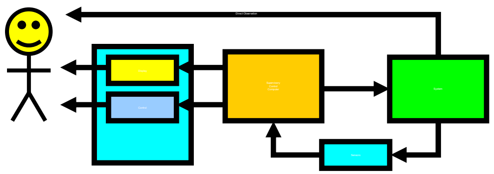
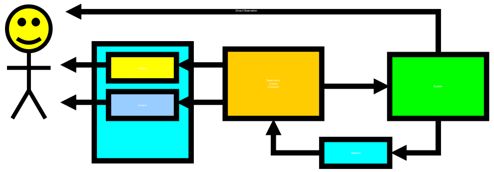


16.422 Information & Signal Detection Theory

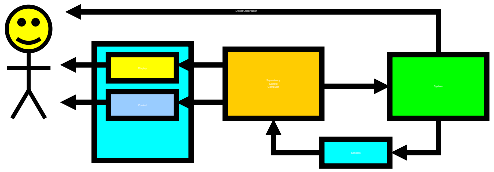
Prof. R. John Hansman

Acknowledgements to Profs Tom Sheridan and Jim Kuchar whose notes are the core of this lecture



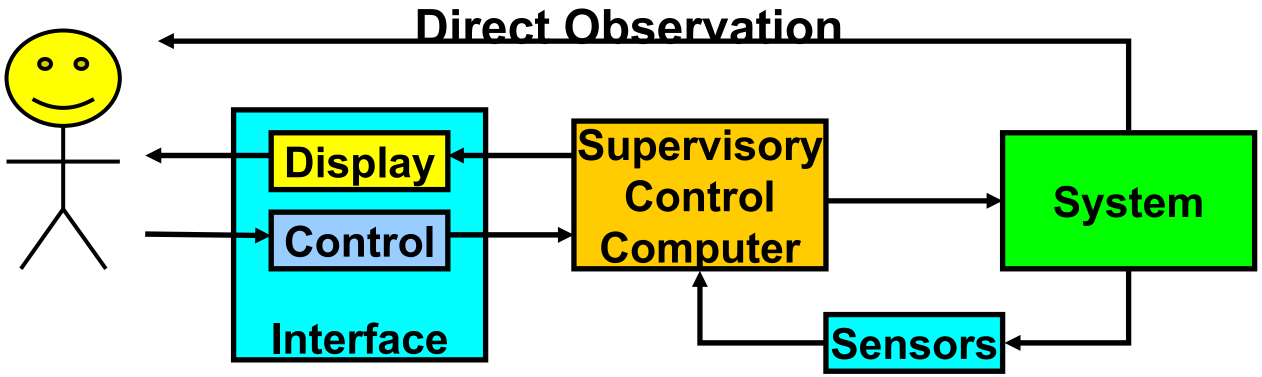
Outline

- **Information Theory**
- **Signal Detection Theory**
- **Alerting Introduction**

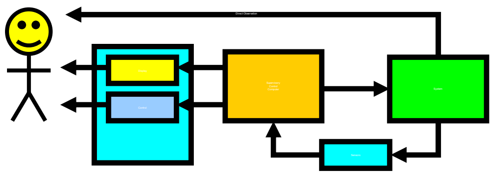


Information Theory

- **What is information?**
- **Control Theoretic View**
  - ☐ Lines in Control Block Diagram



- **Bayesian View**
  - ☐ Information is something which reduces uncertainty in a world model



Bayes Theory

H = Hypothesis, D = Data

$$p(H \mid D) = p(D \mid H)\, p(H)/p(D)$$

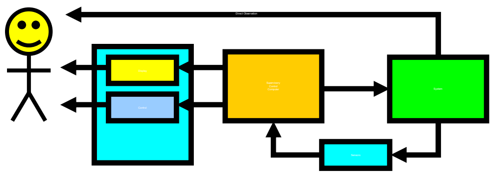
With new data

$$p(H \mid D1,D2) = p(D2 \mid H)\, [p(D1 \mid H)\, p(H)/p(D1)\; /p(D2)$$

With 2 hypotheses

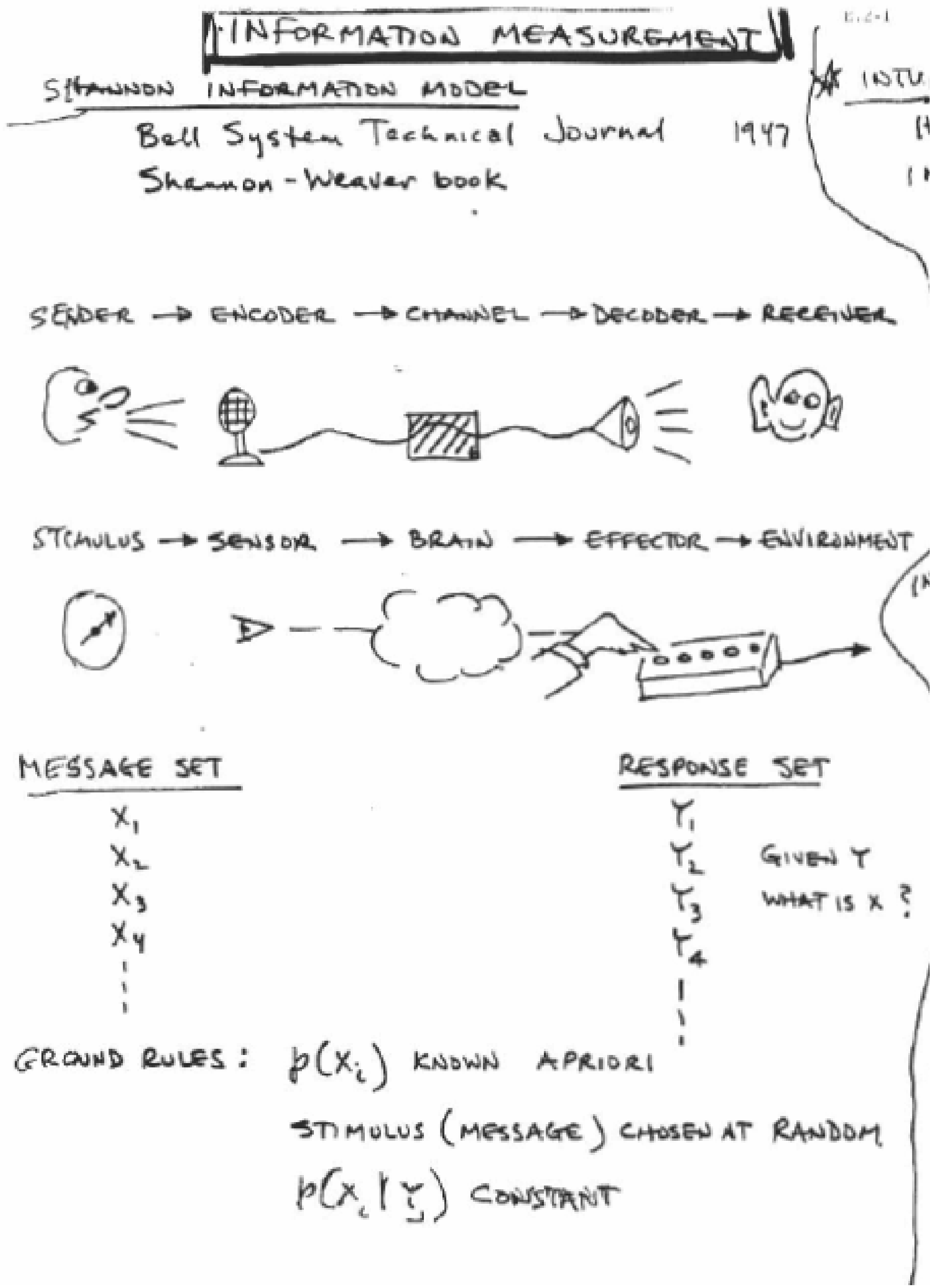
$$\frac{p(H1 \mid D1,D2)}{p(H2 \mid D1,D2)} = \frac{p(D2 \mid H1)}{p(D2 \mid H2)}\;\frac{p(D1 \mid H1)}{p(D1 \mid H2)}\;\frac{p(H1)}{p(H2)}$$

Posterier odds ratio prior odds ratio



- **Shannon Information Theory**
  - ☐ Bell Labs
  - ☐ Telephony

- **Tom Sheridan Notes**



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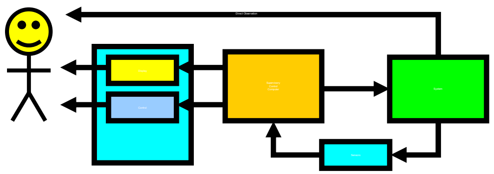
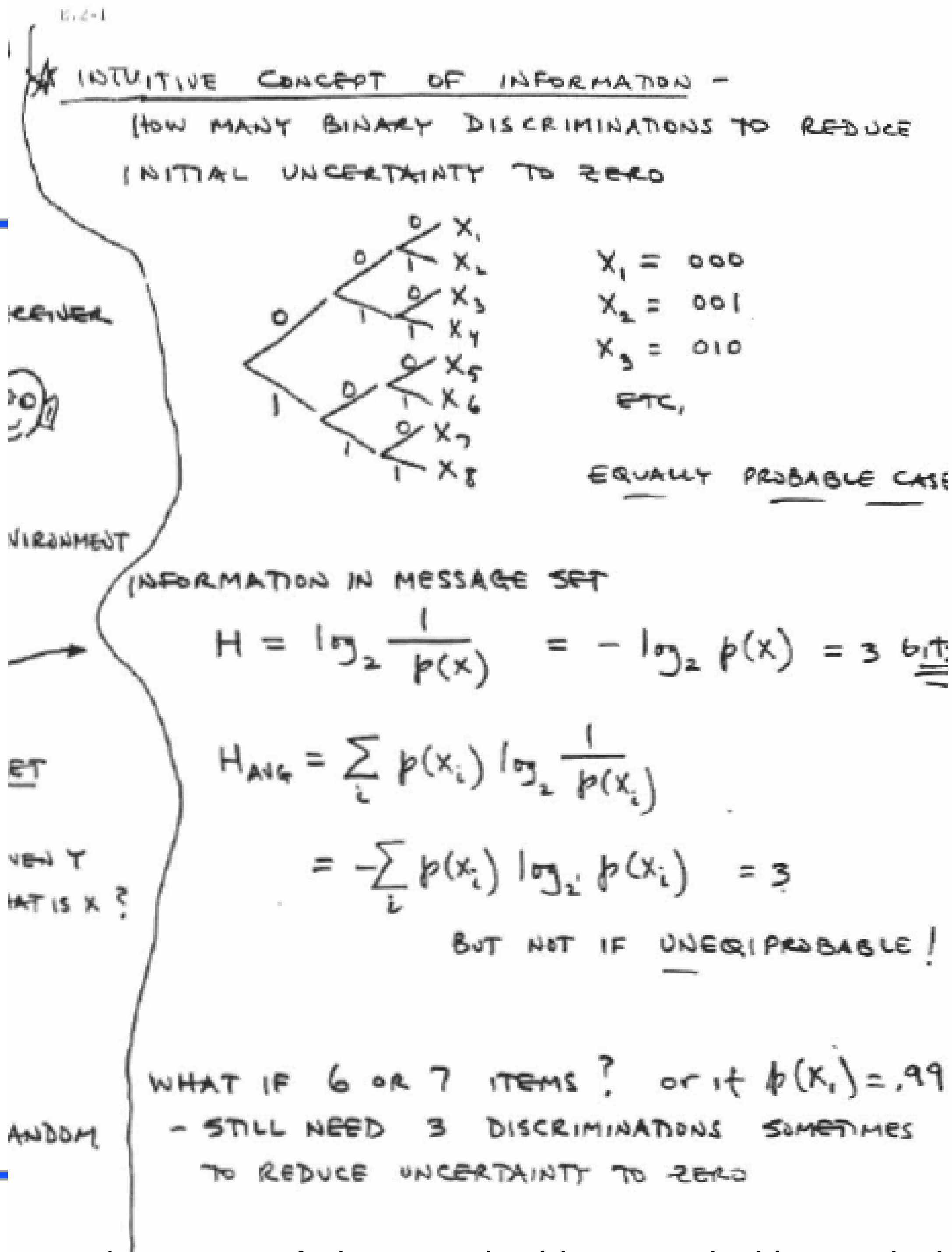


- **Bit view of Information**
  - ☐ # of bits to disambiguate
  - ☐ Bit = binary discrimination
  - ☐ Drive uncertainty to zero



INTUITIVE CONCEPT OF INFORMATION –
(HOW MANY BINARY DISCRIMINATIONS TO REDUCE INITIAL UNCERTAINTY TO ZERO

X_1 = 000
X_2 = 001
X_3 = 010
ETC.

EQUALLY PROBABLE CASE

INFORMATION IN MESSAGE SET

$$H = \log_2 \frac{1}{p(x)} = -\log_2 p(x) = 3 \text{ bit}$$

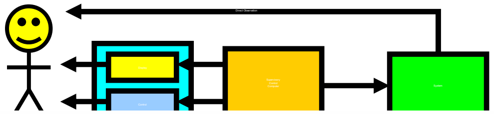
$$H_{AVG} = \sum_i p(x_i) \log_2 \frac{1}{p(x_i)}$$

$$= -\sum_i p(x_i) \log_2 p(x_i) = 3$$

BUT NOT IF UNEQUIPROBABLE !

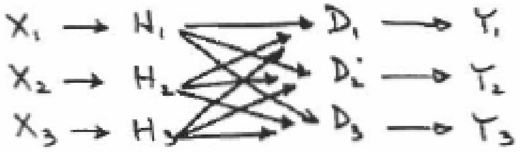
WHAT IF 6 OR 7 ITEMS ? or if $p(x_1) = .99$
– STILL NEED 3 DISCRIMINATIONS SOMETIMES TO REDUCE UNCERTAINTY TO ZERO

(Courtesy of Thomas Sheridan. Used with permission.)



Info Transmission
H = information, D = Data

HOW TO MEASURE INFO. TRANSMITTED?



SEE Y_j WHAT IS X_i?

KNOWN APRIORI $p(H_i)$ IMPROVED TO $p(H_i \mid D_j)$

$p(X_i)$ IMPROVED TO $p(X_i \mid Y_j)$

CRITERIA FOR A GOOD MEASURE

1) WHAT MEASURE $T = f\left[p(X_i), p(X_i \mid Y_j)\right]$

2) INFO. GAINED BY KNOWING Y_1 ADDS TO INFO. GAINED BY KNOWING Y_2, ETC.

$$T\left[p(X_a), p(X_a \mid Y_k)\right] + T\left[p(X_b), p(X_b \mid Y_L)\right] = T\left[p(X_a X_b), p(X_a X_b \mid Y_k Y_L)\right]$$

T MUST BE OF FORM $\left[\log p(X_i \mid Y_j) - \log p(X_i)\right]$

$$= \log \frac{p(X_i \mid Y_j)}{p(X_i)}$$

SIMPLEST EXAMPLES

$p(Y_j)$ above columns (.5, .5); $p(X_i)$ beside rows (.5, .5):

$p(X_i)$ \ $p(Y_j)$	.5	.5
.5	.5	0
.5	0	.5

$p(X_i Y_j)$ JOINT

SAME AS

$p(X_i)$ \ $p(Y_j)$	.5	.5
.5	1	0
.5	0	1

$p(X_i \mid Y_j)$ CONDITIONAL

$$H(X) = \sum_i p(X_i) \log_2 \frac{1}{p(X_i)}$$

$$= .5(\log_2 2) + .5(\log_2 2) = 1$$

$H(Y)$ = SAME

T:

$X_1 \rightarrow Y_1$ $\quad T_1 = \log_2 p(X_1 \mid Y_1) - \log_2 p(X_1)$

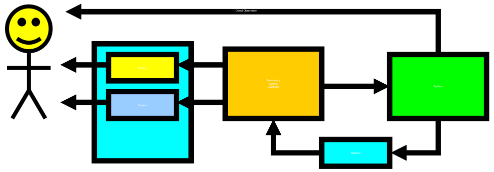
$= \log_2(1) - \log_2(.5) = 1$

$X_2 \rightarrow Y_2$ $\quad T_2$ = SAME $= 1$

$$T_{AVE} = .5(T_1) + .5(T_2) = 1$$

PERFECT INFO. TRANSMISSION

(Courtesy of Thomas Sheridan. Used with permission.)



