

**PART 2**

**POWER AND PROPULSION CYCLES**

## PART 2 – POWER AND PROPULSION CYCLES

### 2A – Gas Power and Propulsion Cycles

[SB&VW - 11.8, 11.9, 11.10, 11.11, 11.12, 11.13, 11.14]

In this section we analyze several gas cycles used in practical applications for propulsion and power generation, using the air standard cycle. The air standard cycle is an approximation to the actual cycle behavior, and the term specifically refers to analysis using the following assumptions:

- Air is the working fluid (the presence of combustion products is neglected)
- Combustion is represented by heat transfer from an external heat source
- The cycle is ‘completed’ by heat transfer to the surroundings
- All processes are internally reversible
- Air is a perfect gas with constant specific heats

#### 2.A.1 The Internal combustion engine (Otto Cycle)

The different processes of an idealized Otto cycle (internal combustion engine) are shown in Figure 2A-1:

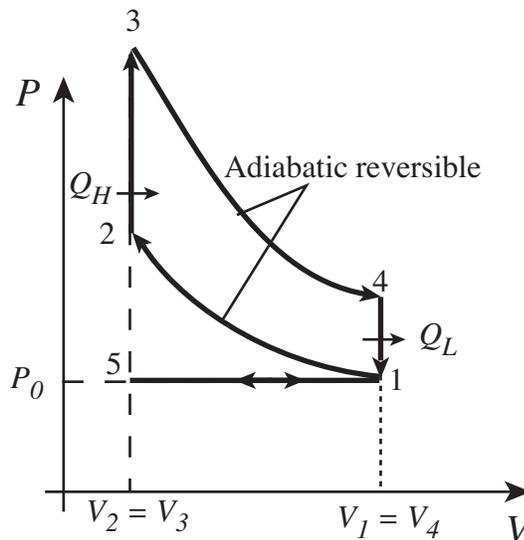


Figure 2A-1: Ideal Otto cycle

- i. Intake stroke, gasoline vapor and air drawn into engine (5 -> 1)
- ii. Compression stroke,  $P$ ,  $T$  increase (1->2)
- iii. Combustion (spark), short time, essentially constant volume (2->3)  
Model: heat absorbed from a series of reservoir at temperatures  $T_2$  to  $T_3$
- iv. Power stroke: expansion (3 ->4)
- v. Valve exhaust: valve opens, gas escapes
- vi. (4->1) Model: rejection of heat to series of reservoirs at temperatures  $T_4$  to  $T_1$
- vii. Exhaust stroke, piston pushes remaining combustion products out of chamber 1->5

The actual cycle does not have these sharp transitions between the different processes and might be as sketched in Figure 2A-2

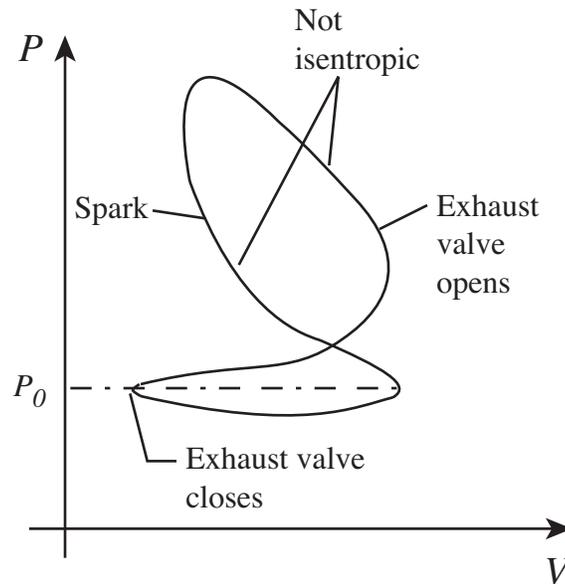


Figure 2A-2: Sketch of actual Otto cycle

#### Efficiency of an ideal Otto cycle

The starting point is the general expression for the thermal efficiency of a cycle:

$$\eta = \frac{\text{work}}{\text{heat input}} = \frac{Q_H + Q_L}{Q_H} = 1 + \frac{Q_L}{Q_H}.$$

The convention, as previously, is that heat exchange is positive if heat is flowing into the system or engine, so  $Q_L$  is negative. The heat absorbed occurs during combustion when the spark occurs, roughly at constant volume. The heat absorbed can be related to the temperature change from state 2 to state 3 as:

$$\begin{aligned} Q_H = Q_{23} &= \Delta U_{23} \quad (W_{23} = 0) \\ &= \int_{T_2}^{T_3} C_v dT = C_v (T_3 - T_2) \end{aligned}$$

The heat rejected is given by (for a perfect gas with constant specific heats)

$$Q_L = Q_{41} = \Delta U_{41} = C_v (T_1 - T_4)$$

Substituting the expressions for the heat absorbed and rejected in the expression for thermal efficiency yields

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

We can simplify the above expression using the fact that the processes from 1 to 2 and from 3 to 4 are isentropic:

$$T_4 V_1^{\gamma-1} = T_3 V_2^{\gamma-1}, \quad T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

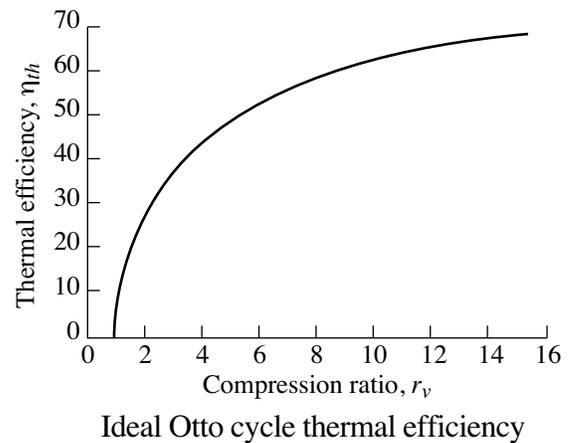
$$(T_4 - T_1) V_1^{\gamma-1} = (T_3 - T_2) V_2^{\gamma-1}$$

$$\frac{T_4 - T_1}{T_3 - T_2} = \left( \frac{V_2}{V_1} \right)^{\gamma-1}$$

The quantity  $\frac{V_1}{V_2} = r$  is called the compression ratio. In terms of compression ratio, the efficiency of an ideal Otto cycle is:

$$\eta_{Otto} = 1 - \frac{1}{\left( \frac{V_1}{V_2} \right)^{\gamma-1}} = 1 - \frac{1}{r^{\gamma-1}}.$$

The ideal Otto cycle efficiency is shown at the right, as a function of the compression ratio. As the compression ratio,  $r$ , increases,  $\eta_{Otto}$  increases, but so does  $T_2$ . If  $T_2$  is too high, the mixture will ignite without a spark (at the wrong location in the cycle).



Engine work, rate of work per unit enthalpy flux:

The non-dimensional ratio of work done (the power) to the enthalpy flux through the engine is given by

$$\frac{\text{Power}}{\text{Enthalpy flux}} = \frac{\dot{W}}{\dot{m} c_p T_1} = \frac{\dot{Q}_{23} \eta_{Otto}}{\dot{m} c_p T_1}$$

There is often a desire to increase this quantity, because it means a smaller engine for the same power. The heat input is given by

$$\dot{Q}_{23} = \dot{m}_{fuel} (\Delta h_{fuel}),$$

where

- $\Delta h_{fuel}$  is the heat of reaction, ie the chemical energy liberated per unit mass of fuel
- $\dot{m}_{fuel}$  is the fuel mass flow rate.

