16.422

Human Supervisory Control

Classical Decision Theory & Bayes' Theorem

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Decision Theory & Supervisory Control

- Two broad areas of decision theory
  - Normative
    - Prescriptive
  - Descriptive
- Why & how do we make decisions?
  - How does technology support or detract from "optimal" decision making
- Informing Design

- Normative: How decisions should be made
- Prescriptive: How decisions can be made, given human limitations
- Descriptive: How decisions are made

Elements of Decision Making

- The rational decision making process
  - Define the problem
  - Information gathering
  - Identify alternatives
  - Evaluate alternatives
  - Select and implement decision
- Why decisions often go wrong:
  - Certainty vs. uncertainty
  - Bounded rationality
  - Nonlinearity
  - Habits & heuristics
  - Path of least resistance

Classic Decision Theory

- Maximizing expected value of the outcome
  - Primary assumption of rationality
- Mathematical models of human decision making
- Following assumptions are made about how people make decisions:
  - All alternatives are considered
  - Information acquisition is perfect
  - Probabilities are calculated correctly
- Problems:
  - Humans are not rational decision makers
  - No universal agreement on the worth associated with various outcomes.

Basic Concepts in Decision Analysis

- State: A description of the world
- Events: A set of states $\{S_1, S_2, \ldots, S_j\}$
- Consequences: A set of states
- Acts: Results from decisions

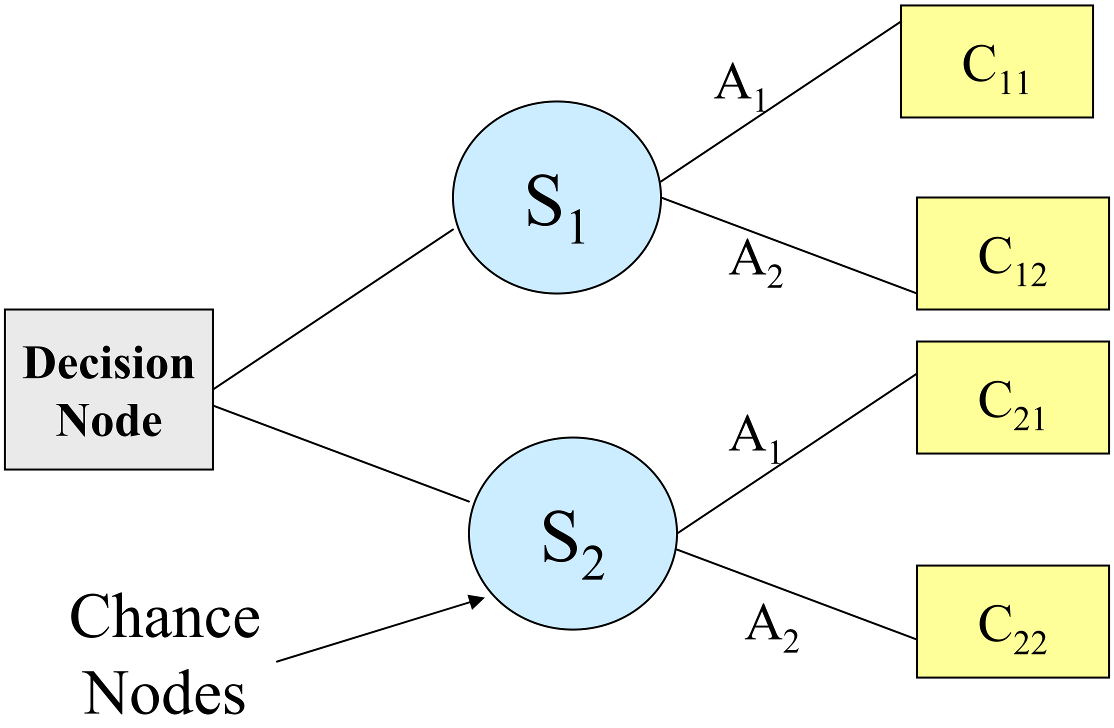
	S_1	S_2
Act 1 (A_1)	C_{11}	C_{12}
Act 2 (A_2)	C_{21}	C_{22}

Basic Terminology

- Ordering of preferences
  - Alternatives can be quantified and ordered
  - A > B, A preferred to B
  - A = B, A is equivalent to B
  - $A \geq B$, B is not preferred to A
- Transitivity of preference
  - if $A_1 \geq A_2$, & $A_2 \geq A_3$, then $A_1 \geq A_3$,

Decision Trees

- Decisions over time and/or events



Decision Making Under Certainty

- Each alternative leads to one and only one consequence
  - Consequences are known
- Lexicographic ordering
- Dominance
- Satisficing
- Maximin
- Minimax

Lexicographic Ordering

- All options are first compared in terms of the criterion deemed most important.
  - If there is a unique best performing option, that option is selected as the most preferred.
- In case of a tie, the selection process moves to the second ranked criterion
  - Seeks the remaining option which scores best on the second criterion.
- In case of a tie on the second criterion, the process is repeated for options tying in terms of both the first and second ranked criteria,
  - And so on until a unique option is identified or all criteria have been considered.