$p(Y_j)$: .5, .5

$p(X_i)$: .5, .5

	.5	.5
.5	.25	.25
.5	.25	.25

$p(X_i Y_j)$ JOINT

SAME AS

$p(Y_j)$: .5, .5

$p(X_i)$	.5	.5
.5	.5	.5
.5	.5	.5

$p(X_i | Y_j)$

TRANSMIT
- $X_1 \rightarrow Y_1$ $\quad T = 1 - 1 = 0$
- $X_2 \rightarrow Y_2$ $\quad T = 1 - 1 = 0$
- $X_1 \rightarrow Y_2$ $\quad T = 1 - 1 = 0$
- $X_2 \rightarrow Y_1$ $\quad T = 1 - 1 = 0$

NO INFO. TRANSMITTED EVEN THO $H(X) = 1$

* IN GENERAL TRANSMITTED INFO

$$T(X:Y) = \sum_i \sum_j p(X_i Y_j) \log_2 \frac{p(X_i | Y_j)}{p(X_i)}$$

JOINT

MULTIPLY NUMERATOR AND DENOMINATOR BY $p(Y_j)$. THEN

$$T(X:Y) = \sum_i \sum_j p(X_i Y_j) \log_2 p(X_i Y_j) \quad \text{MINUS JOINT INFO} \quad -H(XY)$$

$$- \sum_i \sum_j p(X_i Y_j) \log_2 p(X_i) \quad \text{INPUT INFO} \quad H(X)$$

$$- \sum_i \sum_j p(X_i Y_j) \log_2 p(Y_j) \quad \text{OUTPUT INFO} \quad H(Y)$$

$$= H(X) + H(Y) - H(XY)$$

T = INPUT + OUTPUT − JOINT

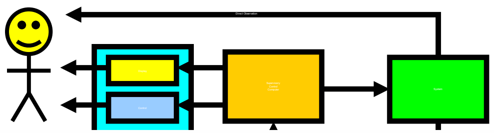
IF X AND Y UNCORRELATED, FROM GENL. EQN.

$$p(X_i | Y_j) = p(X_i) \longrightarrow T = 0$$

IF X AND Y PERFECTLY CORRELATED

$$p(X_i Y_j) = p(X_i) = p(Y_j)$$

$$T(X:Y) = H(X) = H(Y)$$



CAN ALSO REWRITE GENL. EQN.

$$T(X;Y) = \sum_i \sum_j p(X_i, Y_j) \log p(X_i \mid Y_j)$$

MINUS EQUIVOCATION $-H(X|Y)$

$$- \sum_i p(X_i) \log p(X_i)$$

INPUT INFO $H(X)$

$$= H(X) - H(X|Y)$$

T = INPUT − EQUIVOCATION

CAN ALSO REWRITE GENL. EQN.
MULT NUM. + DENOM. BY $p(Y_j)/p(X_i)$

$$T(X:Y) = \sum_i \sum_j p(X_i, Y_j) \log \frac{p(X_i, Y_j)/p(X_i)}{p(Y_j)}$$

$$= \sum_i \sum_j p(X_i, Y_j) \log p(Y_j \mid X_i)$$

MINUS NOISE $-H(Y|X)$

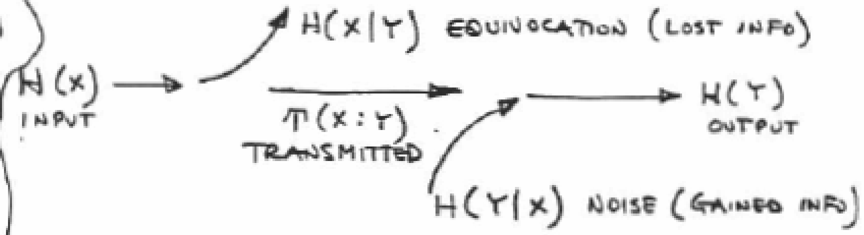
$$- \sum_i \sum_j p(X_i) \log p(Y_j)$$

OUTPUT INFO $H(Y)$

$$= H(Y) - H(Y|X)$$

T = OUTPUT − NOISE

COMBINE IN DIAGRAM



REMEMBER − INFO IS UNCERTAINTY!
(TO BE REDUCED)

$$H(X) = \sum_i p_i \log \frac{1}{p_i} \quad \text{INPUT INFO}$$

$$H(Y) = \sum_j p_j \log \frac{1}{p_j} \quad \text{OUTPUT INFO}$$

$$H(X|Y) = \sum_i \sum_j p(X_i, Y_j) \log \frac{1}{p(X_i \mid Y_j)} \quad \text{EQUIVOCATION}$$

$$H(Y|X) = \sum_i \sum_j p(X_i, Y_j) \log \frac{1}{p(Y_j \mid X_i)} \quad \text{NOISE}$$

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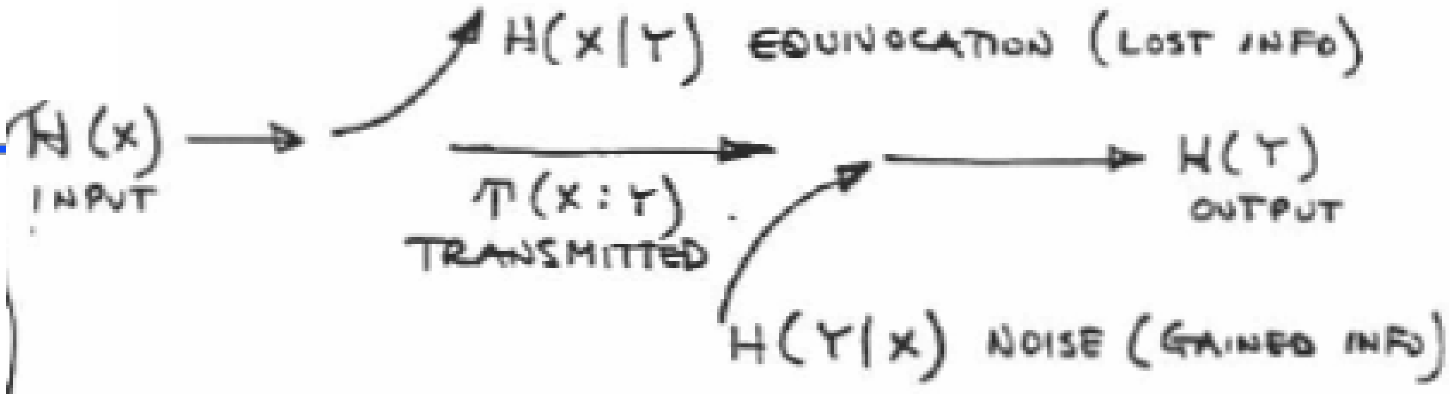


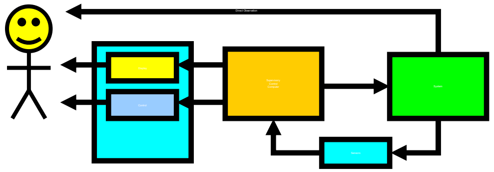
REMEMBER – INFO IS UNCERTAINTY !
(TO BE REDUCED)

$$H(X) = \sum_i p_i \log \frac{1}{p_i}$$ INPUT INFO.

$$H(Y) = \sum_j p_j \log \frac{1}{p_j}$$ OUTPUT INFO.

$$H(X|Y) = \sum_i \sum_j p(X_i Y_j) \log \frac{1}{p(X_i | Y_j)}$$ EQUIVOCATION.

$$H(Y/X) = \sum_i \sum_j p(X_i Y_j) \log \frac{1}{p(Y_j | X_i)}$$ NOISE



Information Value

Assessing the Benefit of Providing Information

Average benefit from having perfect information (indicative of true state x), where one can select u to optimize for each x, is

$$V_P = \Sigma_i \, p(x_i) \, \max_j [V(x_i u_j)].$$

The best one can ever do from just knowing the a priori statistics is

$$V_0 = \max_j [\Sigma_i \, p(x_i) \; V(x_i u_j)].$$

The difference (V_P - V_0) is the net benefit of having the inform-ation if it were free.

As a first approximation, the cost (in complexity of hardware/ software) of providing and/or of accessing the information (in computer or human time /distraction) is

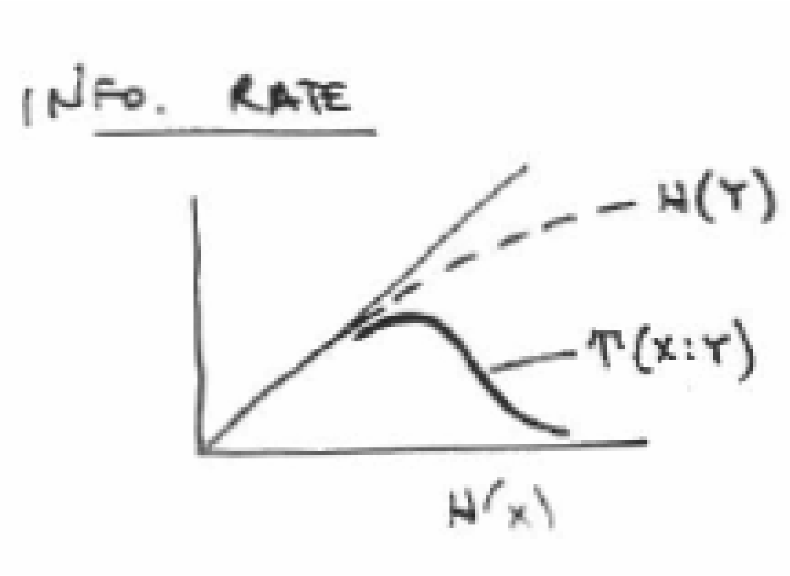
$$C = \Sigma_i \, p(x_i) \, c(x_i) \log_2[1/p(x_i)]$$

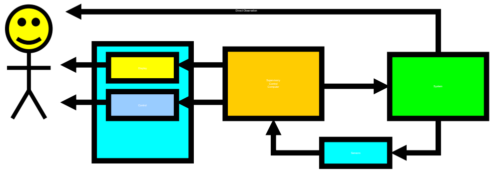
Then should provide the information IFF (V_P - V_0) < C

Information may become imperfect, e.g., state x changes with probability q to each other state before u takes effect. Then

$$V_I = \Sigma_i \, p(x_i) \, .\{(1-q) \max_j [V(x_i u_j)] + q \; \Sigma_{k \neq i} [V(x_{k \neq i} \, u_j \text{ selected for } i)]\}$$

Then should provide information IFF (V_I - V_0) < C


INFO. RATE
H(Y)
T(X:Y)
H(x)



Information Bandwidth

- **Information Rate**
  - ☐ Bits/Sec
- **Information Density**
- **Raster Example**
  - ☐ (# Pixels) (# Bits/Pixel) (Update Rate)

Task Performance & Bandwidth

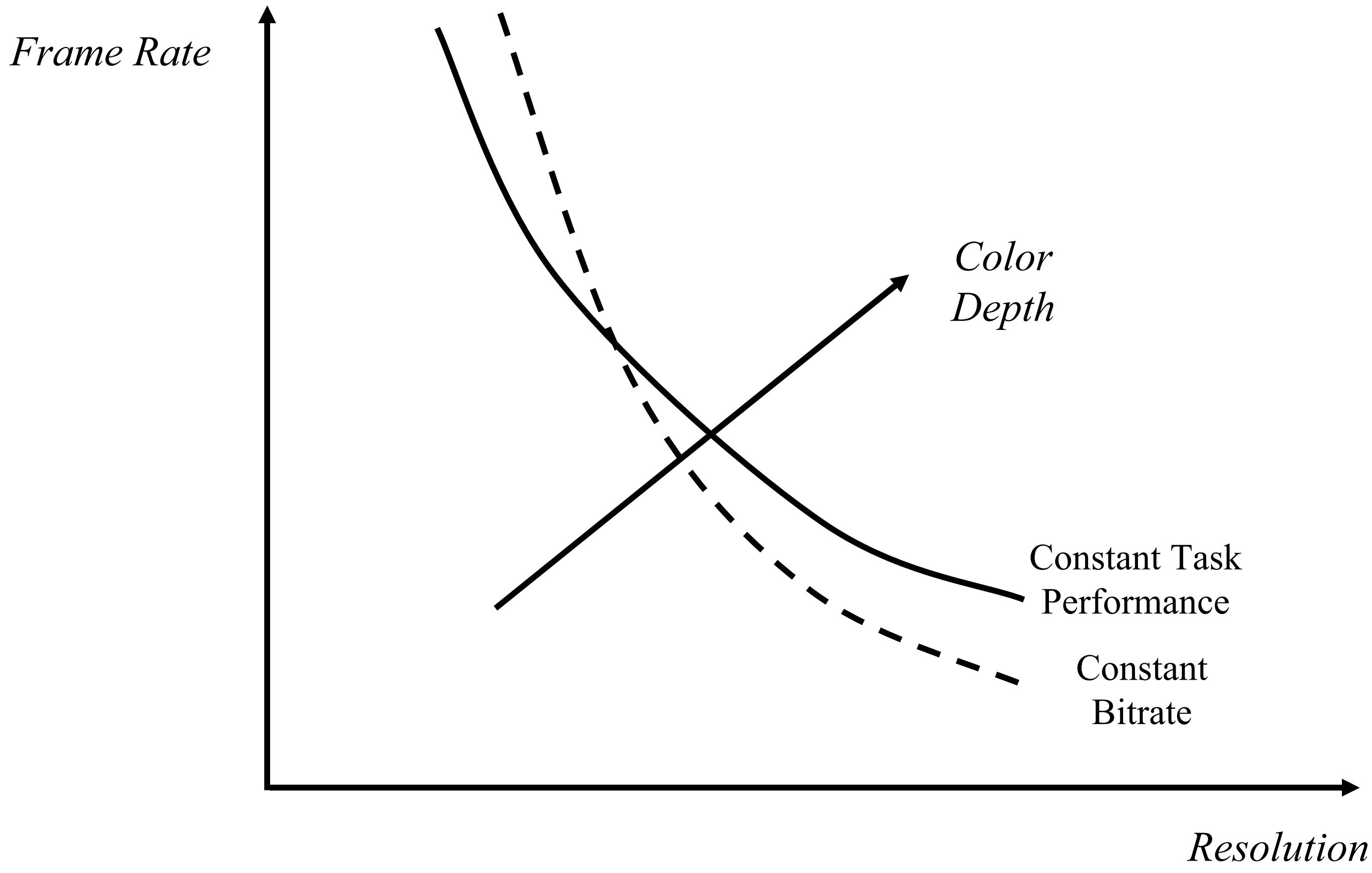
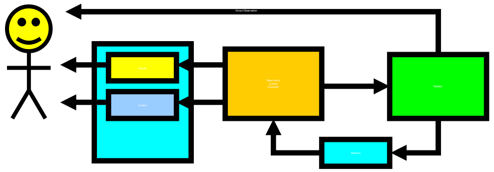


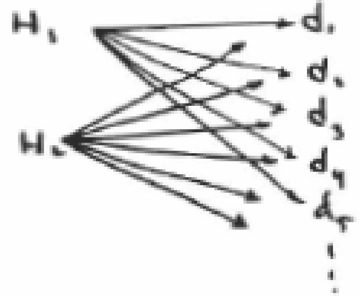
Diagram from Sheridan, Teleoperation



Signal Detection Theory

- **Originally Developed for Radar Threshold Detection**
- **Becomes the Basis for Alerting Theory**
- **Signal versus Noise**
- **A-Scope Example**

SIGNAL DETECTION THEORY



CONSIDER CASE OF ONLY 2 H_i

USING BAYES' THEOREM WE UPDATED OUR ESTIMATE OF $p(H_1)$ AND $p(H_2)$ AS NEW d_j WERE OBSERVED

IN SIGNAL DETECTION THEORY (OR MORE GENERALLY, MAXIMUM LIKELIHOOD DECISION THEORY) WE HAVE A MORE MERCINARY BASIS FOR DECISION ON H_1 OR H_2 –

NAMELY, WHICH WOULD YIELD THE GREATEST PAYOFF.

THIS IS CALLED AN "IDEAL OBSERVER" (TYPICALLY AMERICAN!)

GIVEN A PAYOFF MATRIX

TRUE STATE OF WORLD:

DECISION:	H_1	H_2
$\bar{H}_1$	V_1	C_1
$\bar{H}_2$	C_2	V_2

OVERBAR MEANS "DECISION"

IFF $E(\bar{H}_1) \geq E(\bar{H}_2)$ DECIDE H_1

$$E(\bar{H}_1) = p(H_1)V_1 - p(H_2)C_1$$
$$E(\bar{H}_2) = p(H_2)V_2 - p(H_1)C_2$$

DECIDE H_1 IF $\left[p(H_1)V_1 - p(H_2)C_1\right] \geq \left[p(H_2)V_2 - p(H_1)C_2\right]$

I.E., IFF $\dfrac{p(H_1)}{p(H_2)} \geq \dfrac{V_2 + C_1}{V_1 + C_2}$

LET $p(H_1)$ AND $p(H_2)$ BE PRIOR PROBABILITIES

THEN, AFTER ONE OBSERVATION d_1 CRITERION BECOMES

DECIDE H_1 IFF $\dfrac{p(H_1 \mid d_1)}{p(H_2 \mid d_1)} \geq \dfrac{V_2 + C_1}{V_1 + C_2}$