The non-dimensional power is

$$\frac{\dot{W}}{\dot{m} c_p T_1} = \frac{\dot{m}_{fuel}}{\dot{m}} \frac{\Delta h_{fuel}}{c_p T_1} \left[ 1 - \frac{1}{r^{\gamma-1}} \right].$$

The quantities in this equation, evaluated at stoichiometric conditions are:

$$\frac{\dot{m}_{fuel}}{\dot{m}} \approx \frac{1}{15}, \quad \frac{\Delta h_{fuel}}{c_p T_1} \approx \frac{4 \times 10^7}{10^3 \times 288}$$

so,

$$\frac{\dot{W}}{\dot{m} c_p T_1} \approx 9 \left[ 1 - \frac{1}{r^{\gamma-1}} \right].$$

### Muddy points

How is  $\Delta h_{fuel}$  calculated? (MP 2A.1)

What are "stoichiometric conditions"? (MP 2A.2)

### 2.A.2. Diesel Cycle

The Diesel cycle is a compression ignition (rather than spark ignition) engine. Fuel is sprayed into the cylinder at  $P_2$  (high pressure) when the compression is complete, and there is ignition without a spark. An idealized Diesel engine cycle is shown in Figure 2A-3.

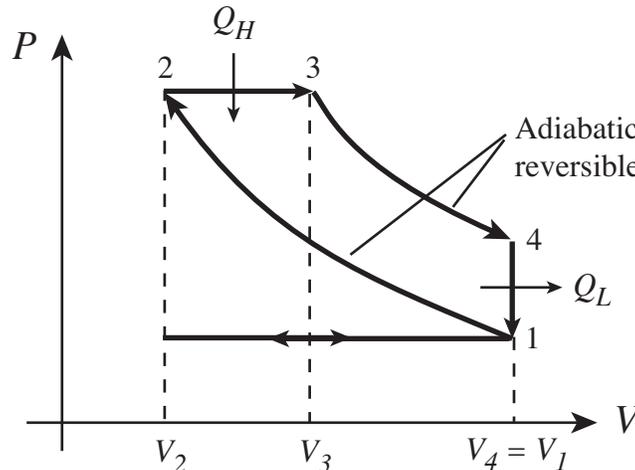


Figure 2A-3 Ideal Diesel cycle

The thermal efficiency is given by:

$$\eta_{Diesel} = 1 + \frac{Q_L}{Q_H} = 1 + \frac{C_v(T_1 - T_4)}{C_p(T_3 - T_2)}$$

$$\eta_{Diesel} = 1 - \frac{T_1 \left( \frac{T_4}{T_1} - 1 \right)}{\gamma T_2 \left( \frac{T_3}{T_2} - 1 \right)}$$

This cycle can operate with a higher compression ratio than Otto cycle because only air is compressed and there is no risk of auto-ignition of the fuel. Although for a given compression ratio the Otto cycle has higher efficiency, because the Diesel engine can be operated to higher compression ratio, the engine can actually have higher efficiency than an Otto cycle when both are operated at compression ratios that might be achieved in practice.

**Muddy points**

When and where do we use  $c_v$  and  $c_p$ ? Some definitions use  $dU=c_vdT$ . Is it ever  $dU=c_pdT$ ? (MP 2A.3)

Explanation of the above comparison between Diesel and Otto. (MP 2A.4)

**2.A.3 Brayton Cycle**

The Brayton cycle is the cycle that represents the operation of a gas turbine engine. The “simple gas turbine” can be operated in open cycle or closed cycle (recirculating working fluid) modes, as shown below.

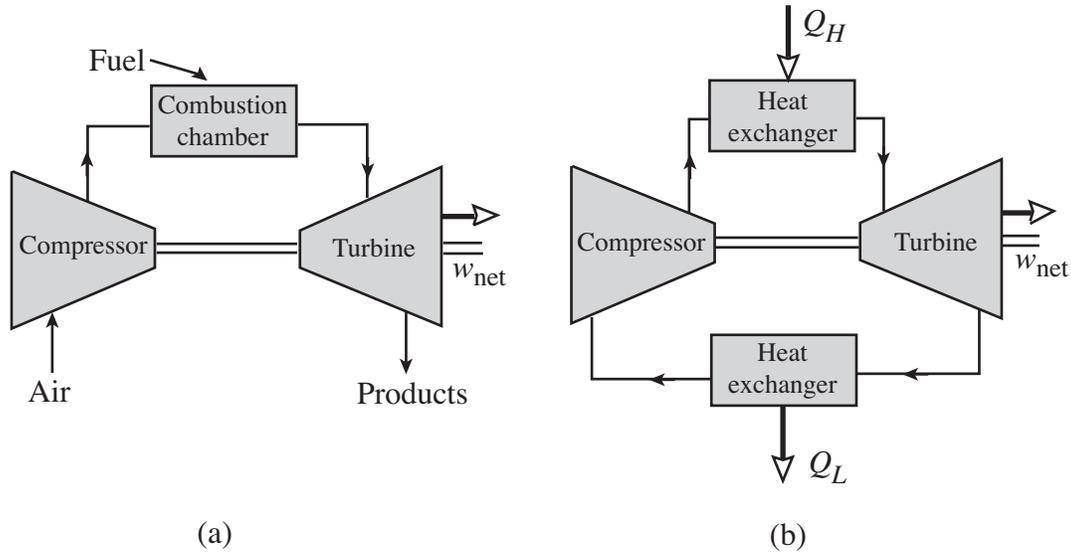


Figure 2A-4: Gas turbine engine operating on the Brayton cycle – (a) open cycle operation, (b) closed cycle operation

Efficiency of the Brayton cycle:

We derived the ideal Brayton cycle efficiency in Section 1.A:

$$\eta_{Brayton} = 1 - \frac{T_{inlet}}{T_{compressor\ exit}} = 1 - \frac{1}{PR^{(\gamma-1)/\gamma}}$$

Net work per unit mass flow in a Brayton cycle:

The net mechanical work of the cycle is given by:

$$\text{Net mechanical work/unit mass} = w_{turbine} - w_{compressor},$$

where

$$w_{compressor} = -\Delta h_{12} = -\Delta h_{comp}$$

$$w_{turbine} = -\Delta h_{34} = -\Delta h_{turb}$$

If kinetic energy changes across the compressor and turbine are neglected, the temperature ratio,  $TR$ , across the compressor and turbine is related to the enthalpy changes:

$$TR - 1 = \frac{\Delta h_{comp}}{h_1} = \frac{|\Delta h_{turb}|}{h_4},$$

$$\Delta h_{turb} = -\Delta h_{comp} \frac{h_4}{h_1}$$

The net work is thus

$$\text{net work} = \Delta h_{comp} \left( \frac{h_4}{h_1} - 1 \right)$$

The turbine work is greater than the work needed to drive the compressor. The thermodynamic states in an enthalpy-entropy ( $h,s$ ) diagram, and the work of the compressor and turbine, are shown below for an ideal Brayton cycle.

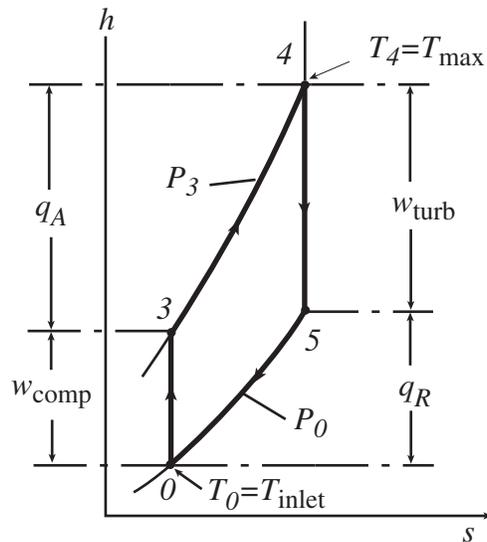


Figure 2A-5: Brayton cycle in enthalpy-entropy ( $h-s$ ) representation showing compressor and turbine work

### Muddy points

What is shaft work? (MP 2A.5)

### 2.A.4 Brayton Cycle for Jet Propulsion: the Ideal Ramjet

A schematic of a ramjet is given in Figure 2A-6 below.