Dominance

- Effective for both quick qualitative and quantitative comparisons
- In the real world, solutions rarely are outwardly dominant

	S_1	S_2	S_3	S_4
A_1	1	0	3	0
A_2	2	0	3	2
A_3	2	-1	3	0

Satisficing

- Otherwise known as Minimum Aspiration
- Select any A_i over all events j such that

  $C_{ij} \geq$ aspiration level
- Cease to search for alternatives when you find an alternative whose expected utility or level of preference satisfaction exceeds some previously determined threshold.
- Stopping rule
- Is this in keeping with the rational approach to decision making?

Maximin

- Select any A_i over all events j such that you minimize the maximum loss
  - Maximum of the (row) minima.
  - Conservative approach but overly pessimistic

	S_1	S_2	S_3
A_1	15	3	-6
A_2	9	4	-2
A_3	3	2	1

Maximin

- Select any A_i over all events j such that you minimize the maximum loss
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	S_1	S_2	S_3	**Row min**
A_1	15	3	-6	-6
A_2	9	4	-2	-2
A_3	3	2	1	1

Minimax (Regret)

- Avoid regrets that could result from making a non-optimal decision.
- Regret: the opportunity loss if A_i is chosen
  - Opportunity loss is the payoff difference between the best possible outcome under S_j and the actual outcome from choosing A_i.
  - Convert payoff matrix to opportunity loss (regret) table

	S_1	S_2	S_3
A_1	15	3	-6
A_2	9	4	-2
A_3	3	2	1
Best	15	4	1

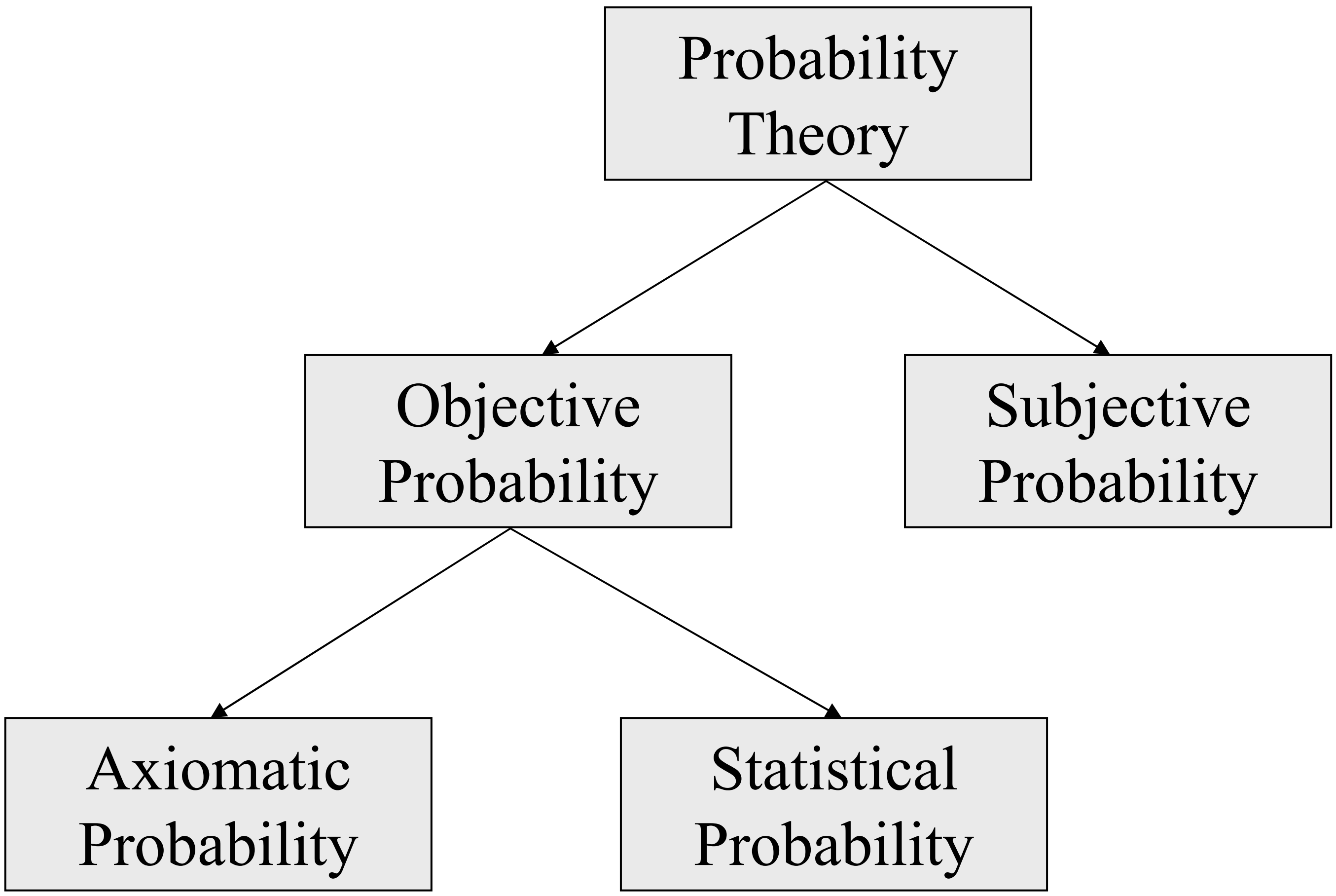
	S_1	S_2	S_3
A_1	0	1	7
A_2	6	0	3
A_3	12	2	0