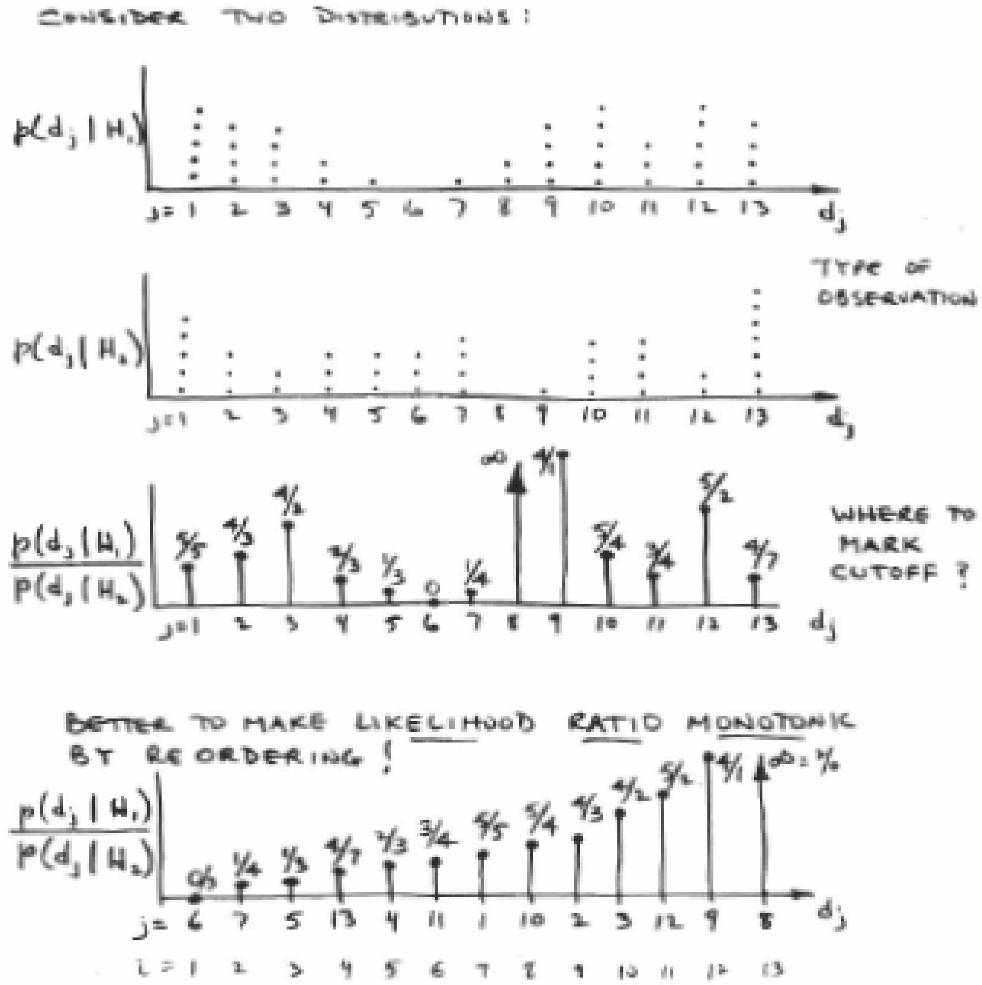
$$\frac{p(H_1|d_i)}{p(H_2|d_i)} = \frac{\dfrac{p(d_i|H_1)\,p(H_1)}{p(d_i)}}{\dfrac{p(d_i|H_2)\,p(H_2)}{p(d_i)}} \geq \frac{V_2+C_1}{V_1+C_2}$$

priors

OR $$\underbrace{\frac{p(d_i|H_1)}{p(d_i|H_2)}}_{\text{LIKELIHOOD FUNCTION OR RATIO}} \geq \underbrace{\frac{p(H_2)}{p(H_1)}\cdot\frac{V_2+C_1}{V_1+C_2}}_{=K}$$

= K CUTOFF VALUE

ALSO CALLED β



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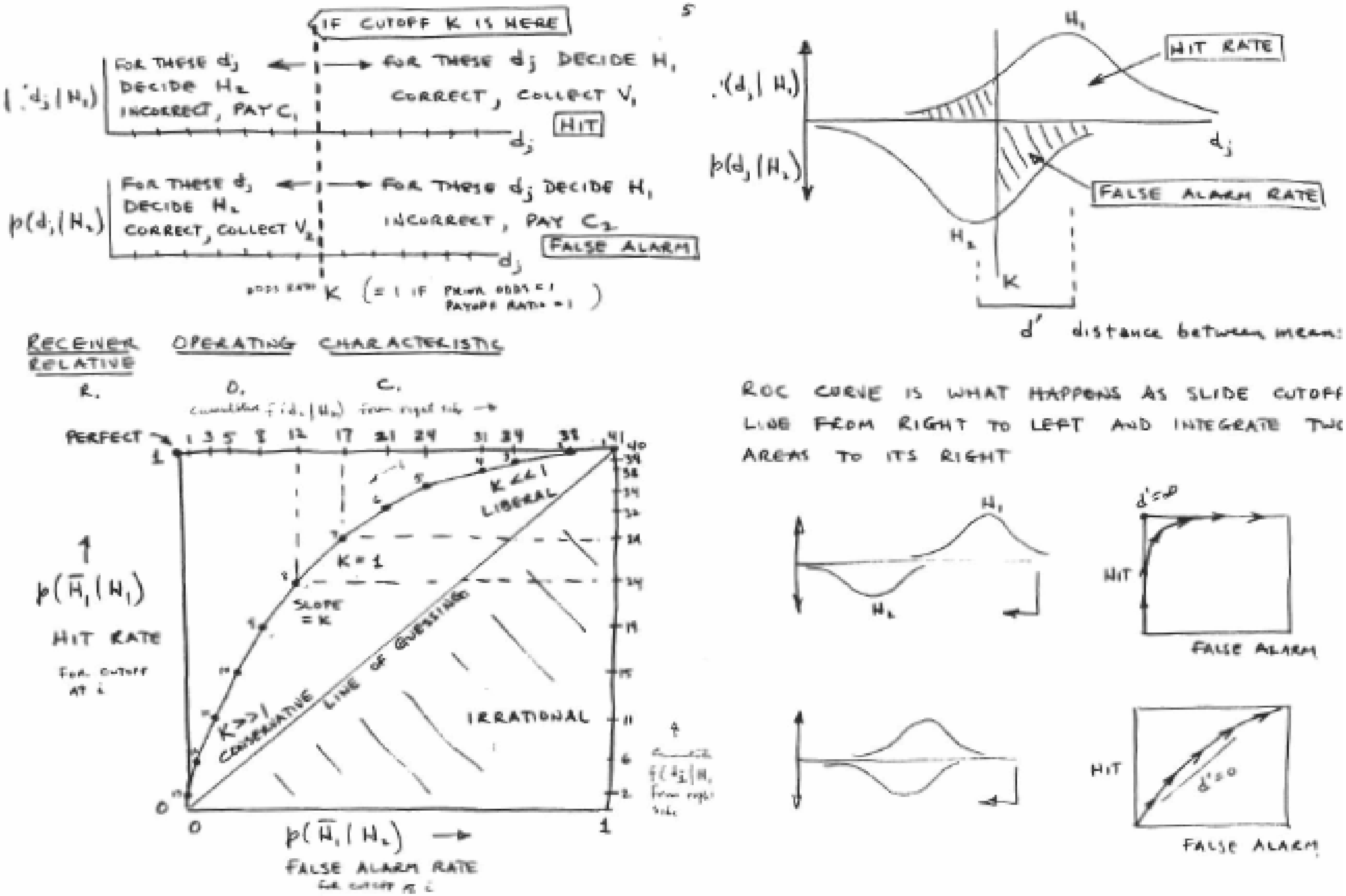

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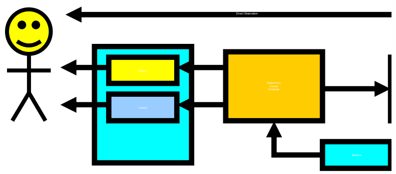

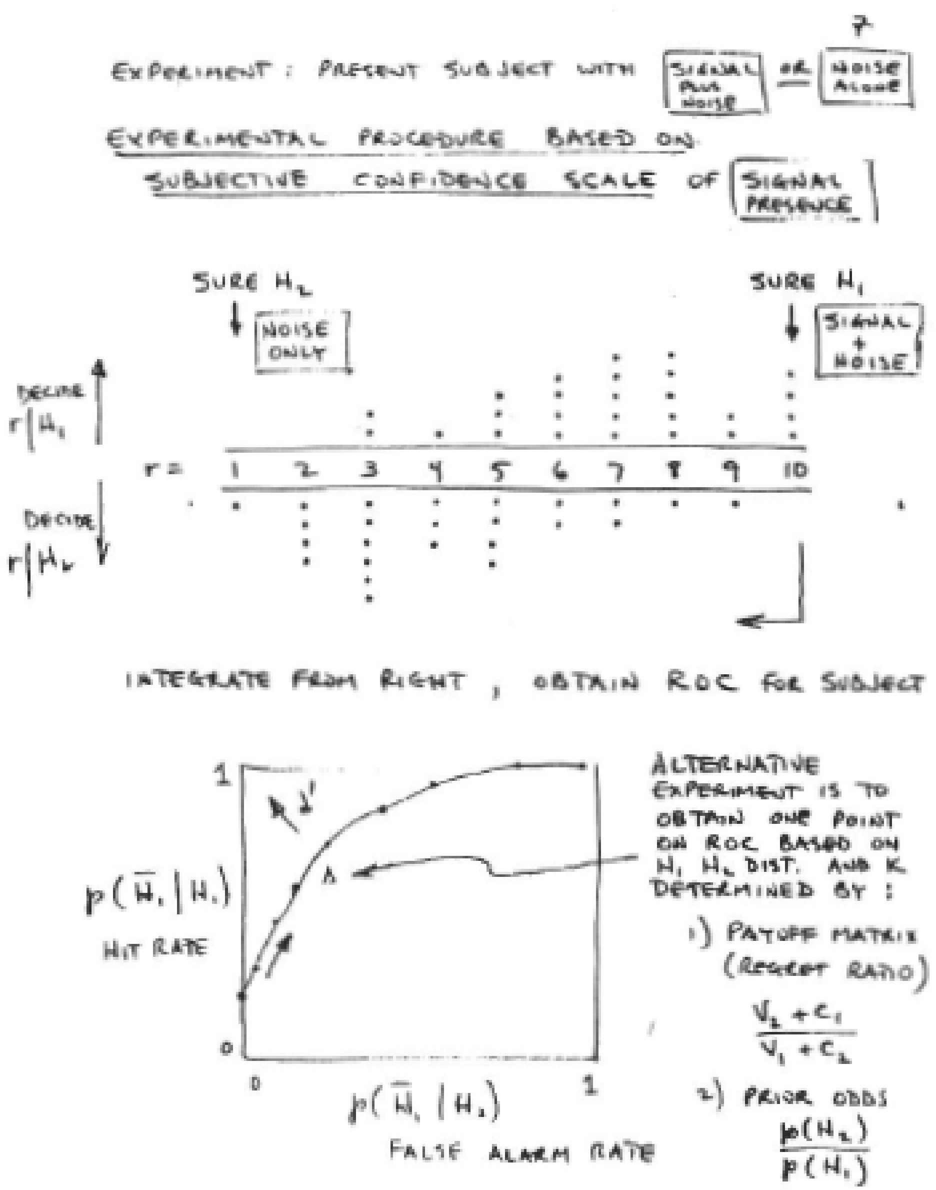

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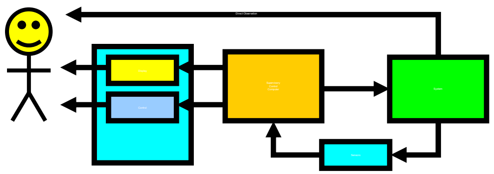
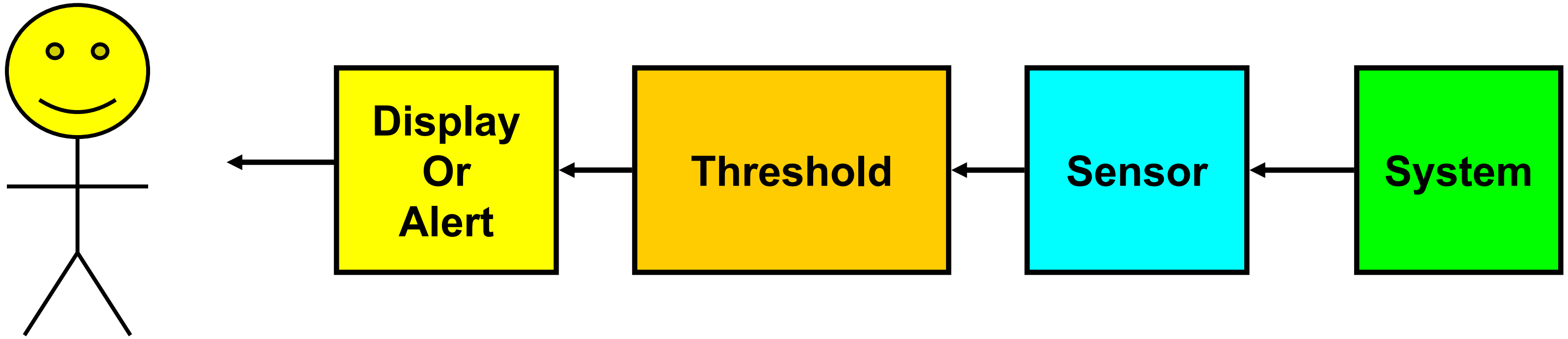
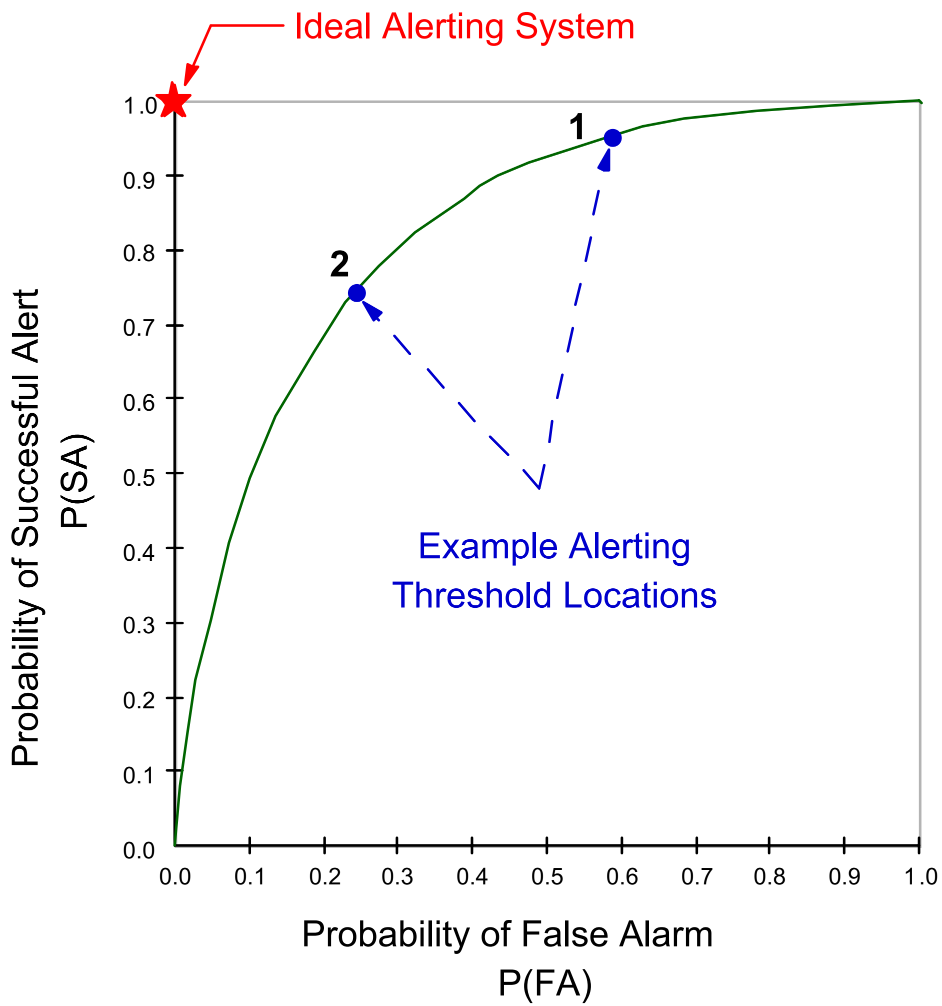


Consider Sensor System



- Radar
- Engine Fire Detection
- Other

Threshold Placement



(Courtesy of James Kuchar. Used with permission.)

Threshold Placement

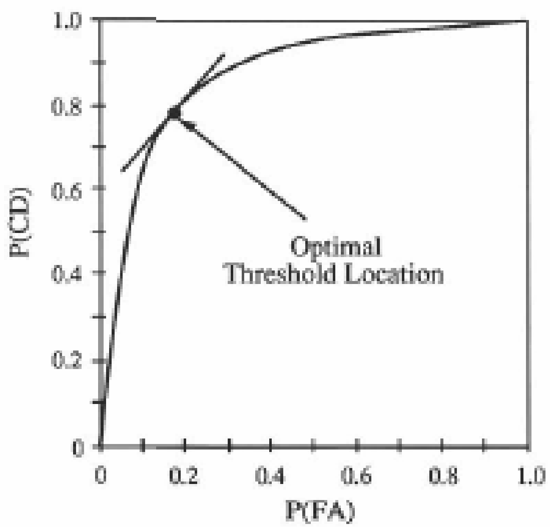
- **Use specified P(FA) or P(MD)**
- **Alerting Cost Function: Define C_{FA}, C_{MD} as alert decision costs**

$$J = P(FA)\,C_{FA} + P(MD)\,C_{MD}$$
$$= P(FA)\,C_{FA} + (1 - P(CD))\,C_{MD}$$

Minimize Cost:

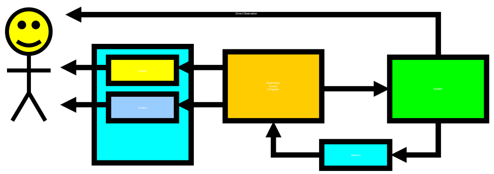
$$dJ = dP(FA)\,C_{FA} - dP(CD)\,C_{MD} = 0$$

$$\frac{dP(CD)}{dP(FA)} = \frac{C_{FA}}{C_{MD}}$$



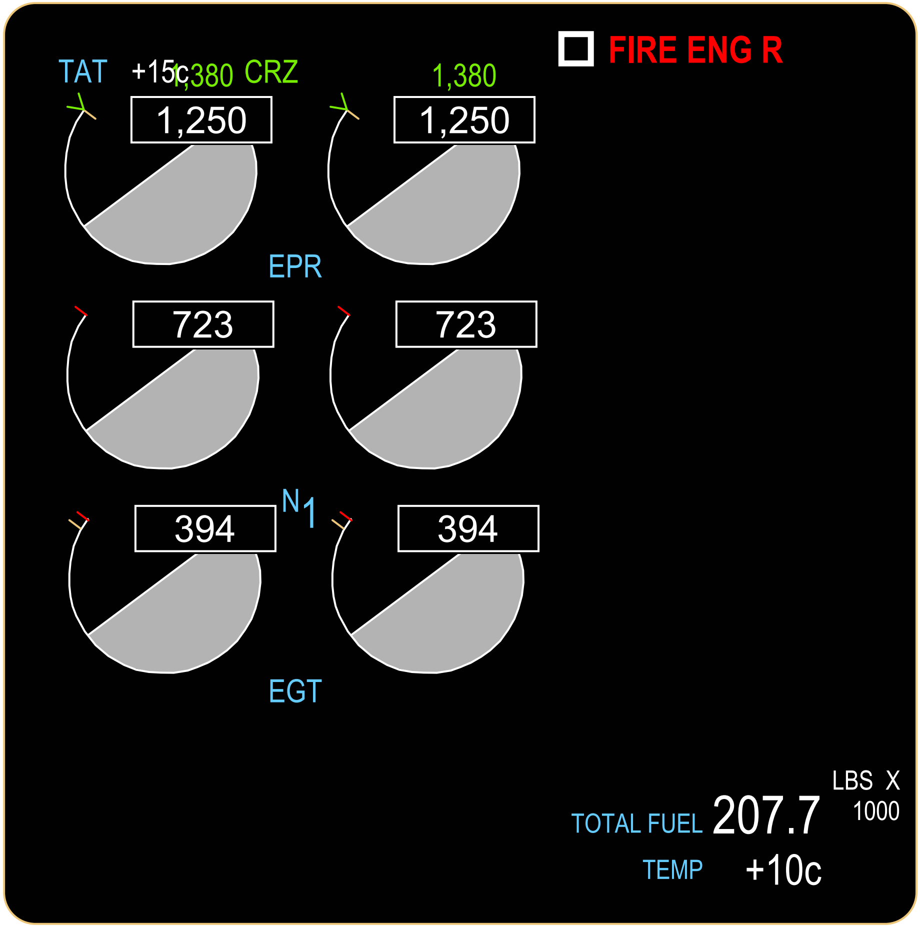
Slope of SOC curve = cost ratio

(Courtesy of James Kuchar. Used with permission.)