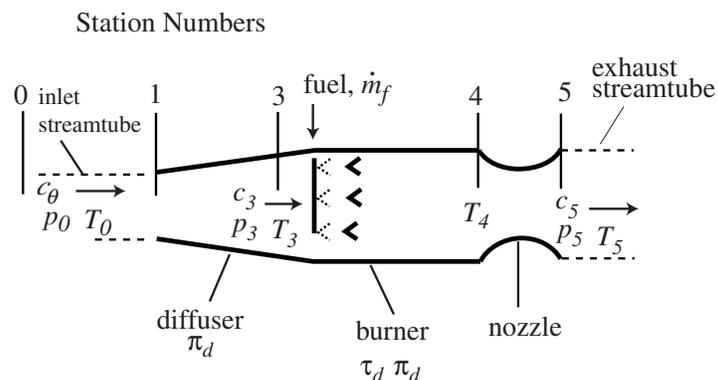


Figure 2A-6: Ideal ramjet [adapted from J. L. Kerrebrock, *Aircraft Engines and Gas Turbines*]

In the ramjet there are “no moving parts”. The processes that occur in this propulsion device are:

0->3 isentropic diffusion (slowing down) and compression, with a decrease in Mach number,  $M_0 \rightarrow M_3 \ll 1$

3->4 Constant pressure combustion,

4->5 Isentropic expansion through the nozzle.

### Thrust of an ideal engine ramjet

The coordinate system and control volume are chosen to be fixed to the ramjet. The thrust,  $F$ , is given by:

$$F = \dot{m}(c_5 - c_0),$$

where  $c_5$  and  $c_0$  are the inlet and exit flow velocities. The thrust can be put in terms of non-dimensional parameters as follows:

$$\frac{F}{\dot{m}a_0} = \frac{c_5}{a_5} \frac{a_5}{a_0} - \frac{c_0}{a_0}, \text{ where } a = \sqrt{\gamma RT} \text{ is the speed of sound.}$$

$$\frac{F}{\dot{m}a_0} = M_5 \frac{a_5}{a_0} - M_0 = M_5 \sqrt{\frac{T_5}{T_0}} - M_0$$

Using  $M_3^2, M_4^2 \ll 1$  in the expression for stagnation pressure,  $\frac{P_T}{P} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}}$ ,

$$P_3 \approx P_{T_3} = P_{T_0}; P_4 \approx P_{T_4} = P_{T_5}; P_5 \approx P_3$$

The ratios of stagnation pressure to static pressure at inlet and exit of the ramjet are:

$$\underbrace{\frac{P_{T_0}}{P_0}}_{\text{determines } M_0} = \frac{P_3}{P_0} = \underbrace{\frac{P_4}{P_0}}_{\text{determines } M_e} = \frac{P_{T_e}}{P_e}$$

The ratios of stagnation to static pressure at exit and at inlet are the same, with the consequence that *the inlet and exit Mach numbers are also the same.*

$$M_5 = M_0.$$

To find the thrust we need to find the ratio of the temperature at exit and the temperature at inlet. This is given by:

$$\frac{T_5}{T_0} = \frac{T_{T_5}}{1 + \frac{\gamma - 1}{2} M_5^2} \frac{1 + \frac{\gamma - 1}{2} M_0^2}{T_0} = \frac{T_{T_5}}{T_0} = \frac{T_{T_4}}{T_{T_3}} = \tau_b,$$

where  $\tau_b$  is the stagnation temperature ratio across the combustor (burner). The thrust is thus:

$$\frac{F}{\dot{m}a_0} = M_0(\sqrt{\tau_b} - 1)$$

Cycle efficiency in terms of aerodynamic parameters:

$$\eta_{Brayton} = 1 - \frac{T_0}{T_{compressor\ exit}} = 1 - \frac{T_0}{T_3} = 1 - \frac{T_0}{T_{T_0}}, \text{ and } \frac{T_0}{T_{T_0}} = \frac{1}{1 + \frac{\gamma - 1}{2} M_0^2}, \text{ so:}$$

$$\eta_{Brayton} = \frac{\frac{\gamma - 1}{2} M_0^2}{1 + \frac{\gamma - 1}{2} M_0^2} : \text{ Ramjet thermodynamic cycle efficiency in terms of flight}$$

Mach number,  $M_0$ .

For propulsion engines, the figures of merit includes more than thrust and  $\eta_{Brayton}$ .

The specific impulse,  $I_{sp}$  measures how effectively fuel is used:

$$I_{sp} = \frac{F}{\dot{m}_f g} = \frac{F}{f \dot{m} g}; \text{ Specific Impulse,}$$

where  $\dot{m}_f = f \dot{m}$  is the fuel mass flow rate.

To find the fuel-air ratio,  $f$ , we employ a control volume around the combustor and carry out an energy balance. Before doing this, however, it is useful to examine the way in which  $I_{sp}$  appears in expressions for range.

### **Muddy points**

What exactly is the specific impulse,  $I_{sp}$ , a measure of? (MP 2A.6)

How is  $I_{sp}$  found for rockets in space where  $g \sim 0$ ? (MP 2A.7)

Why does industry use TSCP rather than  $I_{sp}$ ? Is there an advantage to this? (MP 2A.8)

Why isn't mechanical efficiency an issue with ramjets? (MP 2A.9)

How is thrust created in a ramjet? (MP 2A.10)

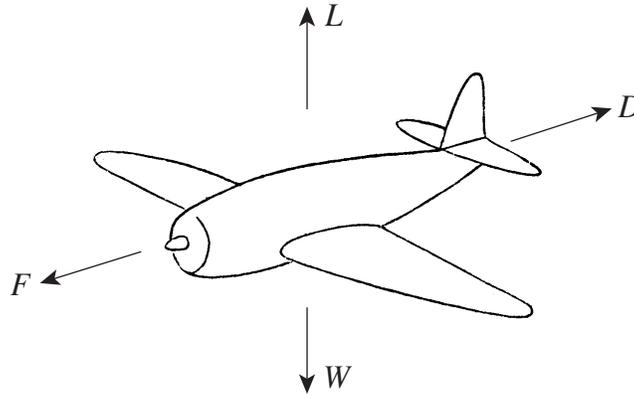
Why don't we like the numbers 1 and 2 for the stations? Why do we go 0-3? (MP 2A.11)

For the Brayton cycle efficiency, why does  $T_3 = T_{10}$ ? (MP 2A.12)

### **2.A.5 The Breguet Range Equation**

[See Waitz Unified Propulsion Notes, No. IV (see the 16.050 Web site)]

Consider an aircraft in level flight, with weight  $W$ . The rate of change of the gross weight of the vehicle is equal to the fuel weight flow:



$$g \dot{m}_f = -\frac{dW}{dt} = -\frac{F}{I_{sp}}$$

$$W = L = D\left(\frac{L}{D}\right) = F\left(\frac{L}{D}\right)$$

The rate of change of aircraft gross weight is thus

$$\frac{dW}{dt} = -\frac{W}{\left(\frac{L}{D}\right)I_{sp}}$$

Suppose  $L/D$  and  $I_{sp}$  remain constant along flight path:

$$\frac{dW}{W} = -\frac{dt}{\left(\frac{L}{D}\right)I_{sp}}$$

We can integrate this equation for the change in aircraft weight to yield a relation between the weight change and the time of flight:

$$\ln \frac{W}{W_i} = -\frac{t}{\left(\frac{L}{D}\right)I_{sp}}, \text{ where } W_i \text{ is the initial weight.}$$

If  $W_f$  is the final weight of vehicle and  $t_{initial}=0$ , the relation between vehicle parameters and flight time,  $t_f$ , is

$$\frac{L}{D} I_{sp} \ln \frac{W_i}{W_f} = t_f$$

The range is the flight time multiplied by the flight speed, or,

$$\text{Range} = c_0 t_f = \underbrace{\left(\frac{L}{D}\right)}_{\text{aircraft designer}} \times c_0 \times \underbrace{\left(I_{sp}\right)}_{\text{propulsion system designer}} \times \ln \underbrace{\left(\frac{W_i}{W_f}\right)}_{\text{structural designer}}$$

The above equation is known as the *Breguet range equation*. It shows the influence of aircraft, propulsion system, and structural design parameters.

