

## **2.B Power Cycles with Two-Phase Media (Vapor Power Cycles)**

[SB&VW – Chapter 3, Chapter 11, Sections 11.1 to 11.7]

In this section, we examine cycles that use two-phase media as the working fluid. These can be combined with gas turbine cycles to provide *combined cycles* which have higher efficiency than either alone. They can also be used by themselves to provide power sources for both terrestrial and space applications. The topics to be covered are:

- i) Behavior of two-phase systems: equilibrium, pressure temperature relations
- ii) Carnot cycles with two-phase media
- iii) Rankine cycles
- iv) Combined cycles

### **2.B.1 Behavior of Two-Phase Systems**

The definition of a phase, as given by SB&VW, is “a quantity of matter that is homogeneous throughout”. Common examples of systems that contain more than one phase are a liquid and its vapor and a glass of ice water. A system which has three phases is a container with ice, water, and water vapor.

We wish to find the relations between phases and the relations that describe the change of phase (from solid to liquid, or from liquid to vapor) of a pure substance, including the work done and the heat transfer. To start we consider a system consisting of a liquid and its vapor in equilibrium, which are enclosed in a container under a moveable piston, as shown in Figure 2B-1. The system is maintained at constant temperature through contact with a heat reservoir at temperature  $T$ , so there can be heat transfer to or from the system.

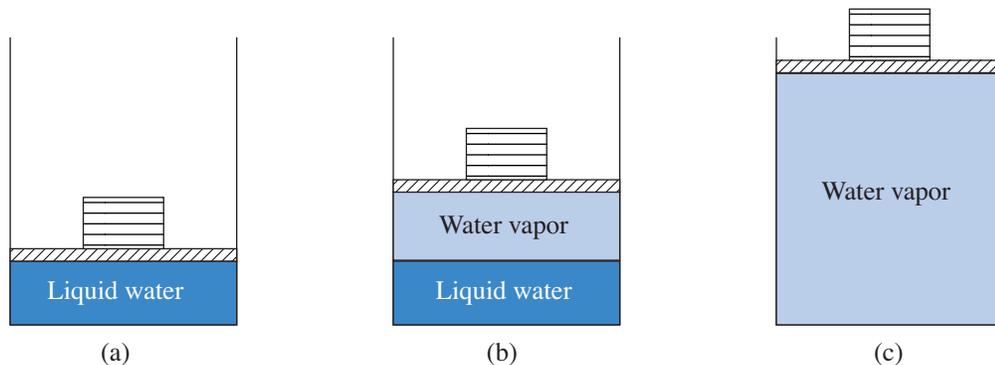
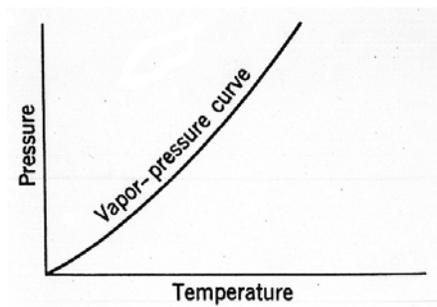


Figure 2B-1: Two-phase system in contact with constant temperature heat reservoir

For a pure substance, as shown at the right, there is a one-to-one correspondence between the temperature at which vaporization occurs and the pressure. These values are called the saturation pressure and saturation temperature (see Ch. 3 in SB&VW).



$P$ - $T$  relation for liquid-vapor system

This means there is an additional constraint for a liquid-vapor mixture, in addition to the equation of state. The consequence is that we only need to specify one variable to determine the state of the system. For example, if we specify  $T$  then  $P$  is set. In summary, for two phases in equilibrium,  $P = P(T)$ . If both phases are present, any quasi-static process at constant  $T$  is also at constant  $P$ .

Let us examine the pressure-volume behavior of a liquid-vapor system at constant temperature. For a single-phase perfect gas we know that the curve would be  $Pv = \text{constant}$ . For the two-phase system the curve looks quite different, as indicated in Figure 2B-2.

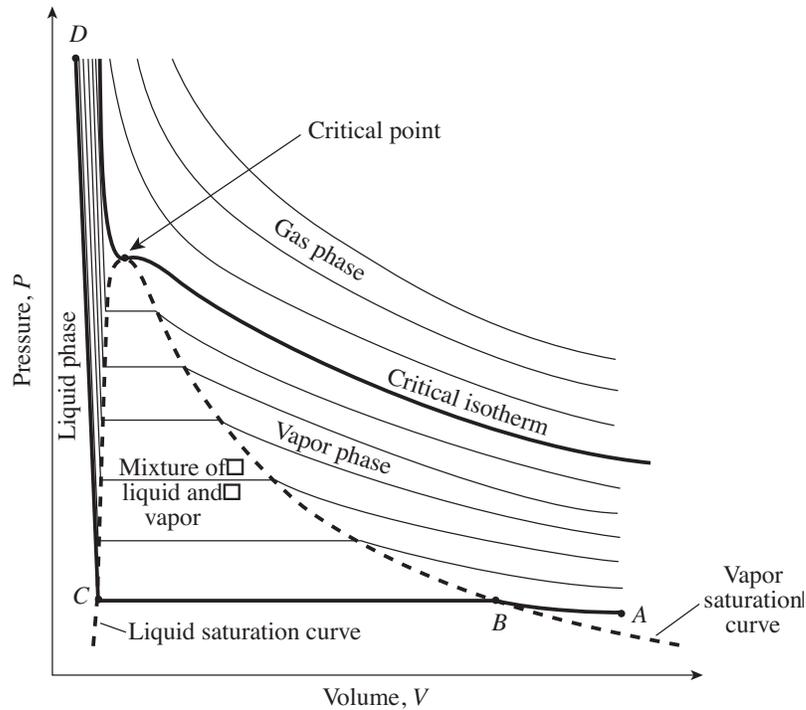


Figure 2B-2 –  $P$ - $v$  diagram for two-phase system showing isotherms

Several features of the figure should be noted. First, there is a region in which liquid and vapor can coexist. This is roughly dome-shaped and is thus often referred to as the “vapor dome”. Outside of this regime, the equilibrium state will be a single phase. The regions of the diagram in which the system will be in the liquid and vapor phases respectively are indicated. Second is the steepness of the isotherms in the liquid phase, due to the small compressibility of most liquids. Third, the behavior of isotherms at temperatures below the “critical point” (see below) in the region to the right of the vapor dome approach those of an ideal gas as the pressure decreases and the ideal gas relation is a good approximation in this region.

The behavior shown is found for all the isotherms that go through the vapor dome. At a high enough temperature, specifically at a temperature corresponding to the pressure at the peak of the vapor dome, there is no transition from liquid to vapor and the fluid goes continuously from a liquid-like behavior to a gas-type behavior. This behavior is unfamiliar, mainly because the temperatures and pressures are not ones that we typically experience; for water the critical temperature is  $374^{\circ}\text{C}$  and the associated critical pressure is 220 atmospheres.

There is a distinct nomenclature used for systems with more than one phase. In this, the terms “vapor” and “gas” seem to be used interchangeably. In the zone where both liquid and vapor exist, there are two bounding situations. When the last trace of vapor condenses, the state becomes saturated liquid. When the last trace of liquid evaporates the state becomes saturated vapor (or dry vapor). If we put heat into a saturated vapor it is referred to as superheated vapor. Nitrogen at room temperature and pressure (at one atmosphere the vaporization temperature of nitrogen is 77 K) is a superheated vapor.