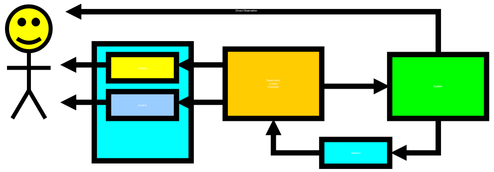


Engine Fire Alerting

- **C(FA) high on takeoff**
- **Alerts suppressed during TO**



Now let's take a quick look at non-normal checklists.
The 777 EICAS message list is similar to other Boeing EICAS airplanes.
[*For 747-400 operators*: It doesn't use the "caret" symbol to indicate a checklist with no QRH items, like the 747-400s do.]
But it has an additional feature, called the "checklist icon". The icon is displayed next to an EICAS message whenever there is an ECL checklist that needs to be completed. Once the checklist is fully complete, the icon is removed from display next to the message. This helps the crew keep track of which checklists remain to be completed.



Crew Alerting Levels

Non-Normal Procedures

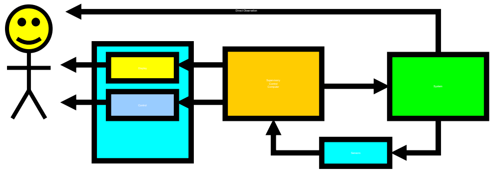
Time Critical	**Operational condition that requires immediate crew awareness and immediate action**
Warning	**Operational or system condition that requires immediate crew awareness and definite corrective or compensatory action**
Caution	**Operational or system condition that requires immediate crew awareness and possible corrective or compensatory action**
Advisory	**Operational or system condition that requires crew awareness and possible corrective or compensatory action**

Alternate Normal Procedures

Comm	**Alerts crew to incoming datalink communication**
Memo	**Crew reminders of the current state of certain manually selected normal conditions**

Source: Brian Kelly Boeing

Don't have time to discuss these levels.
Important thing to know is that we rigorously define and defend these levels
We apply them across all the systems.
The indications are consistent for all alerts at each level.
Thus the pilots instantly know the criticality and nature of an alert even before they know what the problem is

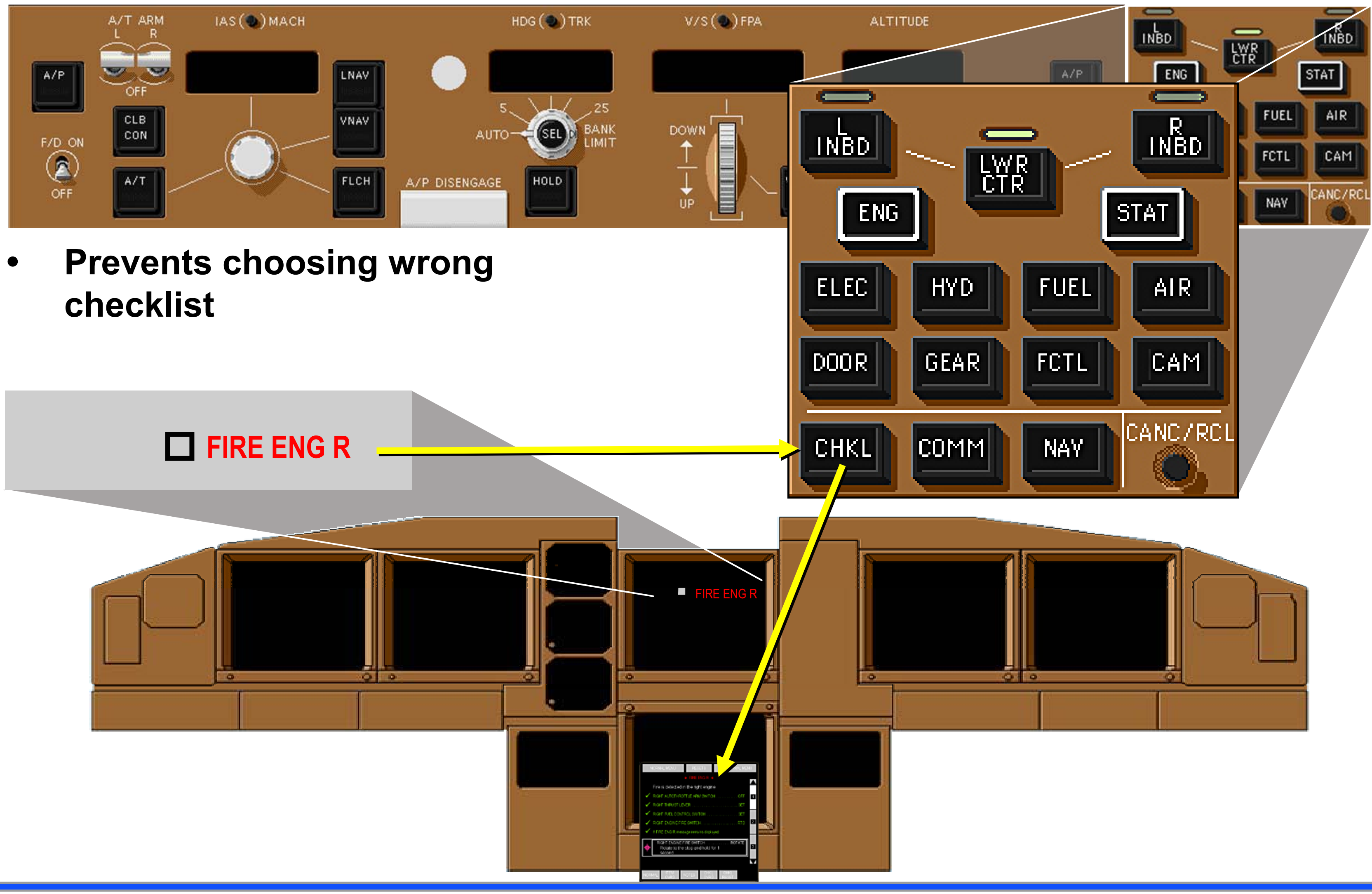


Boeing Color Use Guides

Red	Warnings, warning level limitations
Amber	Cautions, caution level limitations
White	Current status information
Green	Pilot selected data, mode annunciations
Magenta	Target information
Cyan	Background data

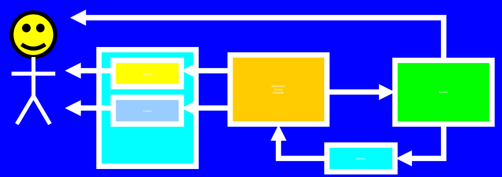
Again, we don't have time to describe these definitions in detail.
The important thing to note is that our philosophy is definite, and as simple as practical.
It fits on one page, in big font no less.

Access To Non-Normal Checklists



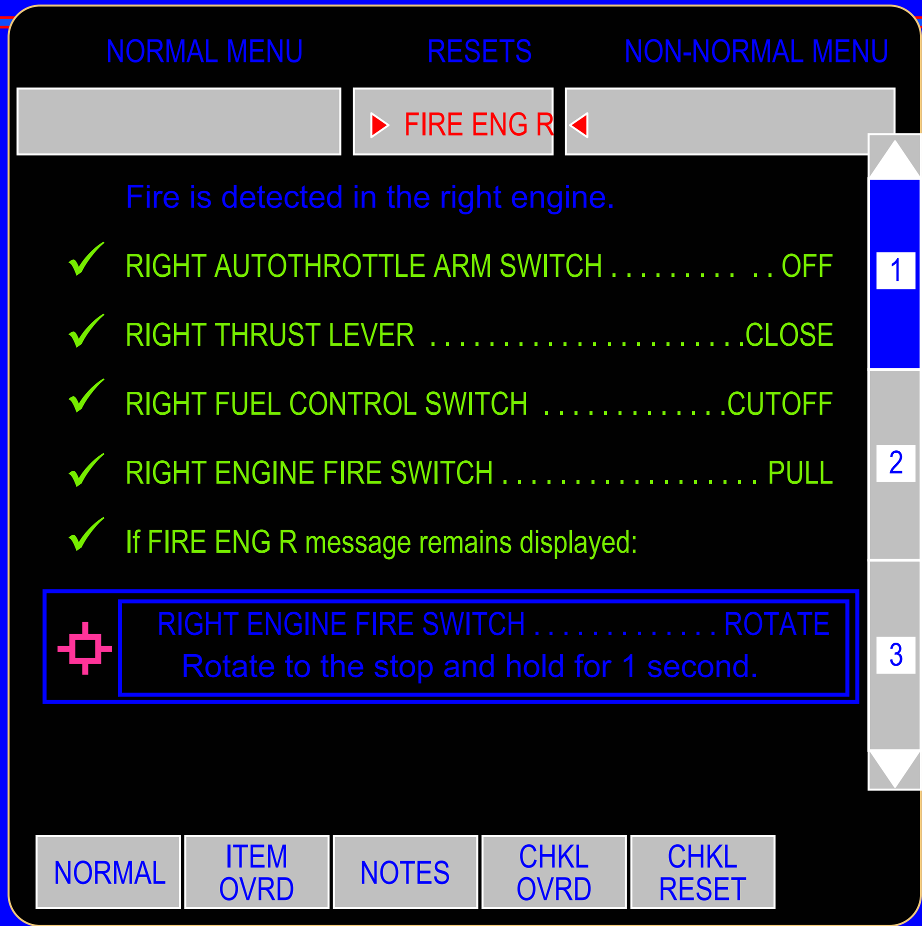
- **Prevents choosing wrong checklist**

When an alert message is displayed, the pilot simply pushes the CHKL button and the correct non-normal checklist is displayed. This prevents the crew from accidentally choosing the wrong checklist.
The non-normal checklists have priority over the normal checklists.

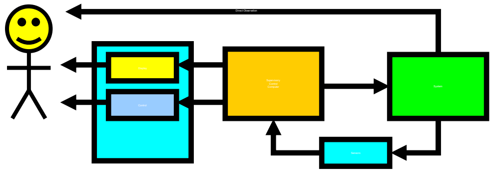


Non-Normal Checklists

- Checklist specific to left or right side
- Exact switch specified
- Memory items already complete
- Closed-loop conditional item
- Page bar

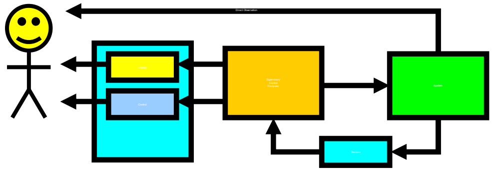


This is what a typical normal checklist looks like. This is the Preflight checkist.
There are two kinds of line items, which we call open-loop and closed-loop items. The open-loop items have a gray check-box in front of them. These are items that the airplane systems cannot sense. The pilot determines whether the items have been completed and clicks the CCD thumbswitch when each item is complete.
Closed-loop items are for switches and selectors that are sensed by the airplane systems. They automatically turn green when the switch has been positioned correctly. If the crew actuates the wrong switch, the closed-loop item will not turn green and the crew will catch their error. In this example, the procedure was already complete, so the last two items are shown in green as soon as the checklist is displayed.
The white current line item box leads the pilot through the checklist and prevents accidentally skipping a line item.
Color is used to indicate line item status. Incomplete items are displayed white and complete items are displayed green. Cyan (or blue) indicates an inapplicable item, or an item that has been intentionally overridden by the crew using the ITEM OVRD button. In this example, the flight is dispatching with autobrakes inoperative, so the crew has overridden the AUTOBRAKE item. Overriding the item allows the checklist to be completed.

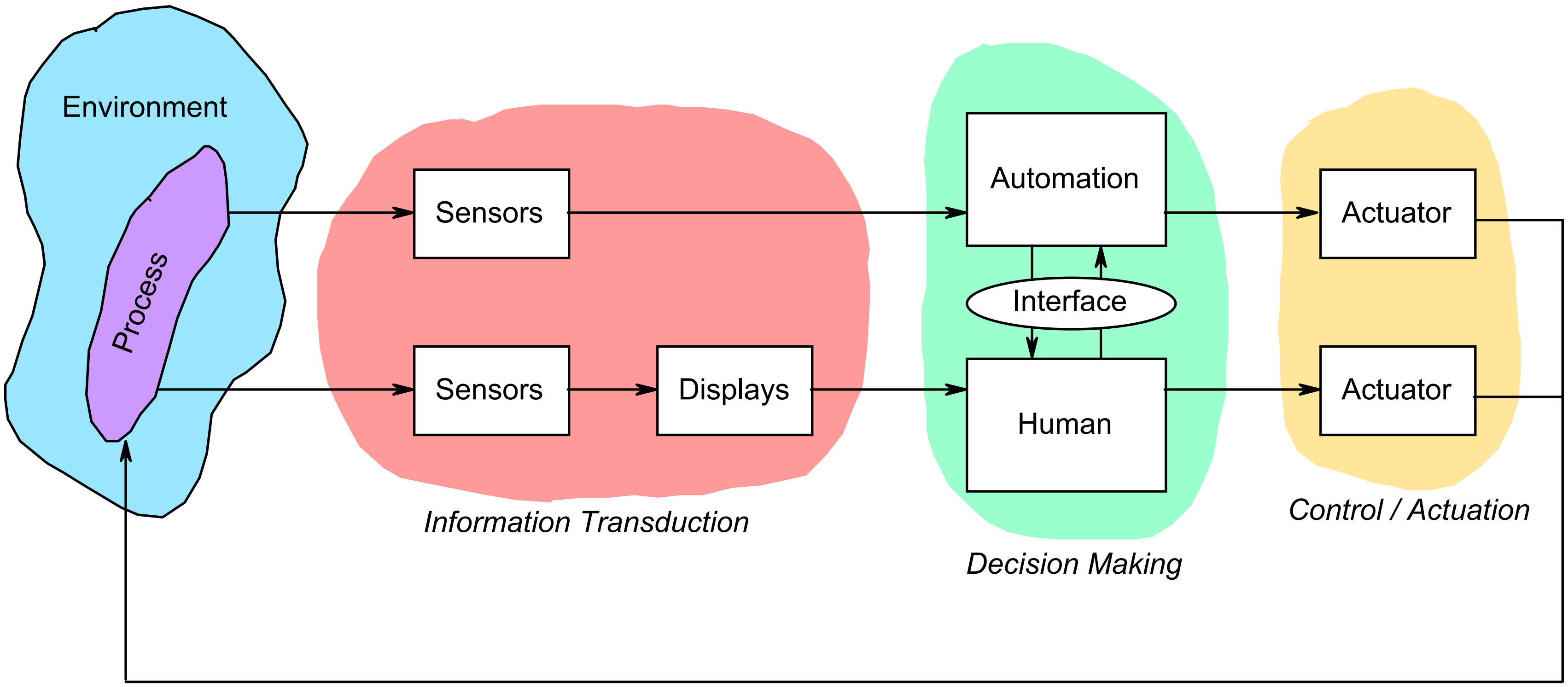


Internal vs External Threat Systems

- **Internal**
  - ☐ System normally well defined
  - ☐ Logic relatively static
  - ☐ Simple ROC approach valid
  - ☐ Examples (Oil Pressure, Fire, Fuel, ...)
- **External**
  - ☐ External environment may not be well defined
    - ◆ Stochastic elements
  - ☐ Controlled system trajectory may be important
    - ◆ Human response
  - ☐ Need ROC like approach which considers entire system
  - ☐ System Operating Characteristic (SOC) approach of Kuchar
  - ☐ Examples (Traffic, Terrain, Weather, …)

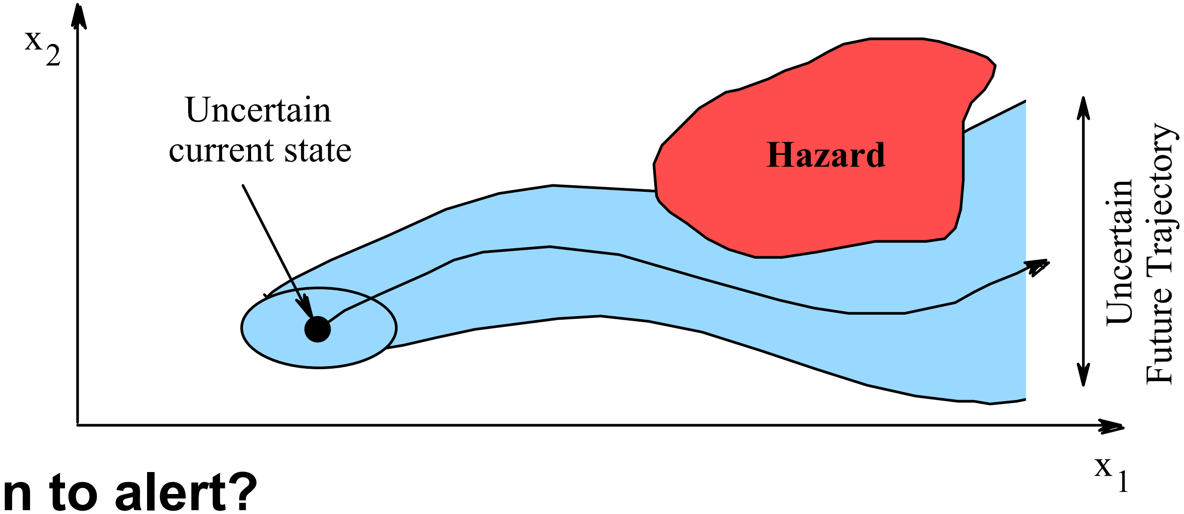


Decision-Aiding / Alerting System Architecture

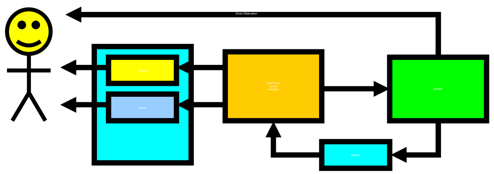


(Courtesy of James Kuchar. Used with permission.)

