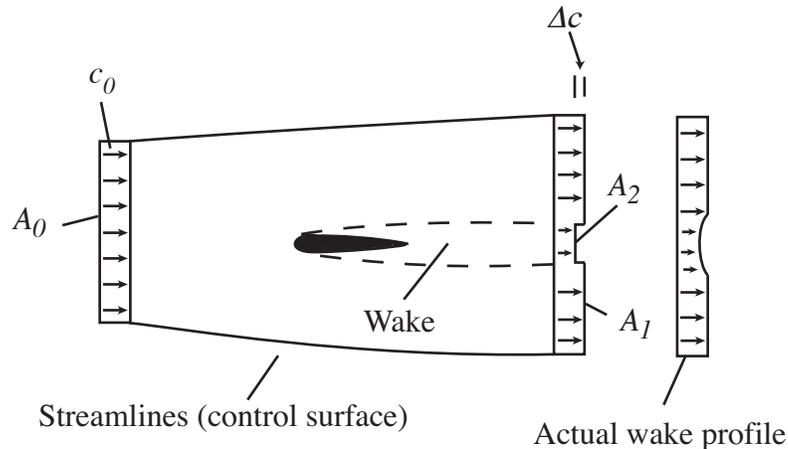


Figure C-3: Airfoil with wake and control volume for analysis of propulsive power requirement

downstream direction. The extent of the wake is also indicated. Because of the lower velocity in the wake the area between the stream surfaces is larger downstream than upstream.

We use a control volume description and take the control surface to be defined by the two stream surfaces and two planes at station  $\theta$  and station  $d$ . This is useful in simplifying the analysis because there is no flow across the stream surfaces. The area of the downstream plane control surface is broken into  $A_1$ , which is area outside the wake and  $A_2$ , which is the area occupied by wake fluid, i.e., fluid that has suffered viscous losses. The control surface is also taken far enough away from the vehicle so that the static pressure can be considered uniform. For fluid which is not in the wake (no viscous forces), the momentum equation is  $\rho c dc = -dP$ . Uniform static pressure therefore implies uniform velocity, so that on  $A_1$  the velocity is equal to the upstream value,  $c_0$ . The downstream velocity profile is actually continuous, as indicated. It is approximated in the analysis as a step change to make the algebra a bit simpler. (The conclusions apply to the more

general velocity profile as well and we would just need to use integrals over the wake instead of the algebraic expressions below.)

The equation expressing mass conservation for the control volume is

$$\rho_0 A_0 c_0 = \rho_0 A_1 c_0 + \rho_2 A_2 c_2. \quad (\text{C.7.5})$$

The vertical face of the control surface is far downstream of the body. By this station, the wake fluid has had much time to mix and the velocity in the wake is close to the free stream value,  $c_0$ . We can thus write,

$$\text{wake velocity} = c_2 = (c_0 - \Delta c); \quad \Delta c / c_0 \ll 1. \quad (\text{C.7.6})$$

(We chose our control surface so the condition  $\Delta c / c_0 \ll 1$  was upheld.)

The integral momentum equation (control volume form of the momentum equation) can be used to find the drag on the vehicle.

$$\rho_0 A_0 c_0^2 = -\text{Drag} + \rho_0 A_1 c_0^2 + \rho_2 A_2 c_2^2. \quad (\text{C.7.7})$$

There is no pressure contribution in Eq. (C.7.7) because the static pressure on the control surface is uniform. Using the form given for the wake velocity, and expanding the terms in the momentum equation out we obtain,

$$\rho_0 A_0 c_0^2 = -\text{Drag} + \rho_0 A_1 c_0^2 + \rho_2 A_2 [c_0^2 - 2c_0 \Delta c + (\Delta c)^2] \quad (\text{C.7.8})$$