Relation of overall efficiency,  $I_{sp}$ , and thermal efficiency

Suppose  $\Delta h_{fuel}$  is the heating value ('heat of combustion') of fuel (i.e., the energy per unit of fuel mass), in J/kg. The rate of energy release is  $\dot{m}_f \Delta h_{fuel}$ , so

$$c_0 I_{sp} = c_0 \frac{F}{\dot{m}_f g} \frac{\Delta h_{fuel}}{\Delta h_{fuel}} = \frac{F c_0}{\dot{m}_f \Delta h_{fuel}} \frac{\Delta h_{fuel}}{g}$$

and  $\frac{F c_0}{\dot{m}_f \Delta h_{fuel}} = \frac{\text{Thrust power (useful work)}}{\text{Ideal available energy}} = \eta_{overall}$  (overall propulsion system efficiency)

$$\eta_{overall} = \frac{g}{\Delta h_{fuel}} c_0 I_{sp} \quad \text{and} \quad \text{Range} = \frac{\Delta h_{fuel}}{g} \eta_{overall} \frac{L}{D} \ln \frac{W_i}{W_f}$$

$$\eta_{overall} = \eta_{thermal} \eta_{propulsive} \eta_{combustion}$$

The Propulsion Energy Conversion Chain

The above concepts can be depicted as parts of the propulsion energy conversion train mentioned in Part 0, which shows the process from chemical energy contained in the fuel to energy useful to the vehicle.

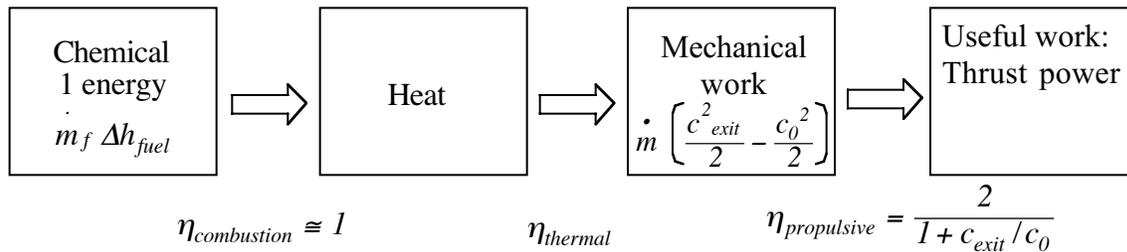


Figure 2A-7: The propulsion energy conversion chain.

The combustion efficiency is near unity unless conditions are far off design and we can regard the two main drivers as the thermal and propulsive<sup>1</sup> efficiencies. The evolution of the overall efficiency of aircraft engines in terms of these quantities is shown below in Figure 2A-8.

<sup>1</sup> The transmission efficiency represents the ratio between compressor and turbine power, which is less than unity due to parasitic frictional effects. As with the combustion efficiency, however, this is very close to one and the horizontal axis can thus be regarded essentially as propulsive efficiency.

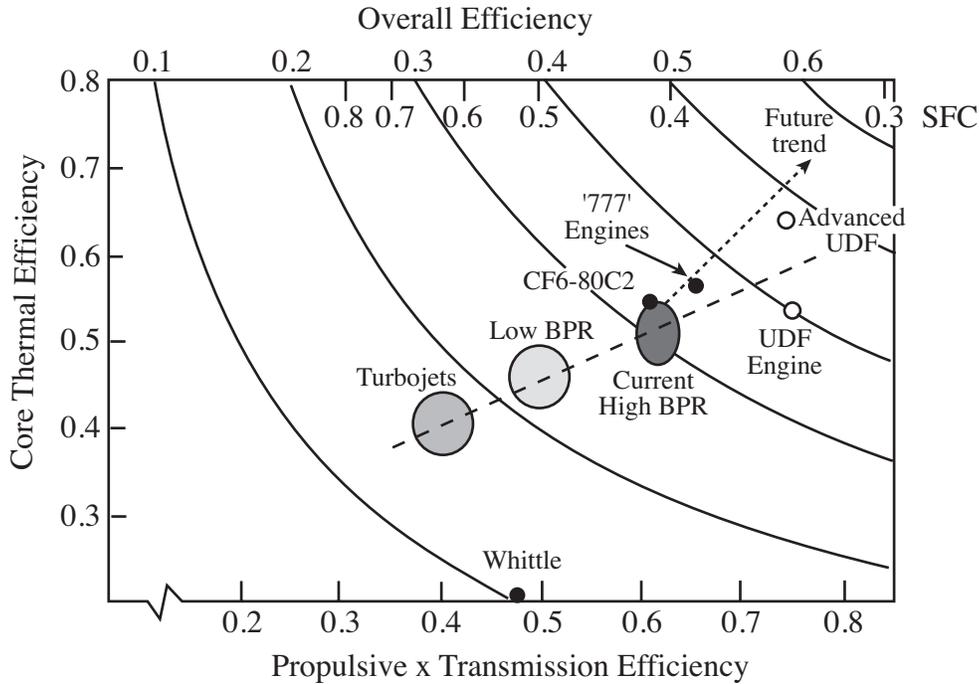


Figure 2A-8: Overall efficiency of aircraft engines as functions of thermal and propulsive efficiencies [data from Koff].

**Muddy points**

How can we idealize fuel addition as heat addition? (MP 2A.13)

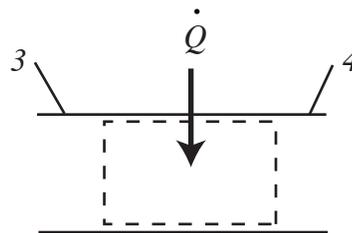
**2.A.6 Performance of the Ideal Ramjet**

We now return to finding the ramjet fuel-air ratio,  $f$ . Using a control volume around the burner, we get:

$$\text{Heat given to the fluid: } \dot{Q} = \dot{m}_f \Delta h_{fuel} = \dot{m} f \Delta h_{fuel}$$

From the steady flow energy equation:

$$\dot{m}_4 h_{t4} - \dot{m}_3 h_{t3} = \dot{m}_3 f \Delta h_{fuel}$$



The exit mass flow is not greatly different from the inlet mass flow,  $\dot{m}_4 = \dot{m}_3(1 + f) \approx \dot{m}_3$ , because the fuel-air ratio is much less than unity (generally several percent). We thus neglect the difference between the mass flows and obtain

$$h_{t4} - h_{t3} = c_p (T_{t4} - T_{t3}) = f \Delta h_{fuel}$$

$$T_{t3} c_p (\tau_b - 1) = f \Delta h_{fuel}, \text{ with } T_{t3} = T_{t0} = T_0 \underbrace{\left(1 + \frac{\gamma-1}{2} M_0^2\right)}_{\theta_0}$$

Fuel-air ratio,  $f$ :

$$f = \frac{\tau_b - 1}{\Delta h_{fuel} / c_p T_0 \Theta_0},$$

The fuel-air ratio,  $f$ , depends on the fuel properties ( $\Delta h_{fuel}$ ), the desired flight parameters ( $\Theta_0$ ), the ramjet performance ( $\tau_b$ ), and the temperature of the atmosphere ( $T_0$ ).