*Figure 2B-3 available from:  
Sonntag, Borgnakke and Van Wylen, Fundamentals of Thermodynamics,  
5th Ed., John Wiley & Sons. Figure 3.3, p35.*

Figure 2B-3: Constant pressure curves in T-v coordinates showing vapor dome

Figure 2B-3 shows lines of constant pressure in temperature-volume coordinates. Inside the vapor dome the constant pressure lines are also lines of constant temperature.

It is useful to describe the situations encountered as we decrease the pressure or equivalently increase the specific volume, starting from a high pressure-low specific volume state (the upper left-hand side of the isotherm in Figure 2B-2). The behavior in this region is liquid-like with very little compressibility. As the pressure is decreased, the volume changes little until the boundary of the vapor dome is reached. Once this occurs, however, the pressure is fixed because the temperature is constant. As the piston is withdrawn, the specific volume increases through more liquid evaporating and more vapor being produced. During this process, since the expansion is isothermal (we specified that it was), heat is transferred to the system. The specific volume will increase at constant pressure until the right hand boundary of the vapor dome is reached. At this point, all the liquid will have been transformed into vapor and the system again behaves as a single-phase fluid. For water at temperatures near room temperature, the behavior would be essentially that of a perfect gas in this region. To the right of the vapor dome, as mentioned above, the behavior is qualitatively like that of a perfect gas.

Referring to Figure 2B-4, we define notation to be used in what follows. The states *a* and *c* denote the conditions at which all the fluid is in the liquid state and the gaseous state respectively.

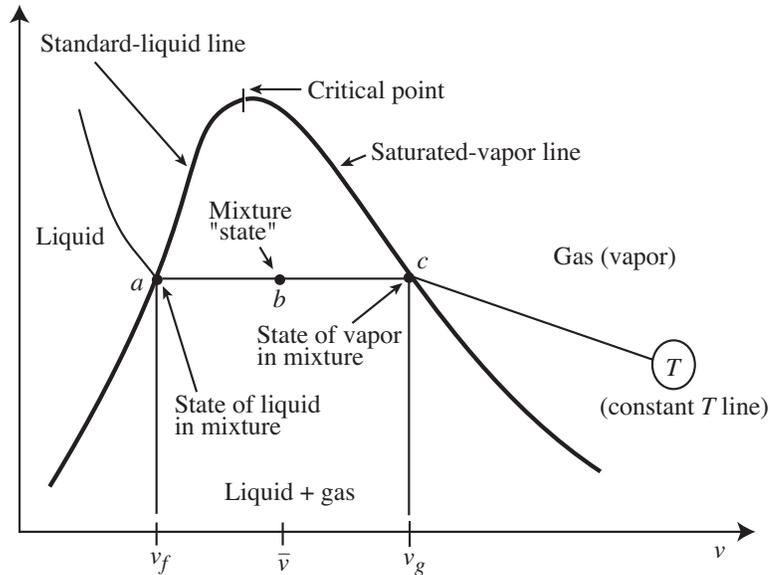


Figure 2B-4: Specific volumes at constant temperature and states within the vapor dome in a liquid-vapor system

The specific volumes corresponding to these states are:

$v_f$  - specific volume of liquid phase

$v_g$  - specific volume of gas phase

For conditions corresponding to specific volumes between these two values, i.e., for state  $b$ , the system would exist with part of the mass in a liquid state and part of the mass in a gaseous (vapor) state. The average specific volume for this condition is

$\bar{v}$  - average specific volume of two-phase system.

We can relate the average specific volume to the specific volumes for liquid and vapor and the mass that exists in the two phases as follows. The total mass of the system is given by

$$\text{total mass} = m = \text{liquid mass} + \text{vapor mass} = m_f + m_g.$$

The volume of the system is

$$\text{Volume of liquid} = V_f = m_f v_f$$

$$\text{Volume of vapor} = V_g = m_g v_g$$

$$\text{Total volume} = V = m_f v_f + m_g v_g.$$

The average specific volume,  $\bar{v}$ , is the ratio of the total volume to the total mass of the system

$$\bar{v} = \frac{m_f v_f + m_g v_g}{m_f + m_g} = \text{average specific volume.}$$

The fraction of the total mass in the vapor phase is called *quality*, and denoted by  $X$ .

$$X = \frac{m_g}{m_f + m_g} = \text{quality of a liquid-vapor system.}$$

In terms of the quality and specific volumes, the average specific volume can be expressed as:

$$\bar{v} = X \cdot v_g + (1 - X) \cdot v_f$$

In reference to Figure 2B-5,  $ab = \bar{v} - v_f$ ,  $ac = v_g - v_f$ .

$$\frac{ab}{ac} = \frac{\bar{v} - v_f}{v_g - v_f} = X = \text{quality.}$$

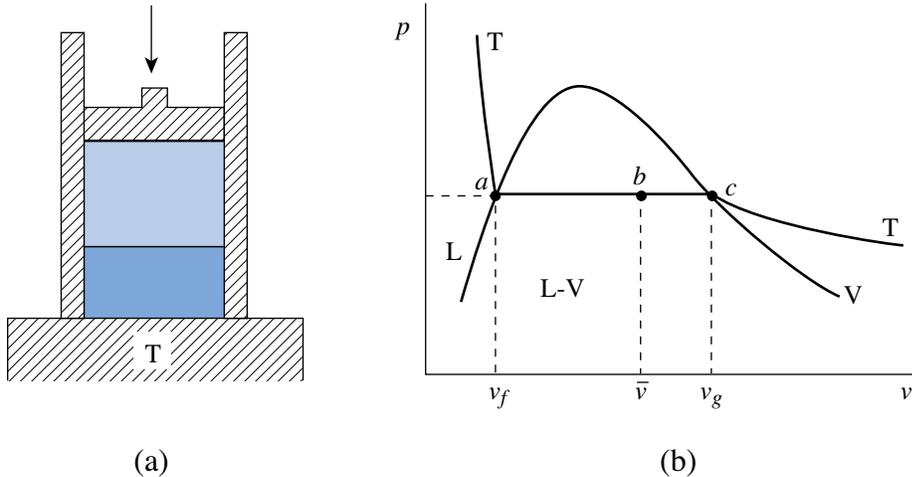


Figure 2B-5: Liquid vapor equilibrium in a two-phase medium

### 2.B.2 - Work and Heat Transfer with Two-Phase Media

We examine the work and heat transfer in quasi-static processes with two-phase systems. For definiteness, consider the system to be a liquid-vapor mixture in a container whose volume can be varied through movement of a piston, as shown in Figure 2B-5. The system is kept at constant temperature through contact with a heat reservoir at temperature  $T$ . The pressure is thus also constant, but the volume,  $V$ , can change. For a fixed mass, the volume is proportional to the specific volume  $\bar{v}$  so that point  $b$  in Figure 2B-5 must move to the left or the right as  $V$  changes. This implies that the amount of mass in each of the two phases, and hence the quality, also changes because mass is transferred from one phase to the other. We wish to find the heat and work transfer associated with the change in mass in each phase. The change in volume can be related to the changes in mass in the two phases as,

$$dV = v_g dm_g + v_f dm_f.$$

The system mass is constant ( $m = m_f + m_g = \text{constant}$ ) so that for any changes

$$dm = 0 = dm_f + dm_g.$$

We can define the quantity  $dm_{fg}$

$$dm_{fg} = dm_g = -dm_f = \text{mass transferred from liquid to vapor.}$$

In terms of  $dm_{fg}$  the volume change of the system is

$$dV = (v_g - v_f)dm_{fg}.$$

The work done is given by

$$\begin{aligned} dW &= PdV \\ &= P(v_g - v_f)dm_{fg}. \end{aligned}$$

The change in internal energy,  $\Delta U$ , can be found as follows. The internal energy of the system can be expressed in terms of the mass in each phase and the specific internal energy (internal energy per unit mass,  $u$ ) of the phase as,

$$\begin{aligned} U &= u_f m_f + u_g m_g \\ dU &= u_f dm_f + u_g dm_g = (u_g - u_f)dm_{fg}. \end{aligned}$$