The last term in the right hand side of the momentum equation,  $\rho_2 A_2 (\Delta c)^2$ , is small by virtue of the choice of control surface and we can neglect it. Doing this and grouping the terms on the right hand side of Eq. (C.7.8) in a different manner, we have

$$c_0 [\rho_0 A_1 c_0] = c_0 [\rho_0 A_1 c_0 + \rho_2 A_2 (c_0 - \Delta c)] + \{-\text{Drag} - \rho_2 A_2 c_0 \Delta c\}$$

The terms in the square brackets on both sides of this equation are the continuity equation multiplied by  $c_0$ . They thus sum to zero leaving the curly bracketed terms as

$$\text{Drag} = -\rho_2 A_2 c_0 \Delta c. \quad (\text{C.7.9})$$

The wake mass flow is  $\rho_2 A_2 c_2 = \rho_2 A_2 (c_0 - \Delta c)$ . All this flow has a velocity defect (compared to the free stream) of  $\Delta c$ , so that the defect in flux of momentum (the mass flow in the wake times the velocity defect) is, to first order in  $\Delta c$ ,

$$\text{Momentum defect in wake} = -\rho_2 A_2 c_0 \Delta c, = \text{Drag}.$$

The combined first and second law gives us a means of relating the entropy and velocity:

$$Tds = dh - dP/\rho.$$

The pressure is uniform ( $dP=0$ ) at the downstream station. There is no net shaft work or heat transfer to the wake so that the mass flux of stagnation enthalpy is constant. We can also approximate that the condition of constant stagnation enthalpy holds locally on all streamlines. Applying both of these to the combined first and second law yields

$$Tds = dh_t - cdc.$$

For the present situation,  $dh_t = 0$ ;  $cdc = c_0\Delta c$ , so that

$$T_0\Delta s = -c_0\Delta c \quad (C.7.10)$$

In Equation (C.7.10) the upstream temperature is used because differences between wake quantities and upstream quantities are small at the downstream control station. The entropy can be related to the drag as

$$\text{Drag} = \rho_2 A_2 T_0 \Delta s \quad (C.7.11)$$

The quantity  $\rho_2 A_2 c_0 \Delta s$  is the entropy flux (mass flux times the entropy increase per unit mass; in the general case we would express this by an integral over the locally varying wake velocity and density).

The power needed to propel the vehicle is the product of drag x flight speed,  $\text{Drag} \times c_o$ . From Eq. (C.7.11), this can be related to the entropy flux in the wake to yield a compact expression for the propulsive power needed in terms of the wake entropy flux:

$$\text{Propulsive power needed} = T_0 (\rho_2 A_2 c_0 \Delta s) = T_0 \times \text{Entropy flux in wake} \quad (C.7.12)$$

This amount of work is dissipated per unit time in connection with sustaining the vehicle motion. Equation (C.7.12) is another demonstration of the relation between lost work and entropy generation, in this case manifested as power that needs to be supplied because of dissipation in the wake.

### **Muddy points**

Is it safe to say that entropy is the tendency for a system to go into disorder? (MP 1C.21)

### **1.C.8 Some Overall Comments on Entropy, Reversible and Irreversible Processes**

[Mainly excerpted (with some alterations) from: *Engineering Thermodynamics*, William C. Reynolds and Henry C. Perkins, McGraw-Hill Book Company, 1977]

***Muddy points***

Isn't it possible for the mixing of two gases to go from the final state to the initial state? If you have two gases in a box, they should eventually separate by density, right? (MP 1C.22)

## Muddiest Points on Part 1C

*1C.1 So, do we lose the capability to do work when we have an irreversible process and entropy increases?*

Absolutely. We will see this in a more general fashion very soon. The idea of *lost work* is one way to view what “entropy is all about”!

*1C.2 Why do we study cycles starting with the Carnot cycle? Is it because it is easier to work with?*

Carnot cycles are the best we can do in terms of efficiency. We use the Carnot cycle as a standard against which all other cycles are compared. We will see in class that we can break down a general cycle into many small Carnot cycles. Doing this we can gain insight in which direction the design of efficient cycles should go.