Fundamental Tradeoff in Alerting Decisions

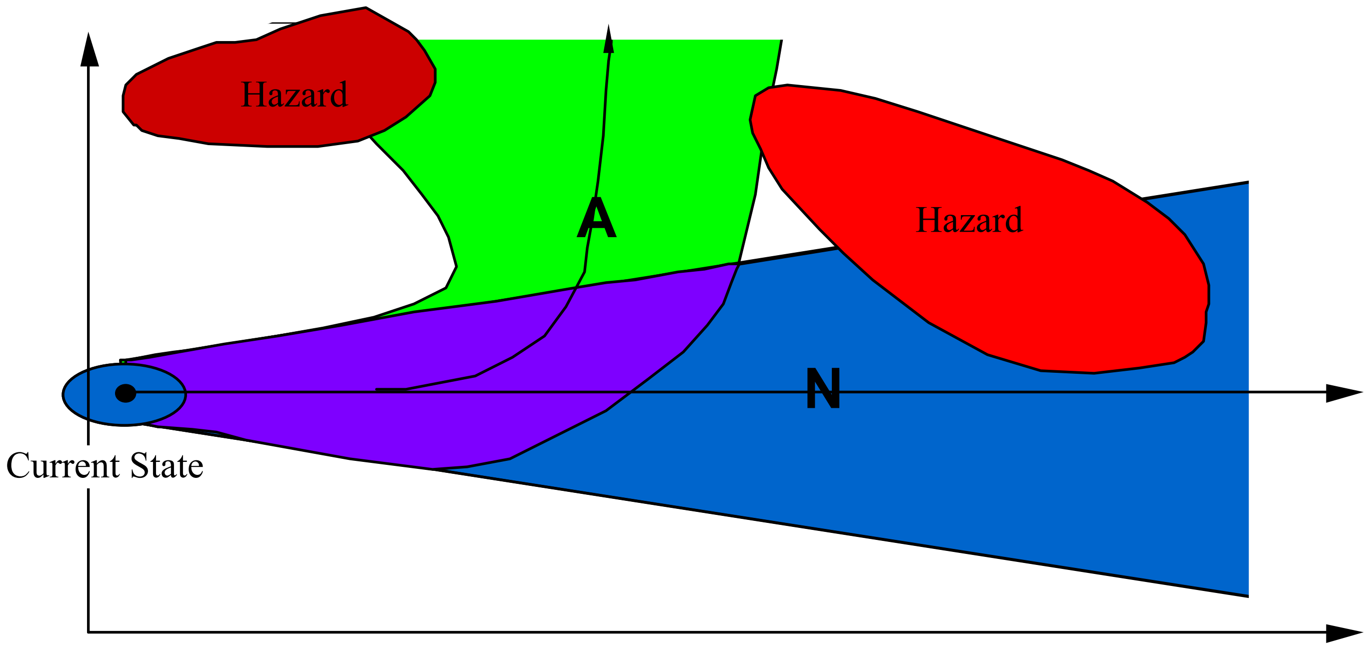


- **When to alert?**
  - ☐ Too early → Unnecessary Alert
    - ◆ Operator would have avoided hazard without alert
    - ◆ Leads to distrust of system, delayed response
  - ☐ Too late → Missed Detection
    - ◆ Incident occurs even with the alerting system
- **Must balance Unnecessary Alerts and Missed Detections**



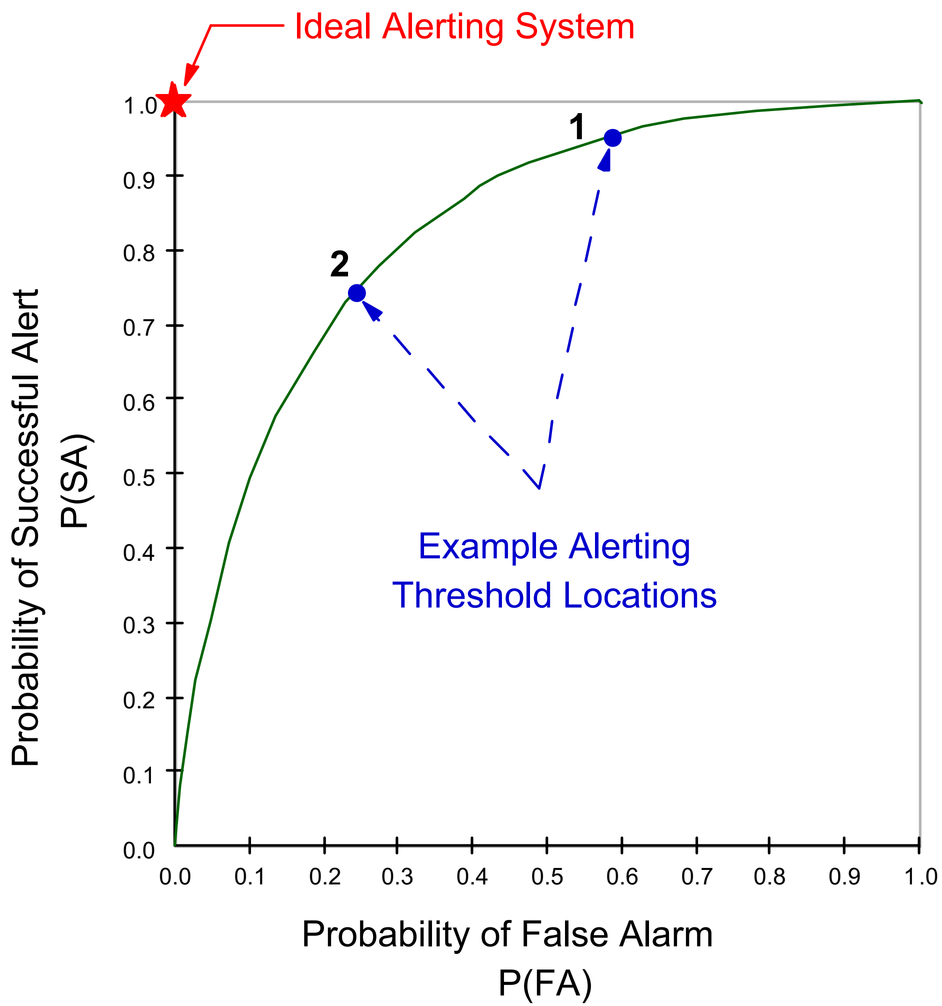
The Alerting Decision

- **Examine consequences of alerting / not alerting**
  - Alert is not issued: Nominal Trajectory (N)
  - Alert is issued: Avoidance Trajectory (A)

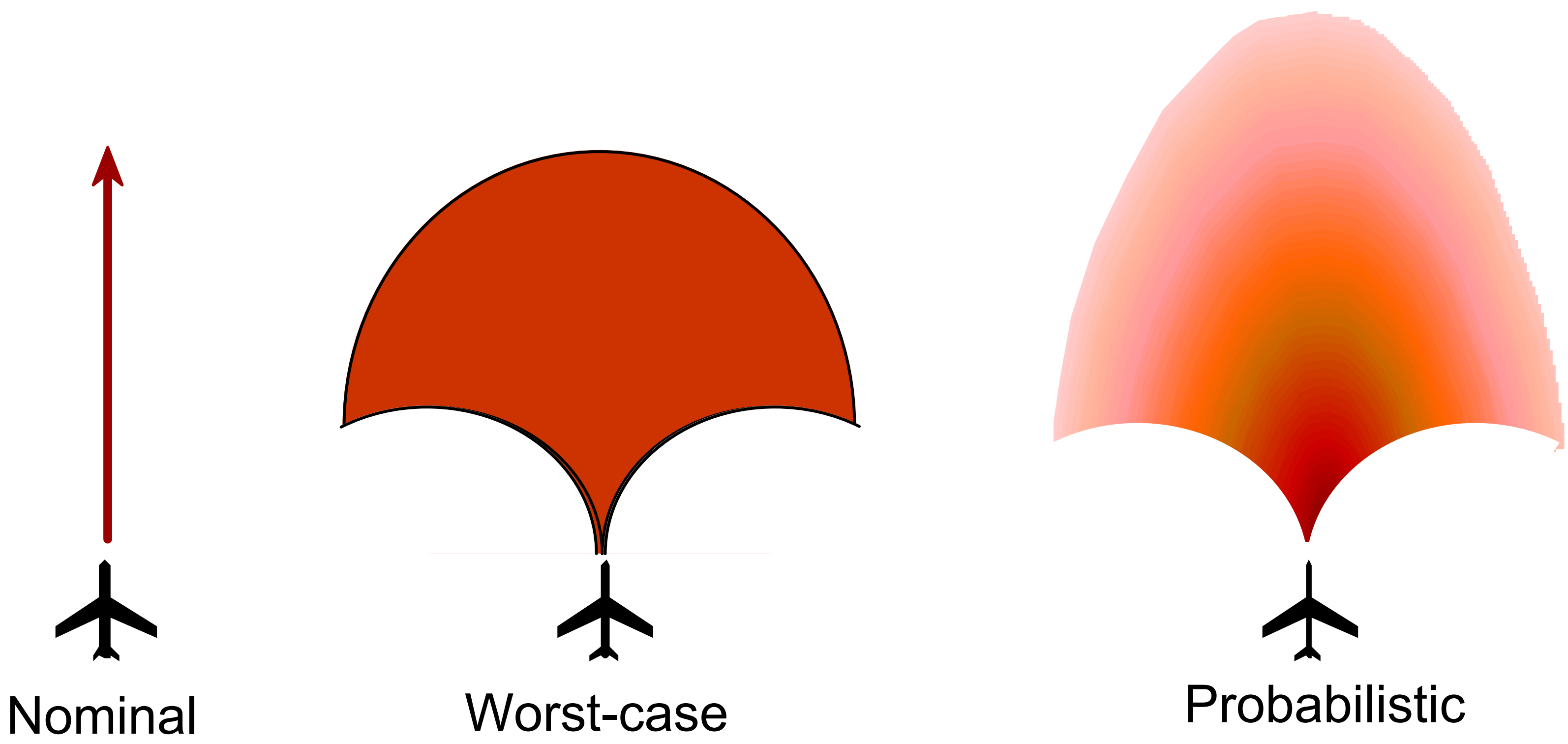


Compute probability of Incident along each trajectory

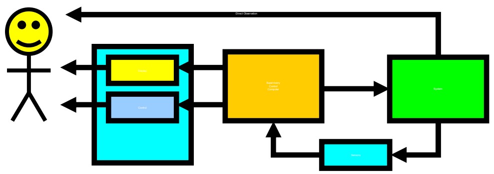
System Operating Characteristic Curve



Trajectory Modeling Methods

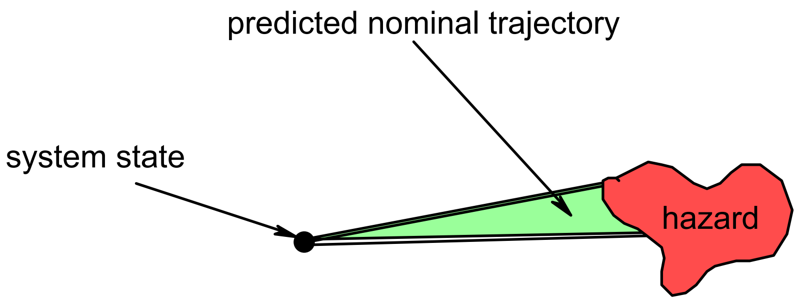


(Courtesy of James Kuchar. Used with permission.)



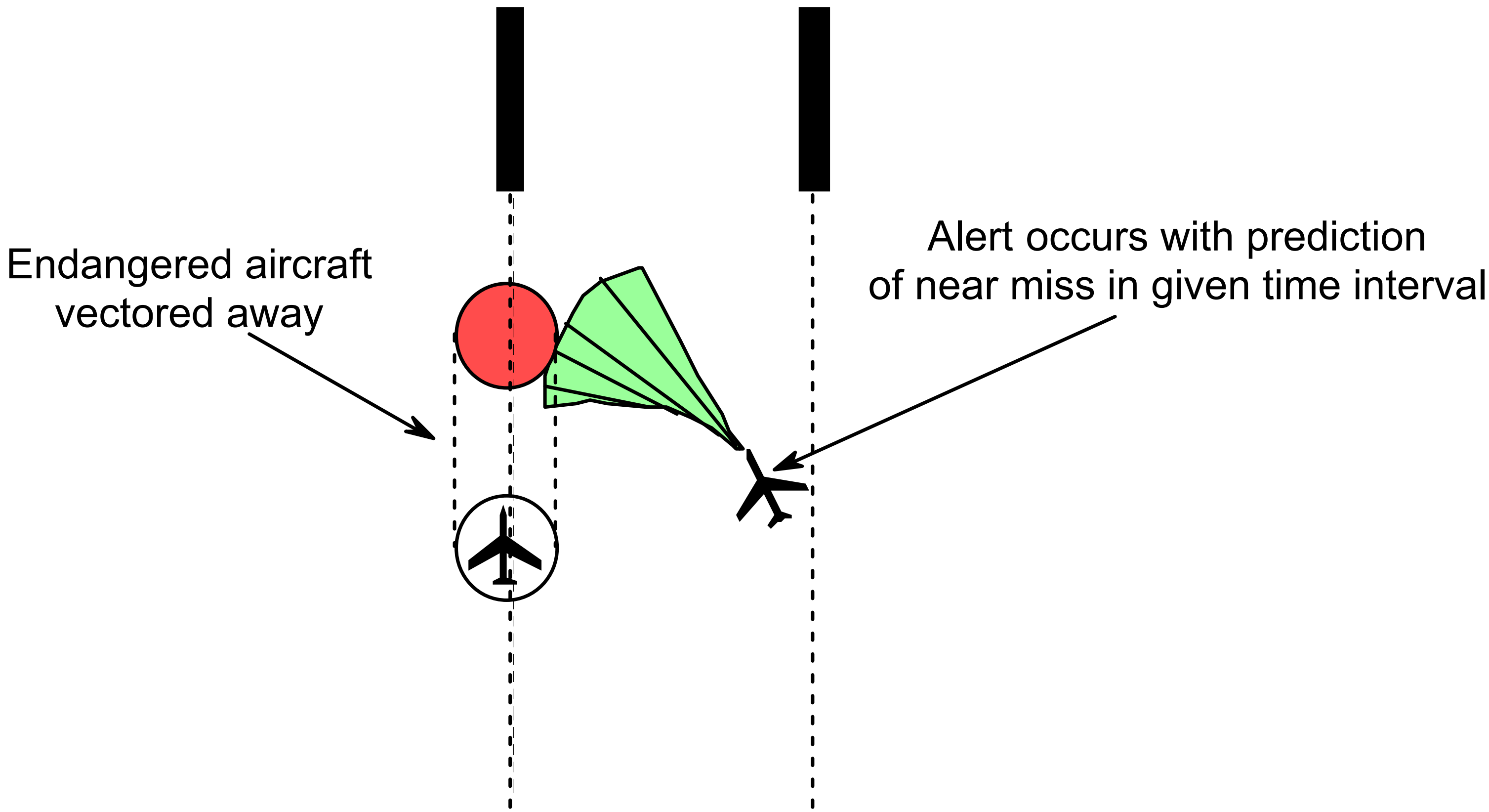
Nominal Trajectory Prediction-Based Alerting