Note that the specific internal energy can be expressed in a similar way as the specific volume in terms of the quality and the specific enthalpy of each phase:

$$u = X \cdot u_g + (1 - X) \cdot u_f$$

Writing the first law for this process:

$$\begin{aligned} dQ &= dU + dW \\ &= (u_g - u_f)dm_{fg} + P(v_g - v_f)dm_{fg}. \\ &= \left[ (u_g + Pv_g) - (u_f + Pv_f) \right] dm_{fg} \\ &= (h_g - h_f)dm_{fg}. \end{aligned}$$

The heat needed for the transfer of mass is proportional to the difference in specific enthalpy between vapor and liquid. The pressure and temperature are constant, so that the specific internal energy and the specific enthalpy for the liquid phase and the gas phase are also constant. For a finite change in mass from liquid to vapor,  $m_{fg}$ , therefore, the quantity of heat needed is

$$Q = (h_g - h_f)m_{fg} = \Delta H \text{ (enthalpy change).}$$

The heat needed per unit mass,  $q$ , for transformation between the two phases is

$$q = \frac{Q}{m_{fg}} = (h_g - h_f) = h_{fg}.$$

The notation  $h_{fg}$  refers to the specific enthalpy change between the liquid state and the vapor state. The expression for the amount of heat needed,  $q$ , is a particular case of the general result that in any reversible process at constant pressure, the heat flowing into, or out of, the system is equal to the enthalpy change. Heat is absorbed if the change is from solid to liquid (heat of fusion), liquid to vapor (heat of vaporization), or solid to vapor (heat of sublimation).

A numerical example is furnished by the vaporization of water at 100°C:

- i) How much heat is needed per unit mass of fluid vaporized?
- ii) How much work is done per unit mass of fluid vaporized?
- iii) What is the change in internal energy per unit mass of fluid vaporized?

In addressing these questions, we make use of the fact that problems involving heat and work exchanges in two-phase media are important enough that the values of the specific thermodynamic properties that characterize these transformations have been computed for many different working fluids. The values are given in SB&VW in Tables B.1.1 and B.1.2 for water at saturated conditions and in Tables B.1.3, B.1.4, and B.1.5 for other conditions, as well as for other working fluids. From these:

- At 100°C, the vapor pressure is 0.1013 MPa,
- The specific enthalpy of the vapor,  $h_g$ , is 2676 kJ/kg and the specific enthalpy of the liquid,  $h_f$ , is 419 kJ/kg
- The difference in enthalpy between liquid and vapor,  $h_{fg}$ , occurs often enough so that it is tabulated also. This is 2257 kJ/kg,
- The specific volume of the vapor is 1.6729 m<sup>3</sup>/kg and the specific volume of the liquid is 0.001044.

The heat input to the system is the change in enthalpy between liquid and vapor,  $h_{fg}$ , and is equal to  $2.257 \times 10^6$  J/kg.

The work done is  $P(v_g - v_f)$  which has a value of

$$P(v_g - v_f) = 0.1013 \times 10^6 \times [1.6729 - 0.001044] = 0.169 \times 10^6 \text{ J/kg.}$$

The change in internal energy per unit mass ( $u_{fg}$ ) can be found from  $\Delta u = q - w$  or from the tabulated values as  $2.088 \times 10^6$  J/kg. This is much larger than the work done. Most of the heat input is used to change the internal energy rather than appearing as work.

### ***Muddy points***

For the vapor dome, is there vapor and liquid inside the dome and outside is it just liquid or just gas? Is it interchangeable? Is it true for the plasma phase? (MP 2B.1)

What is  $h_{fg}$ ? How do we find it? (MP 2B.2)

Reasoning behind the slopes for  $T=\text{cst}$  lines in the  $P$ - $V$  diagram. (MP 2B.3)

For a constant pressure heat addition, why is  $q=\Delta h$ ? (MP 2B.4)

What is latent heat? (MP 2B.5)

Why is  $U$  a function of  $x$ ? (MP 2B.6)

### 2.B.3 The Carnot Cycle as a Two-Phase Power Cycle

A Carnot cycle that uses a two-phase fluid as the working medium is shown below in Figure 2B-6. Figure 2B-6a gives the cycle in  $P$ - $v$  coordinates, 2B-6b in  $T$ - $s$  coordinates, and 2B-6c in  $h$ - $s$  coordinates. The boundary of the region in which there is liquid and vapor both present (the vapor dome) is also indicated. Note that the form of the cycle is different in the  $T$ - $s$  and  $h$ - $s$  representation; it is only for a perfect gas with constant specific heats that cycles in the two coordinate representations have the same shapes.

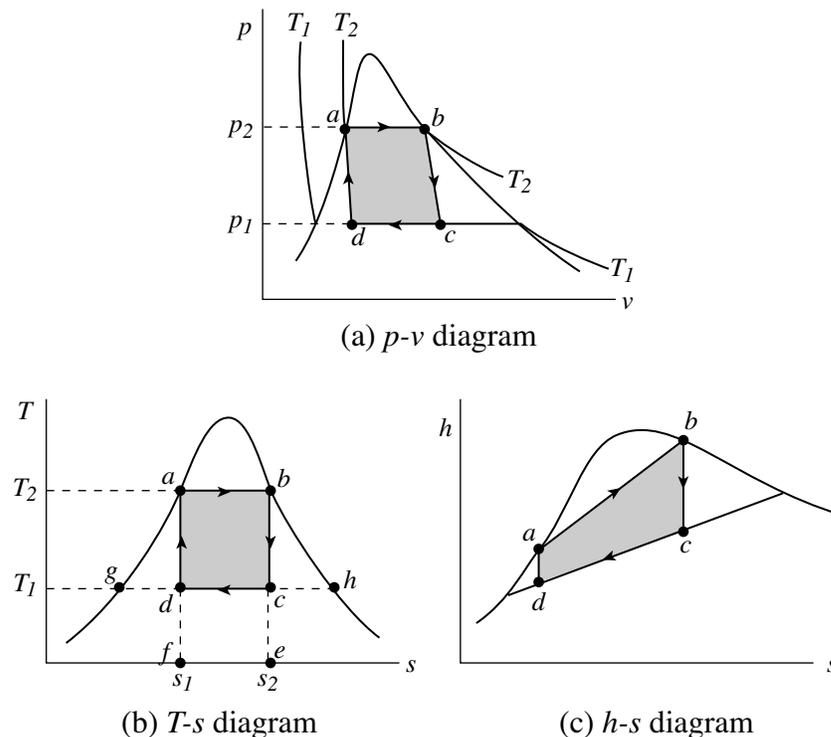


Figure 2B-6: Carnot cycle with two-phase medium. (a) cycle in  $P$ - $v$  coordinates, (b) cycle in  $T$ - $s$  coordinates, (c) cycle in  $h$ - $s$  coordinates

The processes in the cycle are as follows:

- i) Start at state  $a$  with saturated liquid (all of mass in liquid condition). Carry out a reversible isothermal expansion to  $b$  ( $a \rightarrow b$ ) until all the liquid is vaporized. During this process a quantity of heat  $q_H$  per unit mass is received from the heat source at temperature  $T_2$ .
- ii) Reversible adiabatic (i.e., isentropic) expansion ( $b \rightarrow c$ ) lowers the temperature to  $T_1$ . Generally state  $c$  will be in the region where there is both liquid and vapor.