Maximin v. Minimax

- Minimax contains more problem information through opportunity losses (e.g. actual monetary losses plus unrealized potential profits)
  - Still conservative but more middle-of-the-road

	S_1	S_2	S_3	Maximin	Minimax
A_1	15	3	-6	-6	7
A_2	9	4	-2	-2	6
A_3	3	2	1	1	12

The Uncertain State of the World



Decision Making Under Risk

- Each alternative has several possible consequences
  - The probability of occurrence for each consequence, C, is known.
- Expected value: $E(A_i) = \Sigma_j \; p_j \, (C_{ij})$

	S_1	S_2	S_3
$P(S_j)$	.1	.6	.3
A_1	15	3	-6
A_2	9	4	-2
A_3	3	2	1

Decision Making Under Uncertainty

- Alternatives are known
  - Consequences are not known
  - Probability of outcomes are not known
- Minimax/Maximin can still be used.
- Utility a key concept

Other Driver

		Swerve	Don't Swerve
You	Swerve	0,0	0, 10
	Don't Swerve	10, 0	-1000, -1000

Utility Theory

- Utility theory is an attempt to infer subjective value, or utility, from choices.
  - Applies to both decision making under risk & decision making under uncertainty
- Two types
  - Expected utility
    - Same as EV but utility is the value associated with some outcome, not necessarily monetary.
    - Subjective Expected Utility (SEU)
      - Subjective focuses on decision making behavior
        - » Risk neutral/adverse/seeking
  - Multi-attribute utility
    - Multiple objective extension of SEU
    - Interval scale is critical

MAUT Example

- Want to buy a car based on price, fuel economy, & reliability.

	Price	Fuel	Reliability	
Attribute Weight	.5	.8	.7	
BMW Z3	20	95	100	156
Geo Metro	100	99	50	164.2
Subaru Outback	60	85	98	166.6

- Major drawback: Assumption of rationality

Uncertainty, Conditional Probability, & Bayes' Theorem

- Bayes' Theorem

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid \overline{A})P(\overline{A})}$$

  - P(A) - prior probability of A
  - P(A|B) - posterior probability
  - P(B|A), for a specific value of B, is the likelihood function for A
  - P(B) - the prior probability of B
  - $P(A|B) \neq P(B|A)$

Absolute versus conditional

P(Tails) = .5 vs P(heart|red) = .5

Bayesian Networks

- Bayes nets are models which reflect the states of some part of a world and describe how those states are related by probabilities.
  - Causal chains
- If it might rain today and might rain tomorrow, what is the probability that it will rain on both days?
  - Not independent events with isolated probabilities.
  - Joint probabilities
- Useful for monitoring & alerting systems
- Problems
  - Can't represent recursive systems in a straightforward manner
  - Acquiring probabilities
  - The number of paths to explore grows exponentially with each node

Good paper - **Display of Information for Time-Critical (1995)** Eric Horvitz, Matthew Barry

Human probability estimation is unreliable for infrequent events (tails)

References

- *Choices: An Introduction to Decision Theory*, Michael D. Resnick (1987)
- *Making Hard Decisions: An Introduction to Decision Analysis, 2nd ed.*, Robert T. Clemen, (1996)
- Decision Analysis, http://groups.msn.com/DecisionModeling/decisionanalysis1.msnw
- Bayes Net Tutorial

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