- **Alert when projected trajectory encounters hazard**



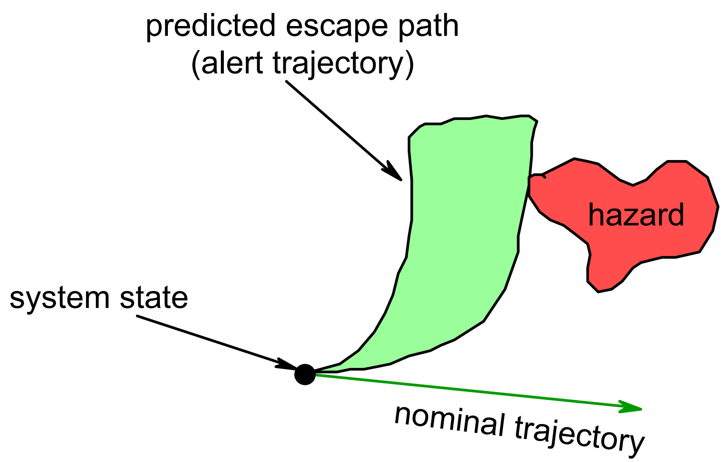
- **Look ahead time and trajectory model are design parameters**
- **Examples: TCAS, GPWS, AILS**

Airborne Information for Lateral Spacing (AILS) (nominal trajectory prediction-based)



Alert Trajectory Prediction-Based Alerting

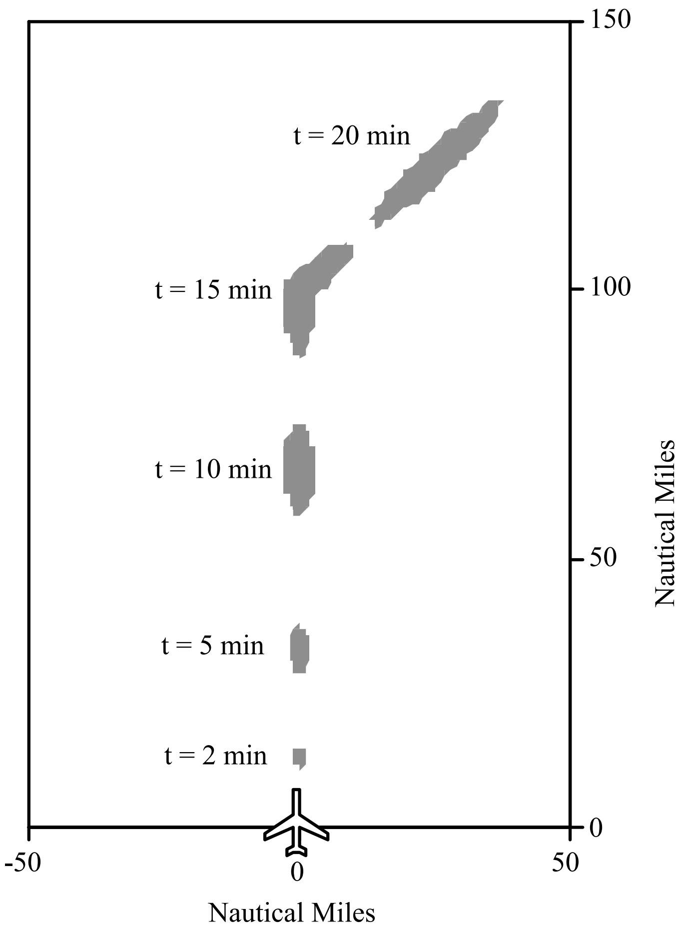
- **Alert is issued as soon as safe escape path is threatened**



- **Attempt to ensure minimum level of safety**
- **Some loss of control over false alarms**
- **Example: Probabilistic parallel approach logic (Carpenter & Kuchar)**

Example State Uncertainty Propagation

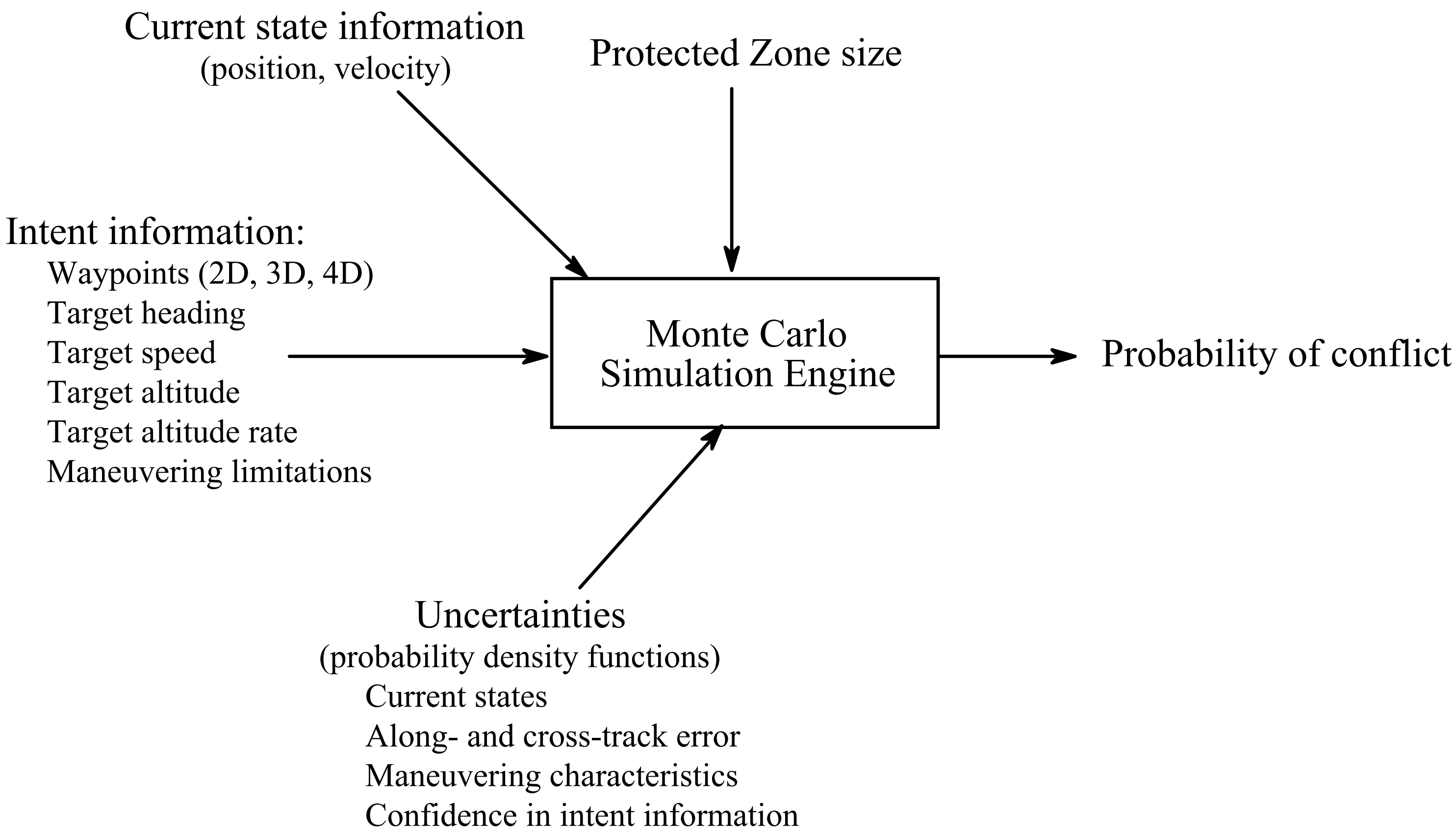
Computed via Monte Carlo



along-track σ = 15 kt
cross-track σ = 1 nmi
(from NASA Ames)

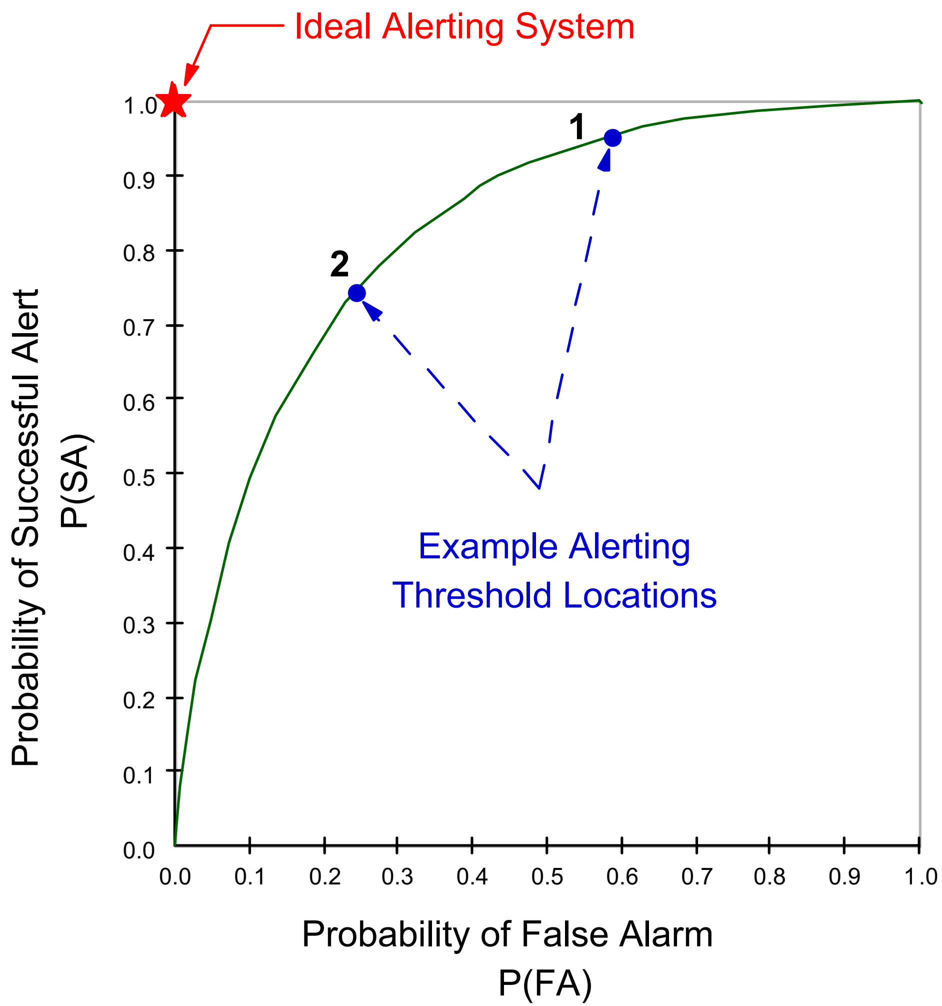
(Courtesy of James Kuchar. Used with permission.)

Monte Carlo Simulation Structure

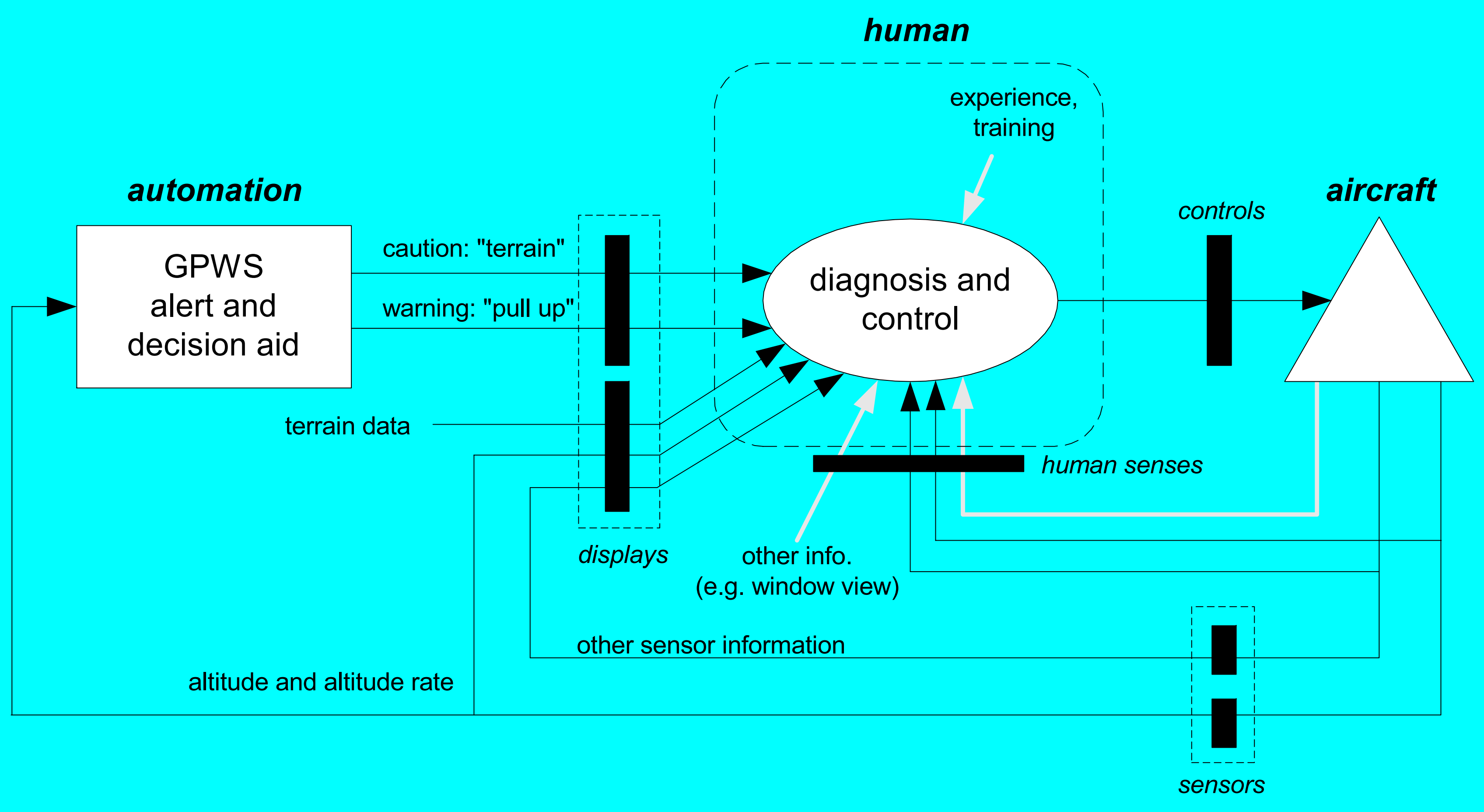


Implemented in real-time simulation studies at NASA Ames
Computational time on the order of 1 sec