Specific impulse,  $I_{sp}$ :

The specific impulse for the ramjet is given by

$$I_{sp} = \frac{F}{f \dot{m} g} = \frac{1}{g} \frac{(c_0 \Delta h_{fuel} / c_p T_0) (\sqrt{\tau_b} - 1)}{\Theta_0 (\tau_b - 1)}$$

The specific impulse can be written in terms of fuel properties and flight and vehicle characteristics as,

$$I_{sp} = \underbrace{\frac{a_0 \Delta h_{fuel}}{g c_p T_0}}_{\text{fuel properties}} \times \underbrace{\frac{M_0}{\Theta_0 (\sqrt{\tau_b} + 1)}}_{\text{flight characteristics, ramjet temp increase}}.$$

We wish to explore the parameter dependency of the above expression, which is a complicated formula. How can we do this? What are the important effects of the different parameters? How do we best capture the ramjet performance behavior?

To make effective comparisons, we need to add some additional information concerning the operational behavior. An important case to examine is when  $f$  is such that all the fuel burns, i.e. when we have stoichiometric conditions. What happens in this situation as the flight Mach number,  $M_0$ , increases?  $T_0$  is fixed so  $T_{t3}$  increases, but the maximum temperature does not increase much because of dissociation: the reaction does not go to completion at high temperature. A useful approximation is therefore to take  $T_{t4}$  constant for stoichiometric operation. In the stratosphere, from 10 to 30 km,  $T_0 \approx \text{constant} \approx 212K$ . The maximum temperature ratio is

$$\tau_{max} = \frac{T_{max}}{T_0} = \frac{T_{t4}}{T_0} = \text{const},$$

$$\tau_b = \frac{T_{t4}}{T_{t3}} = \frac{T_{t4}/T_0}{T_{t3}/T_0} = \frac{\tau_{max}}{\Theta_0}$$

For the stoichiometric ramjet:

$$I_{sp} = \frac{F}{f \dot{m} g} = \frac{F}{\dot{m} a_0 f_{stoich} g} = M_0 (\sqrt{\tau_b} - 1) \frac{a_0}{f_{stoich} g}$$

Using the expression for  $\tau_b$ , the specific impulse is

$$I_{sp} = M_0 \left( \sqrt{\frac{\tau_{max}}{\Theta_0}} - 1 \right) \frac{a_0}{f_{stoich} g}$$

Representative performance values:

A plot of the performance of the stoichiometric ramjet is shown in Figure 2A-9.

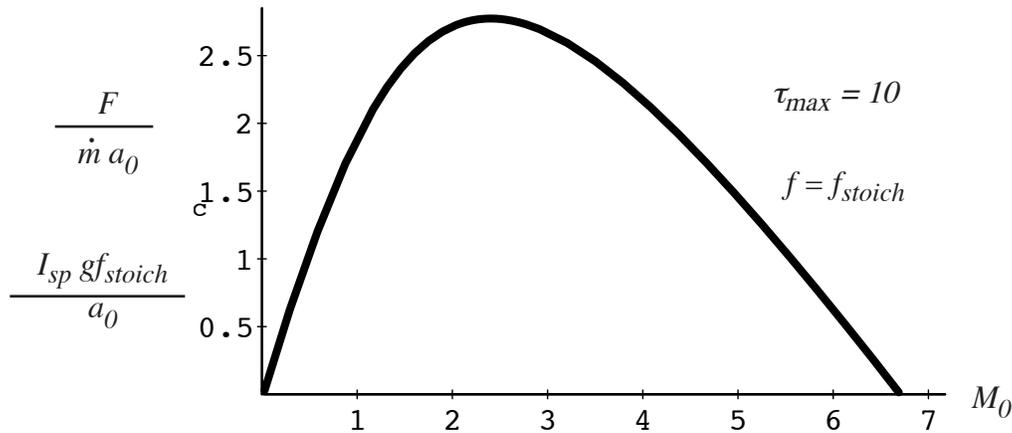


Figure 2A-9: Thrust per unit mass flow and specific impulse for ideal ramjet with stoichiometric combustion [adapted from Kerrebrock]

The figure shows that for the parameters used, the best operating range of a hydrocarbon-fueled ramjet is  $2 \leq M_0 \leq 4$ . The parameters used are  $\tau_{max} = 10$ ,  $a_0 \approx 300 \text{ms}^{-1}$  in the stratosphere,

$$f_{stoich} = 0.067 \text{ for hydrocarbons } \frac{a_0}{g f_{stoich}} \approx 450 \text{ s.}$$

Recapitulation:

In this section we have:

- Examined the Brayton Cycle for propulsion

- Found  $\eta_{Brayton}$  as a function of  $M_0$

- Found  $\eta_{overall}$  and the relation between  $\eta_{overall}$  and  $\eta_{Brayton}$

- Examined  $\frac{F}{\dot{m} a_0}$  and  $I_{sp}$  as a function of  $M_0$  for the ideal ramjet.

### **Muddy points**

What is the relation between  $h_{t4} - h_{t3} = f \Delta h_f$  and the existence of the maximum value of  $T_{t4}$ ? (MP 2A.14a)

Why didn't we have a 2s point for the Brayton cycle with non-ideal components? (MP 2A.14b)

What is the variable  $f_{stoich}$ ? (MP 2A.15)

### 2.A.7 Effect of Departures from Ideal Behavior - Real Cycle behavior

[See also charts 69-82 in 16.050: Gas Turbine Engine Cycles]

What are the sources of non-ideal performance and departures from reversibility?

- Losses (entropy production) in the compressor and the turbine
- Stagnation pressure decrease in the combustor
- Heat transfer

We take into account here only irreversibility in the compressor and in the turbine. Because of these irreversibilities, we need more work,  $\Delta h_{comp}$  (the changes in kinetic energy from inlet to exit of the compressor are neglected), to drive the compressor than in the ideal situation. We also get less work,  $\Delta h_{turb}$ , back from the turbine. The consequence, as can be inferred from Figure 2A-10 below, is that the net work from the engine is less than in the cycle with ideal components.

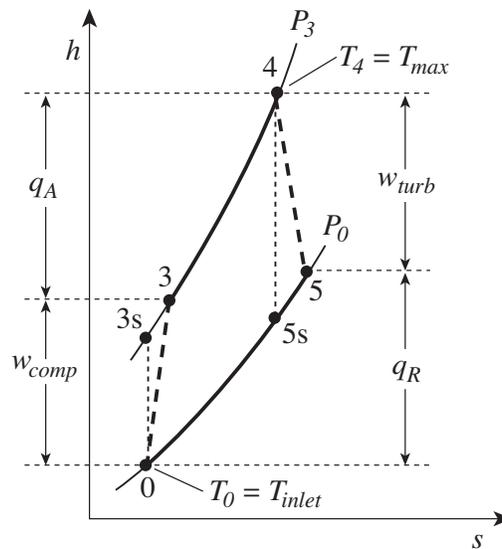


Figure 2A-10: Gas turbine engine (Brayton) cycle showing effect of departure from ideal behavior in compressor and turbine

To develop a quantitative description of the effect of these departures from reversible behavior, consider a perfect gas with constant specific heats and neglect kinetic energy at the inlet and exit of the turbine and compressor. We define the turbine adiabatic efficiency as:

$$\eta_{turb} = \frac{w_{turb}^{actual}}{w_{turb}^{ideal}} = \frac{h_4 - h_5}{h_4 - h_{5s}}$$

where  $w_{turb}^{actual}$  is specified to be at the same pressure ratio as  $w_{turb}^{ideal}$ . (See charts 69-76 in 16.050 Gas Turbine Cycles.)