*1C.3 How does one interpret h-s diagrams?*

I find *h-s* diagrams useful, especially in dealing with propulsion systems, because the difference in stagnation *h* can be related (from the Steady Flow Energy Equation) to shaft work and heat input. For processes that just have shaft work (compressors or turbines) the change in stagnation enthalpy is the shaft work. For processes that just have heat addition or rejection at constant pressure, the change in stagnation enthalpy is the heat addition or rejection.

*1C.4 Is it always OK to "switch" T-s and h-s diagram?*

No! This is only permissible for perfect gases with constant specific heats. We will see, when we examine cycles with liquid-vapor mixtures, that the *h-s* diagrams and the *T-s* diagrams look different.

*1C.5 What is the best way to become comfortable with T-s diagrams?*

I think working with these diagrams may be the most useful way to achieve this objective. In doing this, the utility of using these coordinates (or *h-s* coordinates) should also become clearer. I find that I am more comfortable with *T-s* or *h-s* diagrams than with *P-v* diagrams, especially the latter because it conveys several aspects of interest to propulsion engineers: work produced or absorbed, heat produced or absorbed, and loss.

*1C.6 What is a reversible adiabat physically?*

Let's pick an example process involving a chamber filled with a compressible gas and a piston. We assume that the chamber is insulated (so no heat-transfer to or from the chamber) and the process is thus adiabatic. Let us also assume that the piston is ideal, such that there is no friction between the walls of the chamber and the piston. The gas is

at some Temperature  $T_1$ . We now push the piston in and compress the gas. The internal energy of the system will then increase by the amount of work we put in and the gas will heat up and be at higher pressure. If we now let the piston expand again, it will return to its original position (no friction, ideal piston) and the work we took from the environment will be returned (we get the exact same amount of work back and leave no mark on the environment). Also, the temperature and the pressure of the gas return to the initial values. We thus have an adiabatic reversible process. For both compression and expansion we have no change in entropy of the system because there is no heat transfer and also no irreversibility. If we now draw this process in the h-s or T-s diagram we get a vertical line since the entropy stays constant:  $S = \text{constant}$  or  $\Delta S = 0$  and we can also call this process an isentropic process.

1C.7 If there is an ideal efficiency for all cycles, is there a maximum work or maximum power for all cycles?

Yes. As with the Brayton cycle example, we could find the maximum as a function of the appropriate design parameters.

1C.8 Do we ever see an absolute variable for entropy? So far, we have worked with deltas only.

It is probably too strong a statement to say that for “us” the changes in entropy are what matters, but this has been my experience for the type of problems aerospace engineers work on. Some values of absolute entropy are given in Table A.8 in SB&VW. We will also see, in the lectures on Rankine cycles, that the entropy of liquid water at a temperature of 0.01 C and a pressure of 0.6113 kPa has been specified as zero for problems involving two-phase (steam and water) behavior.

1C.9 I am confused as to  $dS = \frac{dQ_{rev}}{T}$  as opposed to  $dS \geq \frac{dQ_{rev}}{T}$ .

Both of these are true and both can always be used. The first is the definition of entropy. The second is a statement of how the entropy behaves. Section 1C.5 attempts to make the relationship clearer through the development of the equality  $dS = \frac{dQ}{T} + \frac{dW_{lost}}{T}$ .

1C.10 For irreversible processes, how can we calculate  $dS$  if not equal to  $\frac{dQ}{T}$ ?

We need to define a reversible process between the two states in order to calculate the entropy (see muddy point 3, above). See VN Chapter 5 (especially) for discussion of entropy or section 1C.5. If you are still in difficulty, come and see me.

*1C.11 Is  $\Delta S$  path dependent?*

No. Entropy is a function of state (see Gibbs) and thus  $\Delta S$  is path independent. For example we could have three different paths connecting the same two states and therefore have the same change in entropy

$$\Delta S_{\text{path I}} = \Delta S_{\text{path II}} = \Delta S_{\text{path III}} .$$

*1C.12 Are  $Q_{\text{rev}}$  and  $W_{\text{rev}}$  the  $Q$  and  $W$  going from the final state back to the initial state?*

Yes. We have an irreversible process from state 1 to state 2. We then used a reversible process to restore the initial state 1 (we had to do work on the system and extract heat from the system).