- iii) Isothermal compression ( $c \rightarrow d$ ) at  $T_1$  to state  $d$ . During this compression, heat  $q_L$  per unit mass is rejected to the source at  $T_1$ .
- iv) Reversible adiabatic (i.e., isentropic) compression ( $d \rightarrow a$ ) in which the vapor condenses to liquid and the state returns to  $a$ .

In the  $T$ - $s$  diagram the heat received,  $q_H$ , is  $abef$  and the heat rejected,  $q_L$ , is  $dcef$ . The net work is represented by  $abcd$ . The thermal efficiency is given by

$$\eta = \frac{w_{net}}{q_H} = \frac{\text{Area } abcd}{\text{Area } abef} = 1 - \frac{T_1}{T_2}.$$

In the  $h$ - $s$  diagram, the isentropic processes are vertical lines as in the  $T$ - $s$  diagram. The isotherms in the former, however, are not horizontal as they are in the latter. To see their shape we note that for these two-phase processes the isotherms are also lines of constant pressure (isobars), since  $P = P(T)$ . The combined first and second law is

$$Tds = dh - \frac{dp}{\rho}.$$

For a constant pressure reversible process,  $dq_{rev} = Tds = dh$ . The slope of a constant pressure line in  $h$ - $s$  coordinates is thus,

$$\left( \frac{\partial h}{\partial s} \right)_P = T = \text{constant}; \text{ slope of constant pressure line for two-phase medium.}$$

The heat received and rejected per unit mass is given in terms of the enthalpy at the different states as,

$$\begin{aligned} q_H &= h_b - h_a \\ q_L &= h_d - h_c. \quad (\text{In accord with our convention this is less than zero.}) \end{aligned}$$

The thermal efficiency is

$$\eta = \frac{w_{net}}{q_H} = \frac{q_H + q_L}{q_H} = \frac{(h_b - h_a) + (h_d - h_c)}{(h_b - h_a)},$$

or, in terms of the work done during the isentropic compression and expansion processes, which correspond to the shaft work done on the fluid and received by the fluid,

$$\eta = \frac{(h_b - h_c) - (h_a - h_d)}{(h_b - h_a)}.$$

Example: Carnot steam cycle:

Heat source temperature = 300°C

Heat sink temperature = 20°C

What is the (i) thermal efficiency and (ii) ratio of turbine work to compression (pump) work if

a) all processes are reversible?

b) the turbine and the pump have adiabatic efficiencies of 0.8?

Neglect the changes in kinetic energy at inlet and outlet of the turbine and pump.

a) For the reversible cycle,

$$\begin{aligned}\eta_{thermal} &= \eta_{Carnot} = 1 - \frac{T_1}{T_2} \\ &= 1 - \frac{293}{573} = 0.489\end{aligned}$$

To find the work in the pump (compression process) or in the turbine, we need to find the enthalpy changes between states  $b$  and  $c$ ,  $\Delta h_{bc}$ , and the change between  $a$  and  $d$ ,  $\Delta h_{ad}$ . To obtain these the approach is to use the fact that  $s = \text{constant}$  during the expansion to find the quality at state  $c$  and then, knowing the quality, calculate the enthalpy as  $h = Xh_g + (1 - X)h_f$ . We know the conditions at state  $b$ , where the fluid is all vapor, i.e., we know  $T_b$ ,  $h_b$ ,  $s_b$ :

$$\begin{aligned}h_b &= h_{vapor}(300^\circ C) = h_g(300^\circ C) = 2749 \text{ kJ/kg} \\ s_b &= s_{vapor}(300^\circ C) = s_g(300^\circ C) = 5.7045 \text{ kJ/kg} \cdot \text{K} \\ s_b &= s_c \text{ in the isentropic expansion process.}\end{aligned}$$

We now need to find the quality at state  $c$ ,  $X_c$ . Using the definition of quality given in Section 2.B.1, and noting that  $s_c = X_c s_g + (1 - X_c) s_f$ , we obtain,

$$X_c = \frac{s_c - s_f(T_c)}{s_g(T_c) - s_f(T_c)} = \frac{s_c - s_f(T_c)}{s_{fg}(T_c)}.$$

The quantity  $s_c$  is the mass-weighted entropy at state  $c$ , which is at temperature  $T_c$ .

The quantity  $s_f(T_c)$  is the entropy of the liquid at temperature  $T_c$ .

The quantity  $s_g(T_c)$  is the entropy of the gas (vapor) at temperature  $T_c$ .

The quantity  $\Delta s_{fg}(T_c) = \Delta s_{liquid \rightarrow gas}$  at  $T_c$ .

We know:

$$\begin{aligned}s_c &= s_b = 5.7045 \text{ kJ/kg} \cdot \text{K} \\ s_{fg} &= 8.3706 \text{ kJ/kg} \cdot \text{K} \\ s_f &= 0.2966 \text{ kJ/kg} \cdot \text{K}.\end{aligned}$$

The quality at state  $c$  is thus,

$$X_c = \frac{5.7045 - 0.2966}{8.3706} = 0.646.$$

The enthalpy at state  $c$  is,

$$h_c = X_c h_g + (1 - X_c) h_f \text{ at } T_c.$$

Substituting the values,

$$\begin{aligned} h_c &= 0.646 \times 2538.1 + 0.354 \times 83.96 \text{ kJ/kg} \\ &= 1669.4 \text{ kJ/kg.} \end{aligned}$$

The turbine work/unit mass is the difference between the enthalpy at state *b* and state *c*,

$$h_b - h_c = w_{\text{turbine}} = 2749 - 1669.4 = 1079.6 \text{ kJ/kg.}$$

We can apply a similar process to find the conditions at state *d*:

$$X_d = \frac{s_d - s_f(T_d)}{s_g(T_d) - s_f(T_d)} = \frac{s_c - s_f(T_d)}{s_{fg}(T_d)}.$$

We have given that  $T_c = T_d$ . Also  $s_d = s_a = s_f$  at  $300^\circ\text{C}$ . The quality at state *d* is

$$X_d = \frac{3.253 - 0.2966}{8.3706} = 0.353 < X_c$$

The enthalpy at state *d* is

$$\begin{aligned} h_d &= X_d h_g + (1 - X_d) h_f \\ &= 0.353 \times 2538.1 + 0.647 \times 83.96 = 950.8 \text{ kJ/kg.} \end{aligned}$$

The work of compression (pump work) is  $\Delta h_{ad} = h_a - h_d$ . Substituting the numerical values,

$$\Delta h_{ad} = 1344 - 950.8 = 393.3 \text{ kJ/kg.}$$

The ratio of turbine work to compression work (pump work) is  $\frac{w_{\text{turbine}}}{w_{\text{compression}}} = 2.75$

We can check the efficiency by computing the ratio of net work ( $w_{\text{net}} = w_{\text{turbine}} - w_{\text{compression}}$ ) to the heat input ( $T_a s_{fg}$ ). Doing this gives, not surprisingly, the same value as the Carnot equation.