System Operating Characteristic Curve



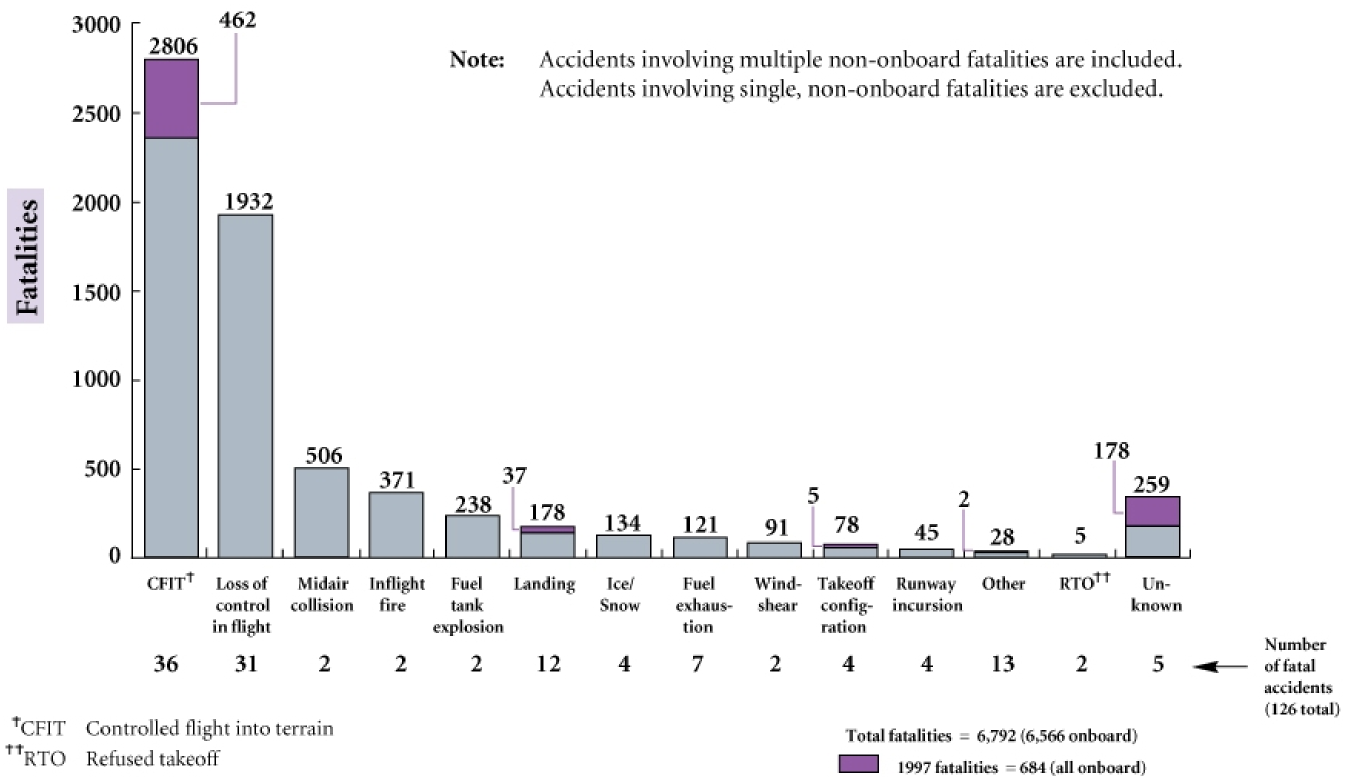
Aircraft Collision Avoidance



Fatal Accident Causes

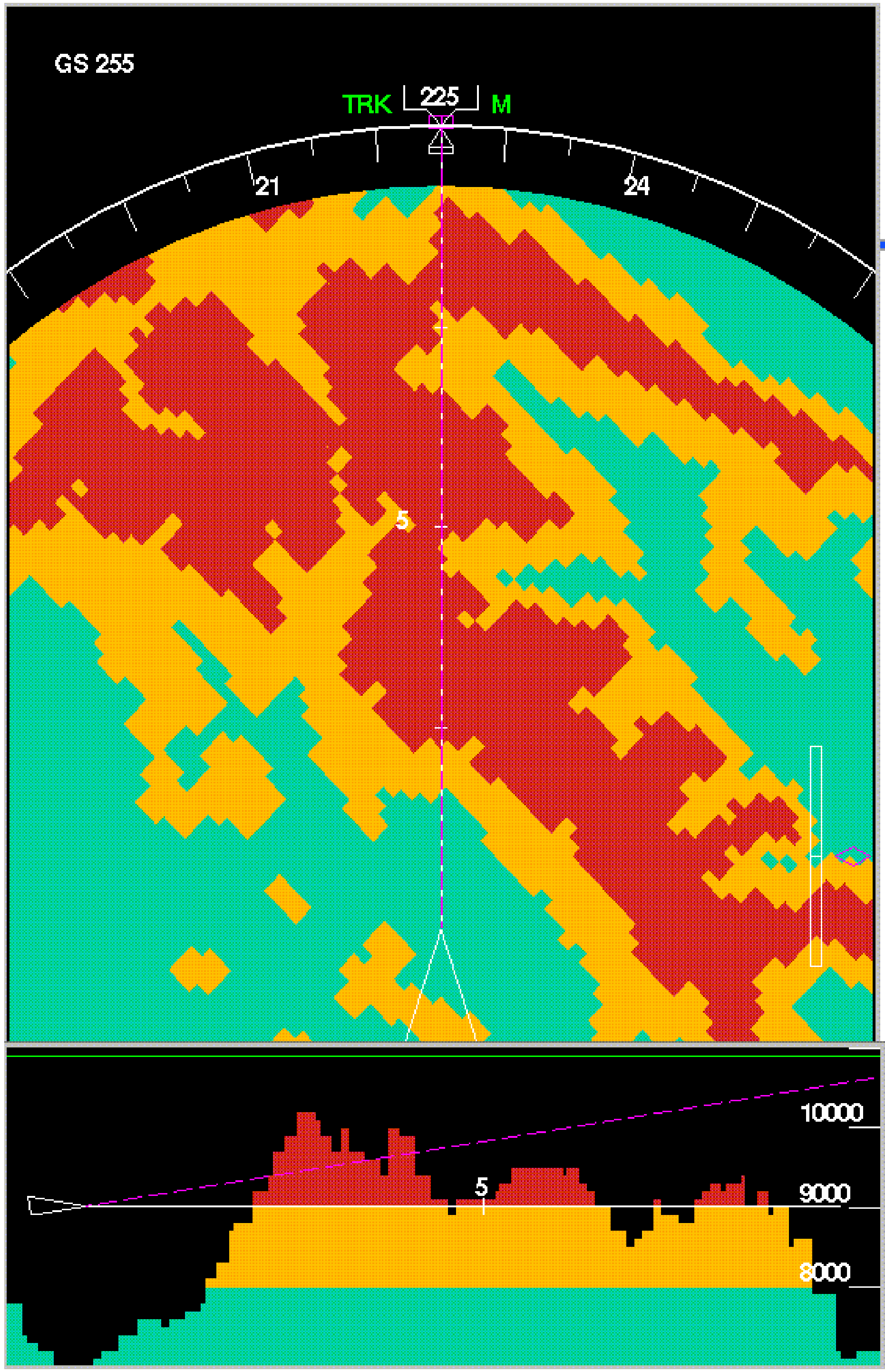
Fatalities by Accident Categories

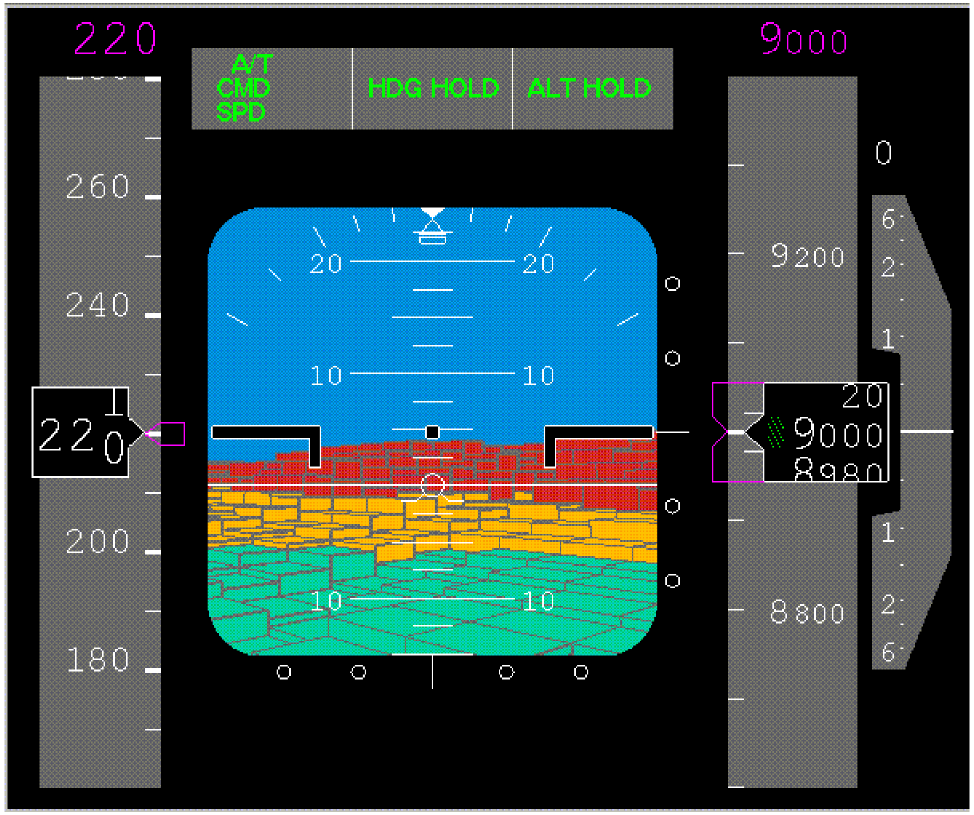
Fatal Accidents - Worldwide Commercial Jet Fleet - 1988 Through 1997



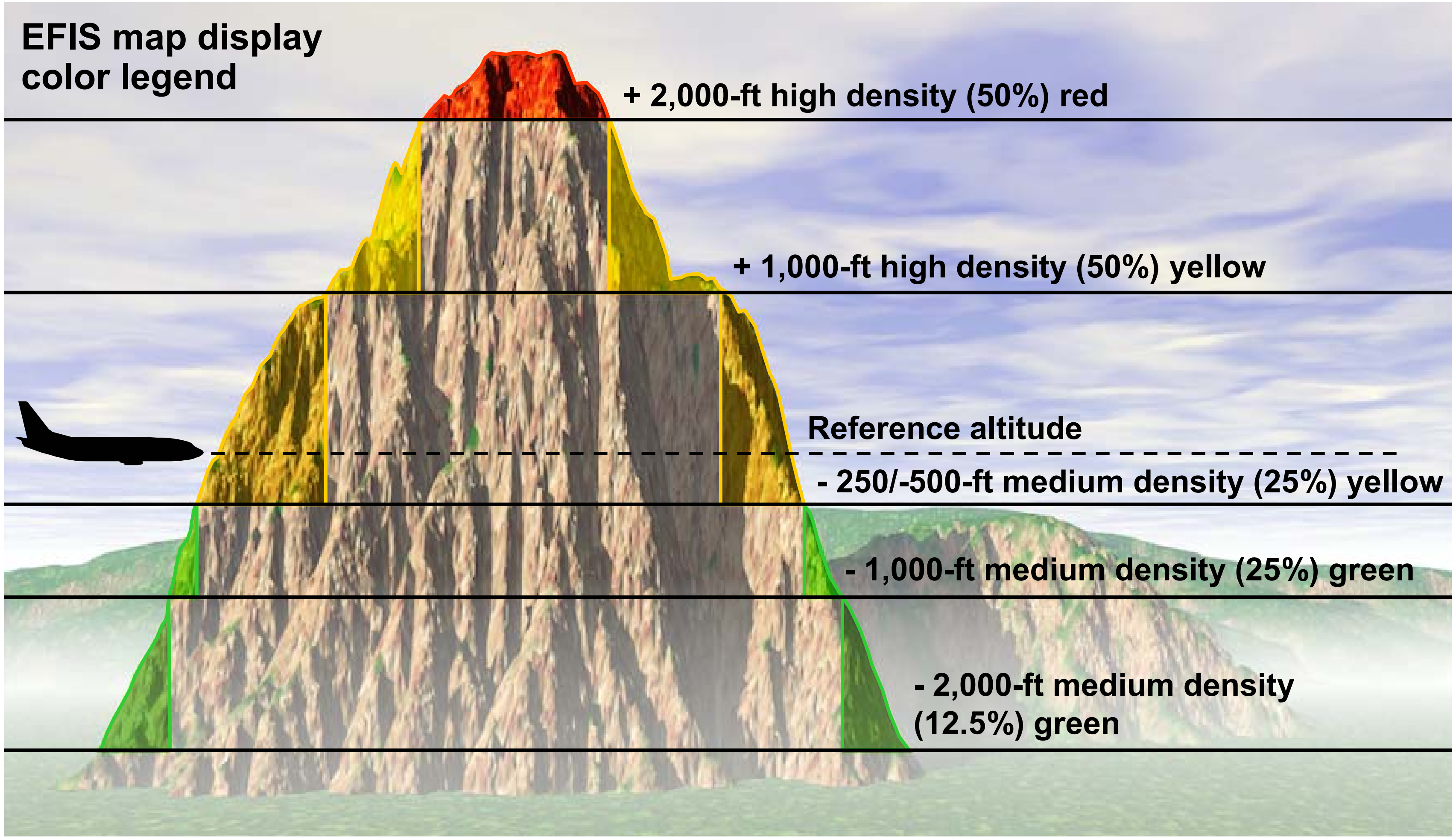
Adapted from The Boeing Company

Prototype MIT Terrain Alerting Displays



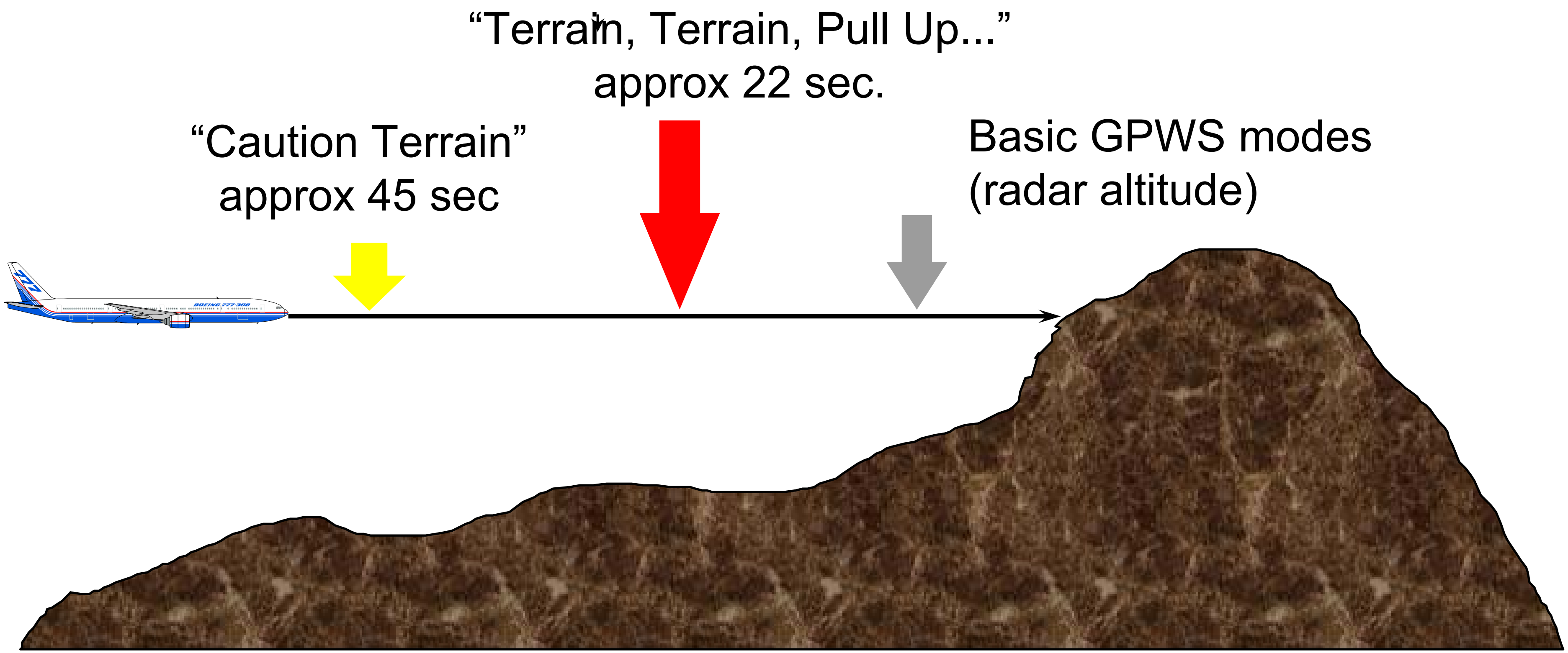


Enhanced GPWS Improves Terrain/Situational Awareness



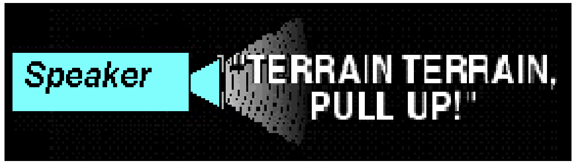
Terrain Alerting

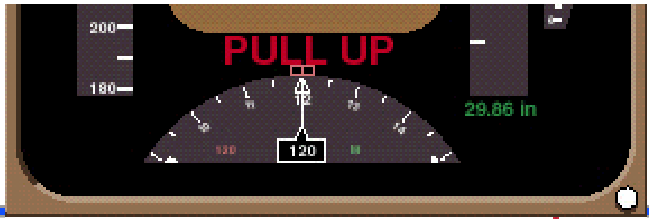
TAWS Look-Ahead Alerts
(Terrain Database)

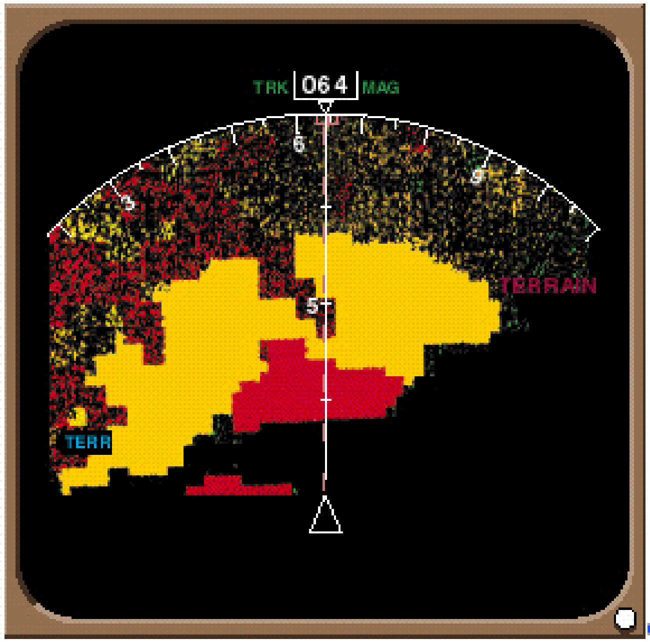


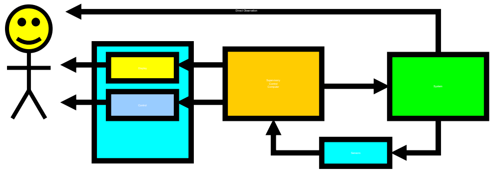
TAWS Look-ahead Warning

- Threat terrain is shown in solid red
- “Pull up” light or PFD message
- Colored terrain on navigation display