*1C.13 Why do we find stagnation enthalpy if the velocity never equals zero in the flow?*

The stagnation enthalpy (or temperature) is a useful reference quantity. Unlike the static temperature it does not vary along a streamline in an adiabatic flow, even if irreversible. It was thus the natural reference temperature to use in describing the throttling process. In addition, changes in stagnation pressure are direct representations of the shaft work or heat associated with a fluid component. The enthalpy is not, unless we assume that changes in KE are small. Measurement of stagnation temperature thus allows direct assessment of shaft work in a turbine or compressor, for example.

*1C.14 Why does  $T_t$  remain constant for throttling?*

Because for a steady adiabatic flow with no shaft work done the Steady Flow Energy Equation yields constant stagnation enthalpy even though the flow processes might not be reversible (see notes). For a perfect gas  $h = c_p T$ , thus the stagnation temperature remains constant.

*1C.15 How do you tell the difference between shaft work/power and flow work in a turbine, both conceptually and mathematically?*

Let us look at the expansion of a flow through a turbine using both the control mass approach and the control volume approach. Using the control mass approach we can model the situation by tracking 1kg of air as follows: state 1 – before the expansion we have 1kg of air upstream of the turbine. We then push the gas into the turbine and expand it through the blade rows. After the expansion we take 1kg of air out of the turbine and the mass of air is downstream of the turbine – state 2. The work done by the gas is work done by the turbine (blades moved around by the gas) **plus** the work done by pressures (flow work). Using the first law we can then write for the change of internal energy of 1kg of air:

$$u_2 - u_1 = - w_{\text{shaft}} + p_1 v_1 - p_2 v_2 \quad (\text{adiabatic turbine: } dq = 0)$$

When entering the turbine, the fluid has to push the surroundings out of the way to make room for itself (it has volume  $v_1$  and is at  $p_1$ ) – the work to do this is  $+p_1v_1$ . When leaving the turbine the fluid is giving up room and the work to keep that volume  $v_2$  at pressure  $p_2$  is freed; thus  $-p_2v_2$ . We can then write for the shaft work

$$w_{\text{shaft}} = u_1 + p_1v_1 - (u_2 + p_2v_2).$$

The right hand side of the above equation is the change in enthalpy ( $h_1 - h_2$ ). This is another example to show how useful enthalpy is (enthalpy is the total energy of a fluid: the internal energy plus the extra energy that it is credited with by having a volume  $v$  at pressure  $p$ ). The shaft work outputted by the turbine is equal to the change in enthalpy (enthalpy contains the flow work!).

$$w_{\text{shaft}} = h_1 - h_2.$$

We can also solve this problem by using the 1<sup>st</sup> law in general form (control volume approach).

$$d/dt\{\Sigma E_{CV}\} = \Sigma \dot{Q} + \Sigma \dot{W}_{\text{shaft}} + \Sigma \dot{W}_{\text{shear}} + \Sigma \dot{W}_{\text{piston}} + \Sigma \dot{m} (h + \frac{1}{2}c^2 + gz).$$

Note that in this form the flow work is buried in  $h$  already! For this turbine, we can drop the unsteady term on the left and neglect heat fluxes (adiabatic turbine), shear work and piston work (no pistons but blades, so we keep the shaft work). Further we assume that changes in potential energy and kinetic energy are negligible and we obtain for 1kg/s air

$$0 = -w_{\text{shaft}} + h_1 - h_2.$$

We obtain the same result as before:  $w_{\text{shaft}} = h_1 - h_2$ .

1C.16 Is the entropy change in the equations two lines above the total entropy change? If so, why does it say  $\Delta S_{\text{atm}}$ ?