**b)** Efficiency and work ratio for a cycle with adiabatic efficiencies of pump and turbine both equal to 0.8 (non-ideal components).

We can find the turbine work using the definition of turbine and compressor adiabatic efficiencies. The relation between the enthalpy changes is

$$w_{\text{turbine}} = h_b - h_{c'} = \eta_{\text{turbine}} (h_b - h_c) = \text{actual turbine work received.}$$

Substituting the numerical values, the turbine work per unit mass is 863.7 kJ/kg.

For the compression process, we use the definition of compressor (or pump) adiabatic efficiency:

$$\begin{aligned}
 w_{\text{compression}} &= h_{a'} - h_d = \frac{1}{(\eta_{\text{compression}})} (h_a - h_d) \\
 &= \text{actual work to achieve given pressure difference} \\
 &= 491.6 \text{ kJ/kg.}
 \end{aligned}$$

The value of the enthalpy at state  $a'$  is 1442.4 kJ/kg.

The thermal efficiency is given by

$$\begin{aligned}
 \eta_{\text{thermal}} &= \frac{w_{\text{net}}}{\text{heat input}} = \frac{w_{\text{turbine}} - w_{\text{compression}}}{\text{heat input}} \\
 &= \frac{(h_b - h_{c'}) - (h_{a'} - h_d)}{(h_b - h_{a'})}.
 \end{aligned}$$

Substituting the numerical values, we obtain for the thermal efficiency with non-ideal components,  $\eta_{\text{thermal}} = 0.285$ .

A question arises as to whether the Carnot cycle can be practically applied for power generation. The heat absorbed and the heat rejected both take place at constant temperature and pressure within the two-phase region. These can be closely approximated by a boiler for the heat addition process and a condenser for the heat rejection. Further, an efficient turbine can produce a reasonable approach to reversible adiabatic expansion, because the steam is expanded with only small losses. The difficulty occurs in the compression part of the cycle. If compression is carried out slowly, there is equilibrium between the liquid and the vapor, but the rate of power generation may be lower than desired and there can be appreciable heat transfer to the surroundings. Rapid compression will result in the two phases coming to very different temperatures (the liquid temperature rises very little during the compression whereas the vapor phase temperature changes considerably). Equilibrium between the two phases cannot be maintained and the approximation of reversibility is not reasonable.

Another circumstance is that in a Carnot cycle all the heat is added at the same temperature. For high efficiency we need to do this at a higher temperature than the critical point, so that the heat addition no longer takes place in the two-phase region. Isothermal heat addition under this circumstance is difficult to accomplish. Also, if the heat source and the cycle are considered together, the products of combustion which provide the heat can be cooled only to the highest temperature of the cycle. The source will thus be at varying temperature while the system requires constant temperature heat addition, so there will be irreversible heat transfer. In summary, the practical application of the Carnot cycle is limited because of the inefficient compression process, the low work per cycle, the upper limit on temperature for operation in the two-phase flow regime, and the irreversibility in the heat transfer from the heat source. In the next section, we examine the Rankine cycle, which is much more compatible with the characteristics of two-phase media and available machinery for carrying out the processes.

### **Muddy points**

What is the reason for studying two-phase cycles? (MP 2B.7)

How did you get thermal efficiency? How does a boiler work? (MP 2B.8)

### **2.B.4 Rankine Power Cycles**

A schematic of the components of a Rankine cycle is shown in Figure 2B-7. The cycle is shown on  $P$ - $v$ ,  $T$ - $s$ , and  $h$ - $s$  coordinates in Figure 2B-8.

The processes in the Rankine cycle are as follows:

- i)  $d \rightarrow e$ : Cold liquid at initial temperature  $T_1$  is pressurized reversibly to a high pressure by a pump. In this, the volume changes slightly.
- ii)  $e \rightarrow a$ : Reversible constant pressure heating in a boiler to temperature  $T_2$
- iii)  $a \rightarrow b$ : Heat added at constant temperature  $T_2$  (constant pressure), with transition of liquid to vapor
- iv)  $b \rightarrow c$ : Isentropic expansion through a turbine. The quality decreases from unity at point  $b$  to  $X_c < 1$
- v)  $c \rightarrow d$ : Liquid-vapor mixture condensed at temperature  $T_1$  by extracting heat.

*Figure 2B-7 Available from:*

*Moran and Shapiro, Fundamentals of Engineering Thermodynamics, 4th Ed. John Wiley & Sons.*

*Figure E8.1 p.328*

Figure 2B-7: Rankine power cycle with two-phase working fluid [Moran and Shapiro, *Fundamentals of Engineering Thermodynamics*]

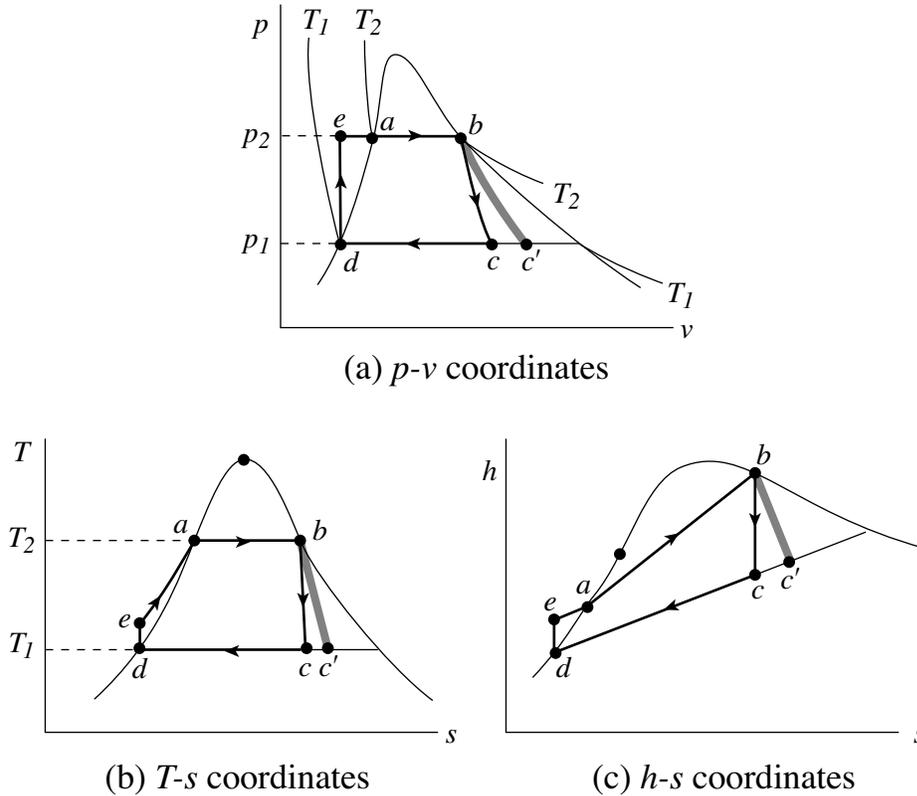


Figure 2B-8: Rankine cycle diagram. (a)  $P$ - $v$  coordinates, (b)  $T$ - $s$  coordinates, (c)  $h$ - $s$  coordinates. Stations correspond to those in Figure 2B-7

In the Rankine cycle, the mean temperature at which heat is supplied is less than the maximum temperature,  $T_2$ , so that the efficiency is less than that of a Carnot cycle working between the same maximum and minimum temperatures. The heat absorption takes place at constant pressure over  $eab$ , but only the part  $ab$  is isothermal. The heat rejected occurs over  $cd$ ; this is at both constant temperature and pressure.