There is a similar metric for the compressor, the compressor adiabatic efficiency:

$$\eta_{comp} = \frac{w_{comp}^{ideal}}{w_{comp}^{actual}} = \frac{h_{3S} - h_0}{h_3 - h_0}$$

again for the same pressure ratio. Note that the ratio is the actual work delivered divided by the ideal work for the turbine, whereas the ratio is the ideal work needed divided by the actual work required for the compressor. These are not thermal efficiencies, but rather measures of the degree to which the compression and expansion approach the ideal processes.

We now wish to find the net work done in the cycle and the efficiency. The net work is given either by the difference between the heat received and rejected or the work of the compressor and turbine, where the convention is that heat received is positive and heat rejected is negative and work done is positive and work absorbed is negative.

$$\text{Net work} = \begin{cases} \underbrace{q_H}_{\text{heat in}} + \underbrace{q_L}_{\text{heat out}} = (h_4 - h_3) - (h_5 - h_0) \\ w_{turb} + w_{comp} = (h_4 - h_5) - (h_3 - h_0) \end{cases}$$

The thermal efficiency is:

$$\eta_{thermal} = \frac{\text{Net work}}{\text{Heat input}}$$

We need to calculate  $T_3$ ,  $T_5$

$$\text{From the definition of } \eta_{comp}, T_3 - T_0 = \frac{(T_{3S} - T_0)}{\eta_c} = T_0 \frac{(T_{3S}/T_0 - 1)}{\eta_c}$$

$$\text{With } (T_{3S}/T_0) = \text{isentropic temperature ratio} = \left( \frac{P_{exit}}{P_{inlet}} \right)_{comp}^{\frac{\gamma-1}{\gamma}} = \Pi_{comp}^{\frac{\gamma-1}{\gamma}}$$

$$T_3 = T_0 \frac{\left( \Pi_{comp}^{\frac{\gamma-1}{\gamma}} - 1 \right)}{\eta_{comp}} + T_0$$

Similarly, by the definition of  $\eta_{turb} = \frac{\text{actual work received}}{\text{ideal work for same } \frac{P_{ext}}{P_{inlet}}}$ , we can find  $T_5$ :

$$T_4 - T_5 = \eta_{turb} (T_4 - T_{5S}) = \eta_{turb} T_4 \left( 1 - \frac{T_{5S}}{T_4} \right) = \eta_{turb} T_4 \left( 1 - \Pi_{Turb}^{\frac{\gamma-1}{\gamma}} \right)$$

$$T_5 = T_4 - \eta_{turb} T_4 \left( 1 - \Pi_{turb}^{\frac{\gamma-1}{\gamma}} \right)$$

The thermal efficiency can now be found:

$$\eta_{thermal} = 1 + \frac{Q_L}{Q_H} = 1 - \frac{T_5 - T_0}{T_4 - T_3}$$

with  $\Pi_c = \frac{1}{\Pi_t} = \Pi$ , and  $\tau_S = \Pi^{\frac{\gamma-1}{\gamma}}$  the isentropic cycle temperature ratio,

$$\eta_{thermal} = 1 - \frac{T_4 \left[ 1 - \eta_{turb} \left( 1 - \frac{1}{\tau_S} \right) \right] - T_0}{T_4 - T_0 \left[ \frac{1}{\eta_{comp}} (\tau_S - 1) + 1 \right]}$$

or,

$$\eta_{thermal} = \frac{\left[ 1 - \frac{1}{\tau_S} \right] \left[ \eta_{comp} \eta_{turb} \frac{T_4}{T_0} - \tau_S \right]}{1 + \eta_c \left[ \frac{T_4}{T_0} - 1 \right] - \tau_S}$$

There are several non-dimensional parameters that appear in this expression for thermal efficiency. We list these just below and show their effects in subsequent figures:

#### Parameters reflecting design choices

$\tau_S = \Pi^{\frac{\gamma-1}{\gamma}}$  : cycle pressure ratio

$\frac{T_4}{T_0}$  : maximum turbine inlet temperature

#### Parameters reflecting the ability to design and execute efficient components

$\eta_{comp}$  : compressor efficiency

$\eta_{turb}$  : turbine efficiency

In addition to efficiency, net rate of work is a quantity we need to examine,

$$\dot{W}_{net} = \dot{W}_{turbine} - \dot{W}_{compressor}$$

Putting this in a non-dimensional form:

$$\frac{\dot{W}_{net}}{\dot{m} c_p T_0} = \underbrace{-\frac{1}{\eta_{comp}} (\tau_S - 1)}_{\text{work to drive compressor}} + \underbrace{\eta_{turb} \frac{T_4}{T_0} \left( 1 - \frac{1}{\tau_S} \right)}_{\text{work extracted from flow by turbine}}$$

$$\frac{\dot{W}_{net}}{\dot{m} c_p T_0} = (\tau_S - 1) \left[ \frac{\eta_{turb} T_4 / T_0}{\tau_S} - \frac{1}{\eta_{comp}} \right]$$

Trends in net power and efficiency are shown in Figure 2A-11 for parameters typical of advanced civil engines. Some points to note in the figure:

- For any  $\eta_{comp}, \eta_{turb} \neq 1$ , the optimum pressure ratio ( $\Pi$ ) for maximum  $\eta_{th}$  is not the highest that can be achieved, as it is for the ideal Brayton cycle. The ideal analysis is too idealized in this regard. The highest efficiency also occurs closer to the pressure ratio for maximum power than in the case of an ideal cycle. Choosing this as a design criterion will therefore not lead to the efficiency penalty inferred from ideal cycle analysis.
  - There is a strong sensitivity to the component efficiencies. For example, for  $\eta_{turb} = \eta_{comp} = 0.85$ , the cycle efficiency is roughly two-thirds of the ideal value.
  - The maximum power occurs at a value of  $\tau_s$  or pressure ratio ( $\Pi$ ) less than that for  $\max \eta$ . (this trend is captured by ideal analysis).
  - The maximum power and maximum  $\eta_{thermal}$  are strongly dependent on the maximum temperature,  $T_4/T_0$ .

### Muddy points

How can  $\frac{T_4}{T_0}$  be the maximum turbine inlet temperature? (MP 2A.16)

When there are losses in the turbine that shift the expansion in T-s diagram to the right, does this mean there is more work than ideal since the area is greater? (MP 2A.17)

*Figure 2A-11 available from:  
Cumpsty, Jet Propulsion. Cambridge University Press,  
February 1998. Figure 4.3 p.33.*

Figure 2A-11: Non-dimensional power and efficiency for a non-ideal gas turbine engine - (a) Non-dimensional work as a function of cycle pressure ratio for different values of turbine entry temperature divided by compressor entry temperature, (b) Overall cycle efficiency as a function of pressure ratio for different values of turbine entry temperature divided by compressor entry temperature, (c) Overall cycle efficiency as a function of cycle pressure ratio for different component efficiencies. [from Cumpsty, *Jet Propulsion*]

A scale diagram of a Brayton cycle with non-ideal compressor and turbine behaviors, in terms of temperature-entropy ( $h-s$ ) and pressure-volume ( $P-v$ ) coordinates is given below as Figure 2A-12.