The entropy change in question is the entropy change due to the heat produced by friction *only*.

$$\left(\Delta S_{\text{atm}}\right)_{\text{work only}}^{\text{due to frictional}} = \frac{Q_{\text{friction}}}{T_{\text{atm}}} = \frac{(W_{\text{system}} - W_{\text{received}})}{T_{\text{atm}}}.$$

1C.17 Why does  $\Delta S^{\text{irrev}} = \Delta S^{\text{total}}$  in this example?

When we wrote this equality, we were considering a system that was returned to its original state, so that there were no changes in any of the system properties. All evidence of irreversibility thus resides *in the surroundings*.

1C.18 In discussing the terms "closed system" and "isolated system", can you assume that you are discussing a cycle or not?

The terms closed system and isolated system have no connection to whether we are discussing a cycle or not. They are attributes of a system (any system), whether undergoing cyclic behavior, one-way transitions, or just sitting there.

*1C.19 Does a cycle process have to have  $\Delta S=0$ ?*

Entropy is a state function. If the process is cyclic, initial and final states are the same. So, for a cyclic process,  $\Delta S = 0$ .

*1C.20 In a real heat engine, with friction and losses, why is  $\Delta S$  still 0 if  $TdS=dQ+d\phi$ ?*

The change in entropy during a real cycle is zero because we are considering a complete cycle (returning to initial state) and entropy is a function of state (holds for both ideal and real cycles!). Thus if we integrate  $dS = dQ/T + d\Phi/T$  around the real cycle we will obtain  $\Delta S_{\text{cycle}} = 0$ . What essentially happens is that all irreversibilities ( $d\Phi$ 's) are turned into additional heat that is rejected to the environment. The amount of heat rejected in the real cycle  $Q_R^{\text{real}}$  is going to be larger than the amount of heat rejected in an ideal cycle  $Q_R^{\text{ideal}}$

$$Q_R^{\text{ideal}} = Q_A T_R/T_A \quad (\text{from } \Delta S_{\text{cycle}} = 0)$$

$$Q_R^{\text{real}} = Q_A T_R/T_A + T_R \Delta S_{\phi} \quad (\text{from } \Delta S_{\text{cycle}} = 0)$$

We will see this better using the T-s diagram. The change of entropy of the surroundings (heat reservoirs) is  $\Delta S_{\text{surr}} = -Q_A/T_A + Q_R^{\text{real}}/T_R = \Delta S_{\phi} > 0$ . So  $\Delta S_{\text{cycle}} = 0$  even for real cycles, but  $\Delta S_{\text{total}} = \Delta S_{\text{cycle}} + \Delta S_{\text{surr}} = \Delta S_{\phi} > 0$ .

*1C.21 Is it safe to say that entropy is the tendency for a system to go into disorder?*

Entropy can be given this interpretation from a statistical perspective, and this provides a different, and insightful view of this property. At the level in which we have engaged the concept, however, we focus on the macroscopic properties of systems, and there is no need to address the idea of order and disorder ; as we will see, entropy is connected to the loss of our ability to do work, and that is sufficient to make it a concept of great utility for the evaluation and design of engineering systems. We will look at this in a later lecture. If you are interested in pursuing this, places to start might be the book by Goldstein and Goldstein referred to above, *Great Ideas in Physics* by Lightman ( paperback book by an MIT professor), or *Modern Thermodynamics*, by Kondepudi and Prigogine.

*1C.22 Isn't it possible for the mixing of two gases to go from the final state to the initial state? If you have two gases in a box, they should eventually separate by density, right?*

Let us assume that gas X is oxygen and gas Y is nitrogen. When the membrane breaks the entire volume will be filled with a mixture of oxygen and nitrogen. This may be considered as a special case of an unrestrained expansion, for each gas undergoes an unrestrained expansion as it fills the entire volume. It is impossible for these two uniformly mixed gases to separate without help from the surroundings or environment. A certain amount of work is necessary to separate the gases and to bring them back into the left and right chambers.