To examine the efficiency of the Rankine cycle, we define a mean effective temperature,  $T_m$  in terms of the heat exchanged and the entropy differences:

$$q_H = T_{m2} \Delta s_2$$

$$q_L = T_{m1} \Delta s_1.$$

The thermal efficiency of the cycle is

$$\eta_{thermal} = \frac{T_{m2}(s_b - s_e) - T_{m1}(s_c - s_d)}{T_{m2}(s_b - s_e)}.$$

The compression and expansion processes are isentropic, so the entropy differences are related by

$$s_b - s_e = s_c - s_d.$$

The thermal efficiency can be written in terms of the mean effective temperatures as,

$$\eta_{thermal} = 1 - \frac{T_{m1}}{T_{m2}}.$$

For the Rankine cycle,  $T_{m1} \approx T_1$ ,  $T_{m2} < T_2$ . From Equation (B.4.1), we see not only the reason that the cycle efficiency is less than that of a Carnot cycle, but the direction to move in terms of cycle design (increased  $T_{m2}$ ) if we wish to increase the efficiency.

There are several features that should be noted about Figure 2B-8 and the Rankine cycle in general:

- i) The T-s and the h-s diagrams are not similar in shape, as they were with the perfect gas. The slope of a constant pressure reversible heat addition line is, as derived in Section 1.C.4,  $\left(\frac{\partial h}{\partial s}\right)_p = T$ . In the two-phase region, constant pressure means also constant temperature, so the slope of the constant pressure heat addition line is constant and the line is straight.
- ii) The effect of irreversibilities is represented by the dashed line from  $b$  to  $c'$ . Irreversible behavior during the expansion results in a value of entropy,  $s_{c'}$ , at the end state of the expansion that is higher than  $s_c$ . The enthalpy at the end of the expansion (the turbine exit) is thus higher for the irreversible process than for the reversible process, and, as seen for the Brayton cycle, the turbine work thus lower in the irreversible case.
- iii) The Rankine cycle is less efficient than the Carnot cycle for given maximum and minimum temperatures, but, as said earlier, it is more effective as a practical power production device.

### **Muddy points**

Where does degrees Rankine come from? Related to Rankine cycles? (MP 2B 9)

### **2.B.5: Enhancements of, and Effect of Design Parameters on, Rankine Cycles**

The basic Rankine cycle can be enhanced through processes such as superheating and reheat. Diagrams for a Rankine cycle with superheating are given in Figure 2B-9. The heat addition is continued past the point of vapor saturation, in other words the vapor is heated so that its temperature is higher than the saturation temperature associated with  $P_a (= P_b = P_c = P_d)$ . This

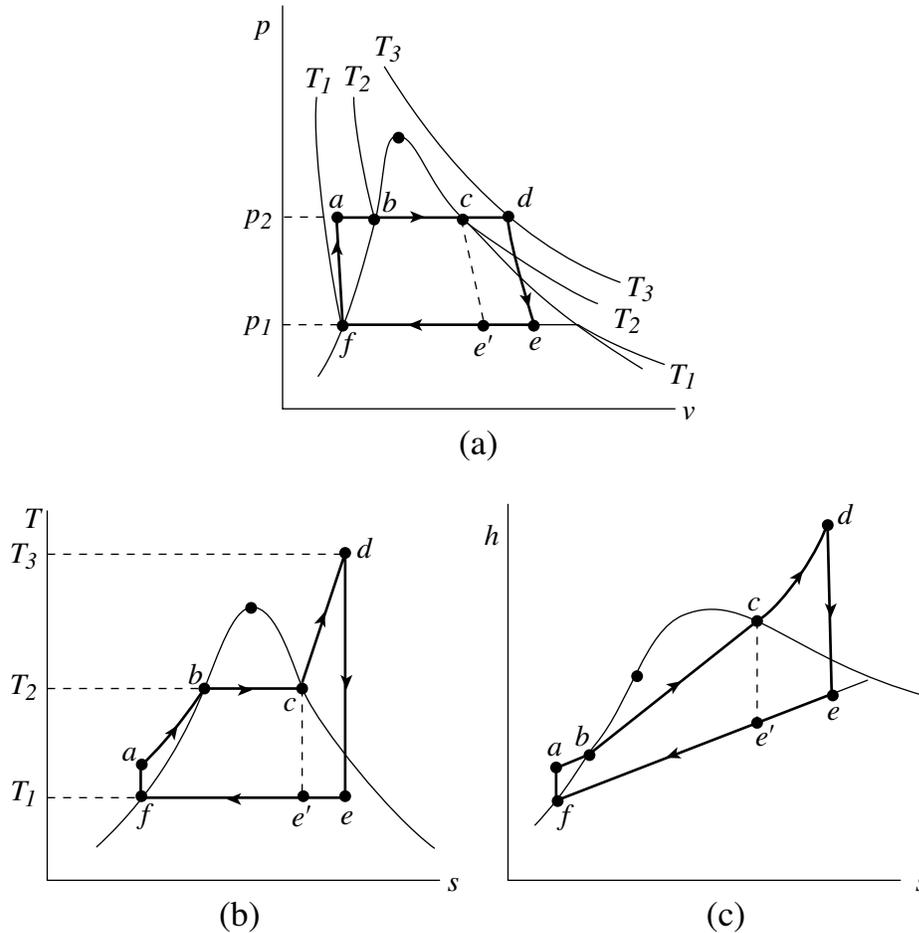


Figure 2B-9: Rankine cycle with superheating

does several things. First, it increases the mean temperature at which heat is added,  $T_{m2}$ , thus increasing the efficiency of the cycle (see Equation B.4.1). Second is that the quality of the two-phase mixture during the expansion is higher with superheating, so that there is less moisture content in the mixture as it flows through the turbine. (The moisture content at  $e$  is less than that at  $e'$ .) This is an advantage in terms of decreasing the mechanical deterioration of the blading.

The heat exchanges in the superheated cycle are;

$$\text{Along } abcd, \text{ which is a constant pressure (isobaric) process: } q_2 = h_d - h_a.$$

$$\text{Along } ef: q_1 = h_f - h_e, (< 0).$$

The thermal efficiency of the ideal Rankine cycle with superheating is

$$\eta_{thermal} = \frac{h_d - h_a - (h_e - h_f)}{h_d - h_a}$$

This can be expressed explicitly in terms of turbine work and compression (pump) work as:

$$\eta_{thermal} = \frac{h_d - h_e - (h_a - h_f)}{h_d - h_a}$$

Compared to the basic cycle, superheating has increased the turbine work, increased the mean temperature at which heat is received,  $T_{m2}$ , and increased the cycle efficiency.

A comparison of the Carnot cycle and the Rankine cycle with superheat is given in Figure 2B-10. The maximum and minimum temperatures are the same, but the average temperature at which heat is absorbed is lower for the Rankine cycle.