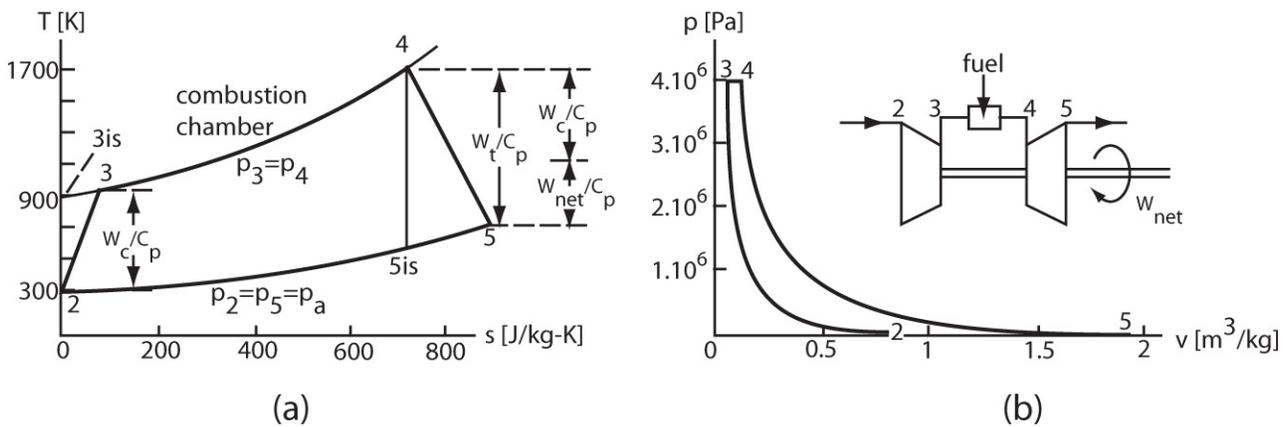


Figure 2A-12: Scale diagram of non-ideal gas turbine cycle. Nomenclature is shown in the figure. Pressure ratio 40,  $T_0 = 288$ ,  $T_4 = 1700$ , compressor and turbine efficiencies = 0.9 [adapted from Cumpsty, *Jet Propulsion*]

### Muddy points

For an afterburning engine, why must the nozzle throat area increase if the temperature of the fluid is increased? (MP 2A.18a)

Why doesn't the pressure in the afterburner go up if heat is added? (MP 2A.18b)

Why is the flow in the nozzle choked? (MP 2A.18c)

What's the point of having a throat if it creates a retarding force? (MP 2A.18d)

Why isn't the stagnation temperature conserved in this steady flow? (MP 2A.18e)

## Muddiest points on part 2A

### 2A.1 How is $\Delta h_{fuel}$ calculated?

For now, we rely on tabulated values. In the lectures accompanying Section 2.C of the notes, we will see how one can calculate the heat,  $\Delta h_{fuel}$ , liberated in a given reaction.

### 2A.2 What are "stoichiometric conditions"?

Stoichiometric conditions are those in which the proportions of fuel and air are such that there is not an excess of each one--all the fuel is burned, and all the air (oxidizer) is used up doing it. See Notes Sections 2.C.

### 2A.3 When and where do we use $c_v$ and $c_p$ ? Some definitions use $dU=c_v dT$ . Is it ever $dU=c_p dT$ ?

The answer is no. The definitions of  $c_p$  and  $c_v$  are derived in the notes on page 0-6.  $c_p$  is the specific heat at constant pressure and for an ideal gas  $dh = c_p dT$  **always holds**. Similarly  $c_v$  is the specific heat at constant volume and for an ideal gas  $du = c_v dT$  **always holds**. A discussion on this is also given in the notes on pages 0-6 and 0-7. If you think about how you would measure the specific heat  $c = q/(T_{final} - T_{initial})$  for a certain known change of state you could do the following experiments.

For a process during which heat  $\Delta q$  is transferred (reversibly) and the volume stays constant (e.g. a rigid, closed container filled with a substance, or the heat transfer in an Otto engine during combustion – the piston is near the top-dead-center and the volume is approximately constant for the heat transfer) the first law is  $du = dq$  since  $v = const$ . Using the definition  $du = c_v dT$  we obtain for the specific heat **at constant volume**

$$c_v = \Delta q / \Delta T ,$$

where both the heat transferred  $\Delta q$  and the temperature difference  $\Delta T$  can be measured.

Similarly we can do an experiment involving a process where the pressure is kept constant during the reversible heat transfer  $\Delta q$  (e.g. a rigid container filled with a substance that is closed by a lid with a certain weight, or the heat transfer in a jet engine combustor where the pressure is approximately constant during heat addition). The first law can be written in terms of enthalpy as  $dh - v dp = dq$ , and since  $p = const$  we obtain  $dh = dq$ . Using the definition  $dh = c_p dT$  we obtain for the specific heat **at constant pressure**

$$c_p = \Delta q / \Delta T .$$

#### 2A.4 Explanation of the above comparison between Diesel and Otto.

Basically we can operate the diesel cycle at much higher compression ratio than the Otto cycle because only air is compressed and we don't run into the auto-ignition problem (knocking problem). Because of the higher compression ratios in the diesel engine we get higher efficiencies.

#### 2A.5 What is shaft work?

I am not sure how best to answer, but it appears that the difficulty people are having might be associated with being able to know when one can say that shaft work occurs. There are several features of a process that produces (or absorbs) shaft work. First of all the view taken of the process is one of control volume, rather than control mass (see the discussion of control volumes in Section 0 or in IAW). Second, there needs to be a shaft or equivalent device (a moving belt, a row of blades) that can be identified as the work carrier. Third, the shaft work is work over and above the "flow work" that is done by (or received by) the streams that exit and enter the control volume.

#### 2A.6 What exactly is the specific impulse, $I_{sp}$ , a measure of?

The specific impulse is a measure of how well the fuel is used in creating thrust. For a rocket engine, the specific impulse is the effective exit velocity divided by the acceleration of gravity,  $g$ . In terms of relating the specific impulse to some characteristic time, we can write the definition of  $I_{sp}$  as

$$I_{sp} \dot{m} g = F .$$

From this, one can regard the specific impulse as the time that it would take to flow a quantity of fuel that has a weight equal to the thrust force

#### 2A.7 How is $I_{sp}$ found for rockets in space where $g \sim 0$ ?

The impulse  $I$  given to a rocket is the thrust force integrated over the burn time. Traditionally, for the case of constant exhaust velocity  $c_{ex}$ , the specific impulse has been used  $I_{sp} = I / (m_p g) = c_{ex} / g_0$ , where  $m_p$  is the propellant mass and  $g_0$  is the Earth's **surface** gravity. Thus  $I_{sp}$  is measured in seconds and is a force per weight flow. Often today, however, specific impulse is measured in units meters/second [m/s], recognizing that force per mass flow is more logical. The specific impulse is then simply equal to the exhaust velocity  $I_{sp} = c_{ex}$ .

#### 2A.8 Why does industry use TSCP rather than $I_{sp}$ ? Is there an advantage to this?

I am not sure why Thrust\*Specific Fuel Consumption was originally used. The gas turbine industry uses TSFC; the rocket propulsion industry uses essentially its inverse, Specific Impulse. Perhaps an advantage is that TSFC is a number of magnitude unity, whereas specific impulse is not.

2A.9 Why isn't mechanical efficiency an issue with ramjets?

As defined, the mechanical efficiency represents bearing friction, and other parasitic torques on the rotating shaft in a gas turbine. The work associated with this needs to be provided by the turbine, but does not go into driving the compressor. The ramjet has no shaft, and hence does not encounter this.

2A.10 How is thrust created in a ramjet?

You can look at thrust in several ways. One is through the integral form of the momentum equation, which relates thrust to the difference between exit and inlet velocities, multiplied by the mass flow. Another way, however, is to look at the forces on the ramjet structure, basically the summation of pressure forces on all the surfaces. I attempted to do this in class using the turbojet with an afterburner. For the ramjet, from the same considerations, we would have an exit nozzle that was larger in diameter than the inlet so that the structural area on which there is a force in the retarding direction is smaller than the area on which there is a force in the thrust direction.

2A.11 Why don't we like the numbers 1 and 2 for the stations? Why do we go 0-3?

A common convention in the industry is that station 0 is far upstream, station 1 is after the shock in the inlet (if there is one), station 2 is at inlet to the compressor (after the inlet/diffuser) and station 3 is after the compressor. In class, when we examined the ramjet we considered no changes in stagnation pressure between 0 and 2, so I have used 0 as the initial state for the compression process. It would be more precise to differentiate between stations 0 and 2, and I will do this where appropriate.

2A.12 For the Brayton cycle efficiency, why does  $T_3 = T_{10}$ ?

The ramjet is operating as a Brayton cycle where  $\eta_b = 1 - T_{\text{inlet}} / T_{\text{compressor exit}}$ . For the ramjet discussed in class the inlet temperature is  $T_0$  and since there is no compressor (no moving parts) the only compression we get is from diffusion. We assumed isentropic diffusion in the diffuser and found for very low Mach numbers that the diffuser exit or combustor inlet temperature  $T_3$  is  $T_{13}$ . From first law we know that for a steady, adiabatic flow where no work is done the stagnation enthalpy stays constant. Assuming perfect gas we thus get  $T_{10} = T_{13} = T_3$ . So we can write for the ramjet thermal efficiency

$$\eta_b = 1 - T_0 / T_{10} .$$

2A.13 How can we idealize fuel addition as heat addition?

The validity of an approximation rests on what the answer is going to be used for. We are seeking basically only one item concerning combustor exit conditions, namely the exit temperature or the exit enthalpy. The final state is independent of *how* we add the heat, and depends only on whether we add the heat. If it is done from an electrical heater or from combustion, and if we neglect the change in the constitution of the gas due to the

combustion products (most of the gas is nitrogen) the enthalpy rise is the same no matter how the temperature rise is achieved.

2A.14a What is the relation between  $h_{t4} - h_{t3} = f\Delta h_f$  and the existence of the maximum value of  $T_{t4}$ ?

The two are very different physical statements. The first is the SFEE (steady flow energy equation) plus the approximation that inlet and exit mass flows to the control volume are the same. The heat received within the volume is represented by the quantity  $f\Delta h_f$ , where  $\Delta h_f$  is the heat liberated per kilogram of fuel. The second statement is a representation of the fact that the degree of completion of the reaction in the combustor depends on temperature, so that even though the inlet temperature increases strongly as the Mach number increases, the combustor exit temperature does not change greatly. This is an attempt to represent a complex physical process (or set of processes) in an approximate manner, not a law of nature.

2A.14b Why didn't we have a 2s point for the Brayton cycle with non-ideal components?

If we didn't, we should have, or I should at least have marked the point at which the compressor exit would be if the compression process was isentropic.

2A.15 What is the variable  $f_{stoich}$ ?

$f_{stoich}$  is the fuel-to-air ratio for stoichiometric combustion, or in other words the fuel-to-air ratio for a chemically correct combustion process during which all fuel is burnt.

2A.16 How can  $\frac{T_4}{T_0}$  be the maximum turbine inlet temperature?

I agree that the  $T_4/T_0$  is a temperature ratio. If we assume constant ambient temperature then this ratio reflects the maximum cycle temperature. The main point was to emphasize that the higher your turbine inlet temperature the higher your power and efficiency levels.

2A.17 When there are losses in the turbine that shift the expansion in T-s diagram to the right, does this mean there is more work than ideal since the area is greater?

We have to be careful when looking at the area enclosed by a cycle or underneath a path in the T-s diagram. Only for a **reversible** cycle, the area enclosed is the work done by the cycle (see notes page 1C-5). Looking at the Brayton cycle with losses in compressor and turbine the net work is the difference between the heat absorbed and the heat rejected (from 1<sup>st</sup> law). The heat absorbed can be found by integrating  $TdS = dQ$  along the heat addition process. The heat rejected during the cycle with losses in compressor and turbine is larger than in the ideal cycle (look at the area underneath the path where heat is rejected, this area is larger than when there are no losses  $ds_{irrev} = 0$  – see also muddy point

1C.1). So we get less net work if irreversibilities are present. It is sometimes easier to look at work and heat (especially shaft work for turbines and compressors) in the h-s diagram because the enthalpy difference between two states directly reflects the shaft work (remember, enthalpy includes the flow work!) and / or heat transfer.

2A.18a For an afterburning engine, why must the nozzle throat area increase if the temperature of the fluid is increased?

The Mach number of the flow is unity at the throat with and without the afterburner lit. The ratio of static pressure to stagnation pressure at the throat is thus the same with and without the afterburner lit. The ratio of static temperature to stagnation temperature at the throat is thus the same with and without the afterburner lit.

$$\frac{T_t}{T} = 1 + \frac{\gamma - 1}{2} M^2 \quad ; \quad \frac{P_t}{P} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\gamma / (\gamma - 1)}$$

The flow through the throat is

$$\dot{m} = \rho c A_{throat} = \rho a A_{throat} = \frac{P}{RT} \sqrt{\gamma RT} A_{throat}$$

The flow through the throat thus scales as

$$\frac{\dot{m}_{A/B}}{\dot{m}_{noA/B}} = \frac{\left( \frac{P}{\sqrt{T}} A_{throat} \right)_{A/B}}{\left( \frac{P}{\sqrt{T}} A_{throat} \right)_{noA/B}}$$

From what we have said, however, the pressure at the throat is the same in both cases. Also, we wish to have the mass flow the same in both cases in order to have the engine operate at near design conditions. Putting these all together, plus use of the idea that the ratio of stagnation to static temperature at the throat is the same for both cases gives the relation

$$\frac{A_{throatA/B}}{A_{throatnoA/B}} = \sqrt{\frac{T_{tA/B}}{T_{tnoA/B}}}$$

The necessary area to pass the flow is proportional to the square root of the stagnation temperature.

If too much fuel is put into the afterburner, the increase in area cannot be met and the flow will decrease. This can stall the engine, a serious consequence for a single engine fighter.

2A.18b Why doesn't the pressure in the afterburner go up if heat is added?

From discussions after lecture, the main point here seems to be that the process of heat addition in the afterburner, or the combustor, is not the same as heat addition to a gas in a box. In that case the density (mass/volume) would be constant and, from  $P = \rho RT$ , increasing the temperature would increase the pressure. In a combustor, the geometry is such that the pressure is approximately constant; this happens because

the fluid has the freedom to expand so the density decreases. From the equation  $P = \rho RT$  if the temperature goes up, the density must go down.

*2A.18c Why is the flow in the nozzle choked?*

As seen in Unified, choking occurs when the stagnation to static pressure ratio ( $P_t/P$ ) gets to a certain value, 1.89 for gas with  $\gamma$  of 1.4. Almost all jet aircraft operate at flight conditions such that this is achieved. If you are not comfortable with the way in which the concepts of choking are laid out in the Unified notes, please see me and I can give some references.

*2A.18d What's the point of having a throat if it creates a retarding force?*

As shown in Unified, to accelerate the flow from subsonic to supersonic, i.e., to create the high velocities associated with high thrust, one *must* have a converging-diverging nozzle, and hence a throat.

*2A.18e Why isn't the stagnation temperature conserved in this steady flow?*

Heat is added in the afterburner, so the stagnation temperature increases.