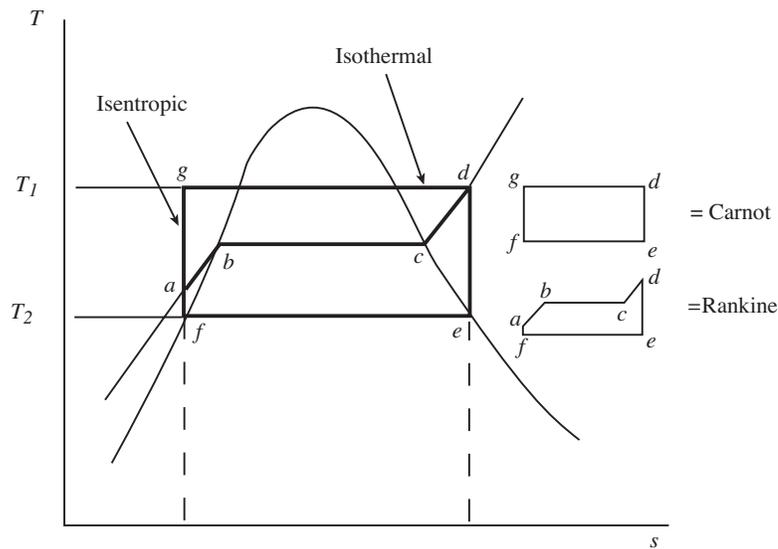


Figure 2B-10: Comparison of Rankine cycle with superheat and Carnot cycle

To alleviate the problem of having moisture in the turbine, one can heat again after an initial expansion in a turbine, as shown in Figure 2B-11, which gives a schematic of a Rankine cycle for space power application. This process is known as reheat. The main practical advantage of reheat (and of superheating) is the decrease in moisture content in the turbine because most of the heat addition in the cycle occurs in the vaporization part of the heat addition process.

We can also examine the effect of variations in design parameters on the Rankine cycle. Consider first the changes in cycle output due to a decrease in exit pressure. In terms of the cycle shown in Figure 2B-12, the exit pressure would be decreased from  $P_{exit}$  to  $(P_{exit} - dP_{exit})$ . The original cycle is  $abcdea$ , and the modified cycle is  $abfgha$ . The consequences are that the cycle work, which is



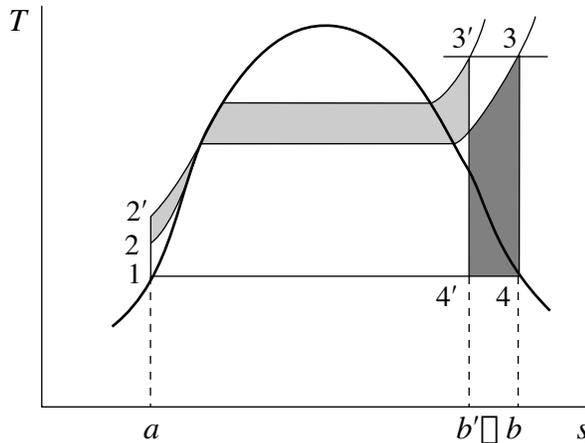


Figure 2B-13: Effect of maximum boiler pressure on Rankine cycle efficiency

### ***Muddy points***

Why do we look at the ratio of pump (compression) work to turbine work? We did not do that for the Brayton cycle. (MP 2B.10)

Shouldn't the efficiency of the super/re-heated Rankine cycle be larger because its area is greater? (MP 2B.11)

Why can't we harness the energy in the warm water after condensing the steam in a power plant? (MP 2B.12)

### **2.B.6 Combined Cycles in Stationary Gas Turbine for Power Production**

The turbine entry temperature in a gas turbine (Brayton) cycle is considerably higher than the peak steam temperature. Depending on the compression ratio of the gas turbine, the turbine exhaust temperature may be high enough to permit efficient generation of steam using the “waste heat” from the gas turbine. A configuration such as this is known as a gas turbine-steam combined cycle power plant. The cycle is illustrated in Figure 2B-14.

Figure 2B-14 available from:

Kerrebrock, *Aircraft Engines and Gas Turbines*, 2nd Ed. MIT Press. Figure 1.10, p14.

Figure 2B-14: Gas turbine-steam combined cycle [Kerrebrock, *Aircraft Engines and Gas Turbines*]

The heat input to the combined cycle is the same as that for the gas turbine, but the work output is larger (by the work of the Rankine cycle steam turbine). A schematic of the overall heat engine,

which can be thought of as composed of an upper and a lower heat engine in series, is given in Figure 2B-15. The upper engine is the gas turbine (Brayton cycle) which expels heat to the lower

*Figure 2B-15 drawn from:  
Lee Langston. Global Gas Turbine News, ASME International Gas Turbine  
Institute.*

Figure 2B-15: Schematic of combine cycle using gas turbine (Brayton cycle) and steam turbine (Rankine cycle) [Langston]

engine, the steam turbine (Rankine cycle). The overall efficiency of the combined cycle can be derived as follows. We denote the heat received by the gas turbine as  $Q_{in}$  and the heat rejected to the atmosphere as  $Q_{out}$ . The heat out of the gas turbine is denoted as  $Q_1$ . The hot exhaust gases from the gas turbine pass through a heat exchanger where they are used as the heat source for the two-phase Rankine cycle, so that  $Q_1$  is also the heat input to the steam cycle. The overall combined cycle efficiency is

$$\eta_{CC} = \frac{W}{Q_{in}} = \frac{W_B + W_R}{Q_{in}},$$

where the subscripts refer to combined cycle (CC), Brayton cycle (B) and Rankine cycle (R) respectively.

From the first law, the overall efficiency can be expressed in terms of the heat inputs and heat rejections of the two cycles as (using the quantity  $|Q_1|$  to denote the magnitude of the heat transferred):

$$\eta_{CC} = \frac{Q_{in} - |Q_1| + (|Q_1| - Q_{out})}{Q_{in}} = \left[ 1 - \frac{|Q_1|}{Q_{in}} \right] + \left[ 1 - \frac{Q_{out}}{|Q_1|} \right] \left( \frac{|Q_1|}{Q_{in}} \right).$$

The first square bracket term on the right hand side is the Brayton cycle efficiency,  $\eta_B$ , the second is the Rankine cycle efficiency,  $\eta_R$ , and the term in parentheses is  $(1 - \eta_B)$ . The combined cycle efficiency can thus be written as,

$$\eta_{CC} = \eta_B + \eta_R - \eta_B \eta_R; \text{ Combined cycle efficiency.} \quad (\text{B.6.1})$$

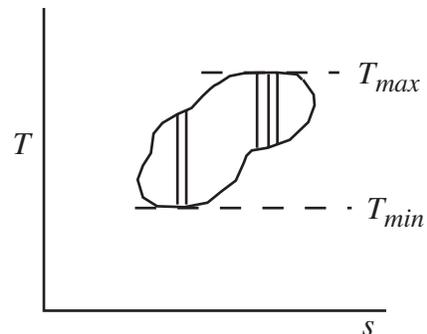
Equation (B.6.1) gives insight into why combined cycles are so successful. Suppose that the gas turbine cycle has an efficiency of 40%, which is a representative value for current Brayton cycle gas turbines, and the Rankine cycle has an efficiency of 30%. The combined cycle efficiency would be 58%, which is a very large increase over either of the two simple cycles. Some representative efficiencies and power outputs for different cycles are shown in Figure 2B-16.

*Figure 2B-16 drawn from:  
Dominic Bartol, Keynote talk, 1997 International Gas Turbine Institute (IGTI) Turbo Expo.*

Figure 2B-16: Comparison of efficiency and power output of various power products [Bartol (1997)]

2.B.7 Some Overall Comments on Thermodynamic Cycles

- i) There are many different power and propulsion cycles, and we have only looked at a few of these. Many other cycles have been devised in the search for ways to increase efficiency and power in practical devices.
- ii) We can view a given cycle in term of elementary Carnot cycles, as sketched in the figure on the right. This shows that the efficiency of any other cycle operating between two given temperatures will be less than that of a Carnot cycle.



iii) If we view the thermal efficiency as

$$\eta_{thermal} = 1 - \frac{(T_{heat\ rejected})_{Average}}{(T_{heat\ absorbed})_{Average}},$$

(derived in Section 2.B.4), this means that we should accept heat at a high temperature and reject it at a low temperature for high efficiency. This objective must be tempered by considerations of practical application.

- iv) The cycle diagrams in  $T$ - $s$  and  $h$ - $s$  coordinates will only be similar if the working medium is an ideal gas. For other media (for example, a two-phase mixture) they will look different.
- v) Combined cycles make use of the rejected heat from a “topping” cycle as heat source for a “bottoming” cycle. The overall efficiency is higher than the efficiency of either cycle.

## Muddiest Points on Part 2B

2B.1 For the vapor dome, is there vapor and liquid inside the dome and outside is it just liquid or just gas? Is it interchangeable? Is it true for the plasma phase?

The vapor dome separates the two-phase region from the single-phase region. Inside, we have a mixture of liquid and vapor. The peak of the vapor dome is called the critical point. The left-hand side leg of the vapor dome (from the critical point) is called the saturated liquid line along which the quality  $x$  is zero (purely liquid). The right-hand side leg is denoted the saturated vapor line and the quality  $x$  is one (purely vapor). For further details see the notes.

Heating of a solid or liquid substance leads to phase transition to a liquid or gaseous state, respectively. This takes place at a constant temperature for a given pressure, and requires an amount of energy known as latent heat. On the other hand, the transition from a gas to an **ionized gas**, i.e., **plasma**, is not a phase transition, since it occurs gradually with increasing temperature. During the process, a molecular gas dissociates first into an atomic gas which, with increasing temperature, is ionized. Resulting plasma consists of a mixture of neutral particles, positive ions (atoms or molecules that have lost one or more electrons), and negative electrons.

2B.2 What is  $h_{fg}$ ? How do we find it?

The quantity  $h_{fg}$  represents the specific enthalpy change between the liquid and vapor phases of a substance at constant temperature, and thus constant pressure, and thus constant temperature. It is therefore the heat input, per unit mass, to vaporize a kilogram of liquid. See the notes, Section 2.B.2.

2B.3 Reasoning behind the slopes for  $T=const$  lines in the  $P$ - $V$  diagram.

The slope of an isotherm in the gaseous phase (to the right of the vapor dome) is similar to the slope we found for the isotherm of an ideal gas ( $PV=const$ ). Inside the vapor dome pressure and temperature are directly related to one another ( $P = P(T)$ , vapor pressure curve) such that an isotherm is a horizontal line (isobar inside the vapor dome). In the liquid phase the isotherms are very steep lines, because for liquids the volume is about constant (very low compressibility).

2B.4 For a constant pressure heat addition, why is  $q = \Delta h$ ?

The combined first and second law is  $dh = Tds + vdP$ . For a *reversible* constant pressure process,  $dP = 0$ , and the heat input,  $dq$ , is  $Tds$ . Thus for a reversible constant pressure process, the answer is yes.

For an irreversible process we can say from the steady flow energy equation:  $dq = dh_t = dh + cdc$ . For a steady flow, the one-dimensional momentum equation is

$$cdc = -\frac{dP}{\rho} + \frac{F_{viscous}}{\rho},$$

where  $F_{viscous}$  represents the viscous forces in an irreversible flow. Combining these two expressions, and using  $dP = 0$  (the condition of constant pressure) gives

$$dq = dh + cdc = dh + \frac{F_{viscous}}{\rho}.$$

Without going into any detail concerning the form of the viscous forces, this equation shows that the equality between heat input and enthalpy change does not hold for general irreversible flow processes at constant pressure.

### 2B.5 What is latent heat?

Latent heat is a term for the enthalpy change needed for vaporization.

### 2B.6 Why is U a function of x?

Inside the vapor dome we have a mixture of liquid and vapor. The internal energy  $U$  of the system (liquid and vapor) can be expressed in terms of the mass in each phase and the specific internal energy  $u$  of each phase as,

$$U = u_f m_f + u_g m_g.$$

Introducing the quality  $x$  as the fraction of the total mass in the vapor phase  $x = m_g/(m_g+m_f)$  we can write

$$U = (1 - x) u_f + x u_g.$$

Since the specific energy of the saturated liquid and the saturated vapor are functions of temperature the internal energy  $U$  of the two-phase system is a function of  $x$  and  $T$  (see also notes on page 2B-6).

### 2B.7 What is the reason for studying two-phase cycles?

Their immense practical utility in a number of industrial devices and their intrinsic interest as applications of the basic principles.

### 2B.8 How did you get thermal efficiency? How does a boiler work?

The thermal efficiency is, as previously, the net work done divided by the heat input. Using the first law for a control volume we can write both of these quantities in terms of the enthalpy at different states of the cycle.

For the steam cycles discussed in class, a boiler is a large (as in the viewgraph of the Mitsubishi power plant) structure with a lot of tubes running through it. The water (or whatever medium is used in the cycle) runs through the tubes. Hot gases wash over the

outside of the tubes. The hot gases could be from a combustor or from the exit of a gas turbine.

2B.9 Where does degrees Rankine come from? Related to Rankine cycles?

I think the answer is yes, although I do not know for sure. If so, this is the same Rankine who has his name on the Rankine-Hugoniot conditions across a shock wave.

2B.10 Why do we look at the ratio of pump (compression) work to turbine work? We did not do that for the Brayton cycle.

If the ratio of compression work to turbine work were close to unity for an ideal cycle, small changes in component efficiencies would have large effects on cycle efficiency and work. For the Rankine cycle this is not true. (The effect of pump efficiency on Rankine cycle efficiency is clearly small in the class example.) For the Brayton cycle, where the *net work* is the difference of two numbers which are of (relatively) similar sizes, the effect of compressor and turbine efficiency on cycle efficiency can be much larger.

I used the word “sensitive” and the meaning was that the cycle performance responded strongly to changes in the compressor and turbine behavior.

2B.11 Shouldn't the efficiency of the super/re-heated Rankine cycle be larger because its area is greater?

The area enclosed by an ideal cycle in a T-s diagram is the net work done, but it does not tell you about efficiency. We saw that for example when we looked at the Brayton cycle for the condition of maximum work, rather than maximum efficiency (Section 1.A.4 in the notes).

2B.12 Why can't we harness the energy in the warm water after condensing the steam in a power plant?

Let's assume the temperature of the warm water after condensing the steam is at a temperature of about 30 to 40 degrees C. If we consider running a heat engine between this heat reservoir (say 35 degrees C) and the surroundings at 20 degrees C, we would get an ideal thermal efficiency of about 5%. In other words, the available useful work is relatively small if we considered the lower heat reservoir to be the surroundings. In general, there is a property that only depends on state variables called **availability**. The change in availability gives the maximum work between two states, where one state is referred to the surroundings (dead state).