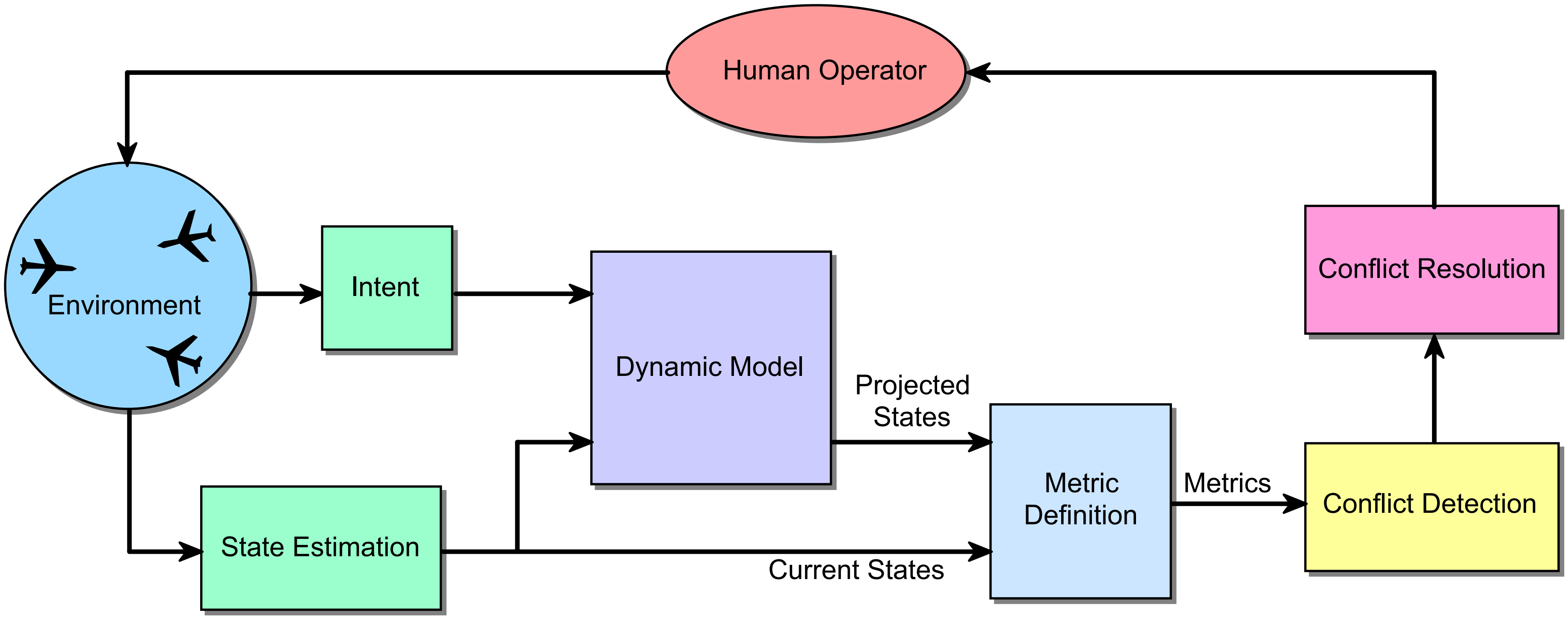






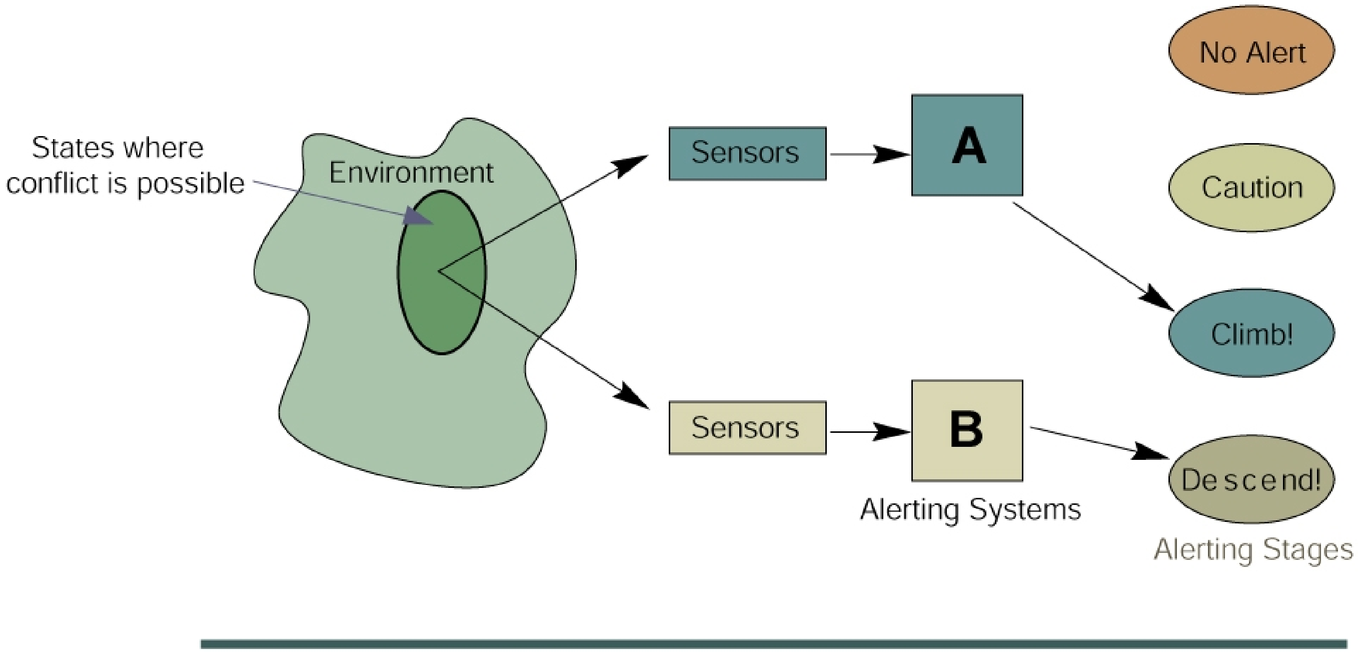


Conflict Detection and Resolution Framework



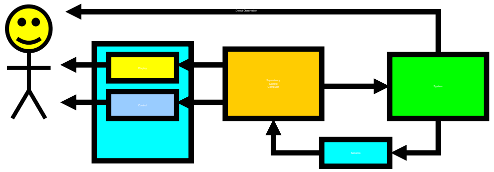
(Courtesy of James Kuchar. Used with permission.)

Multiple Alerting System Conflicts



Adapted from Jim Kuchar.

- Developing formal methods for system analysis
- Identification of conflicts and methods to mitigate
- Drivers / implications for human interaction

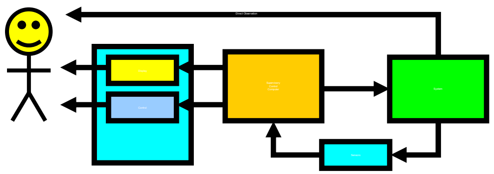


Design Principles for Alerting and Decision-Aiding Systems for Automobiles

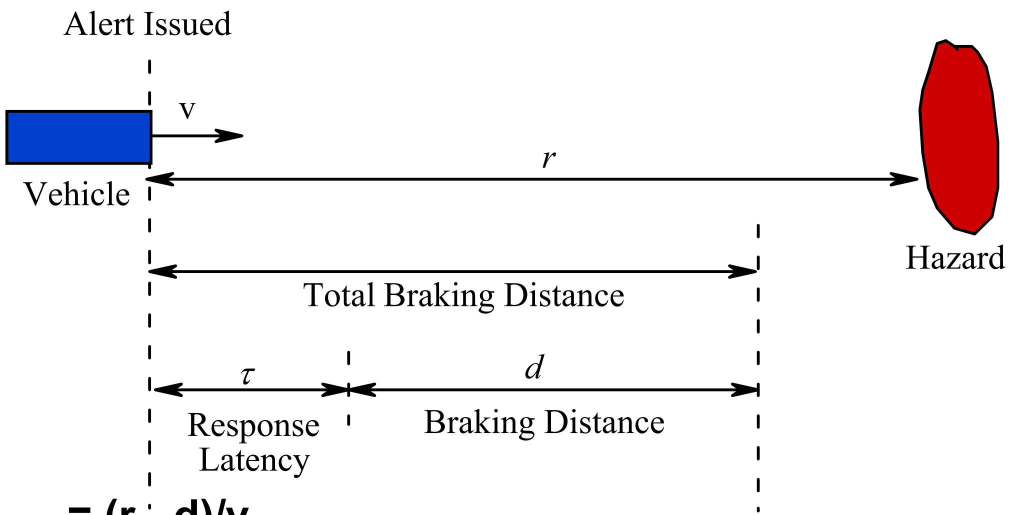
James K. Kuchar

Department of Aeronautics and Astronautics

Massachusetts Institute of Technology



Kinematics



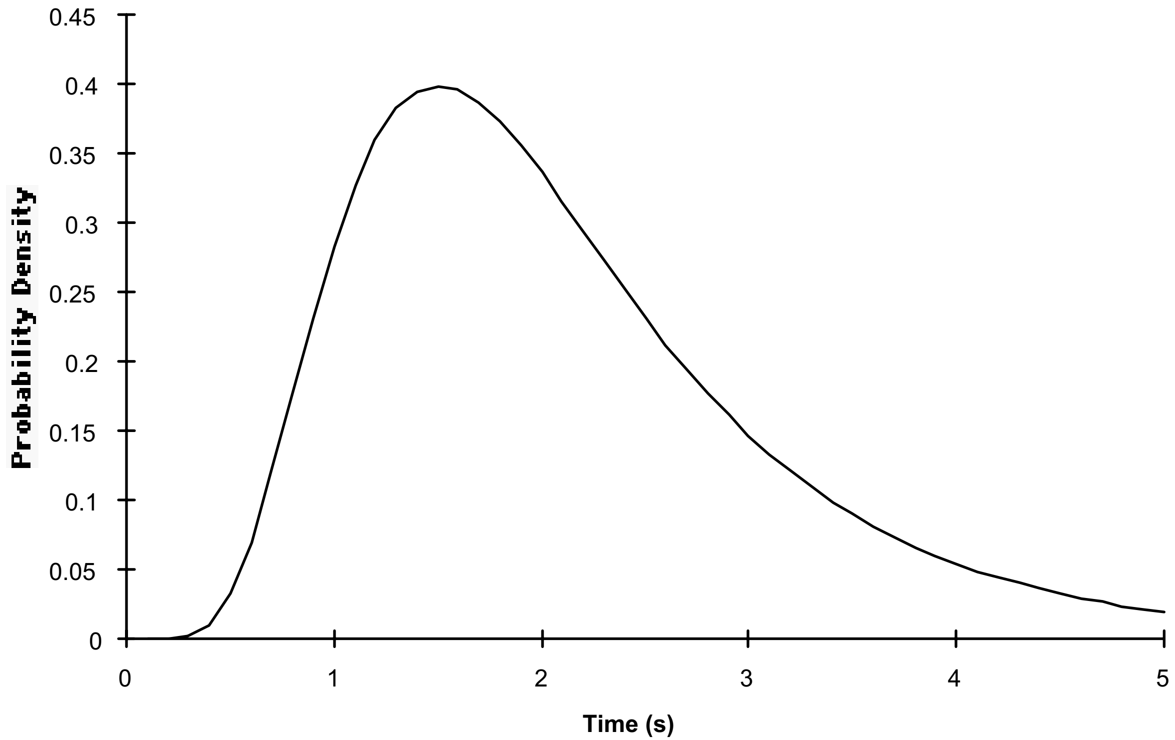
- **Alert time: $t_{alert} = (r - d)/v$**

  $t_{alert} = 0 \rightarrow$ braking must begin immediately

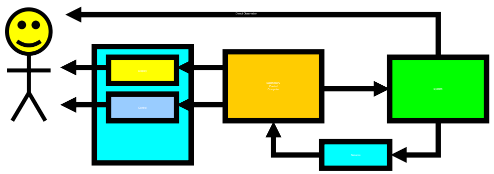
  $t_{alert} = \tau \rightarrow$ alert is issued τ seconds before braking is required

- **Determine P(UA) and P(SA) as function of t_{alert}**

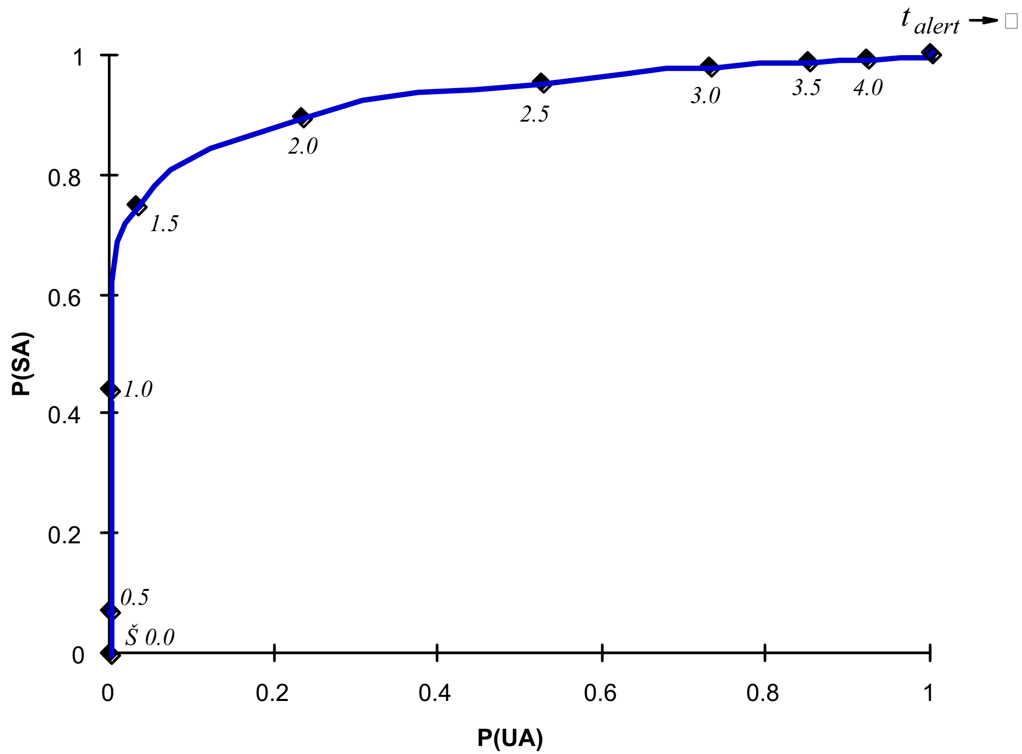
Example Human Response Time Distribution



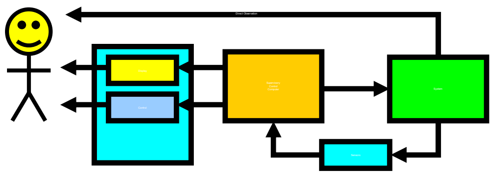
Lognormal distribution (mode = 1.07 s, dispersion = 0.49) [Najm et al.]



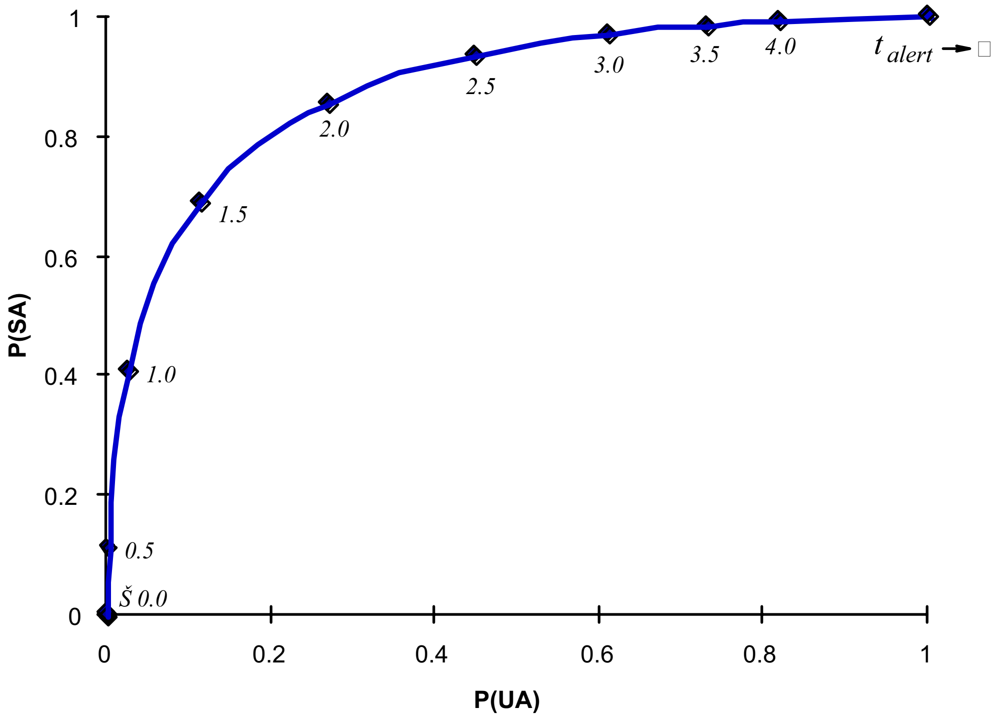
Case 3: Add Response Delay Uncertainty



(Courtesy of James Kuchar. Used with permission.)

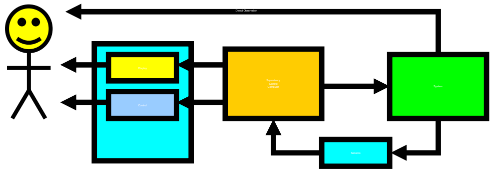


Case 4: Add Deceleration Uncertainty



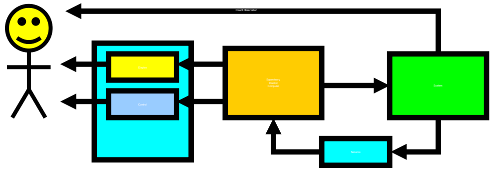
σ_a = 3 ft/s^2

(Courtesy of James Kuchar. Used with permission.)



Conformance Monitoring for Internal and Collision Alerting

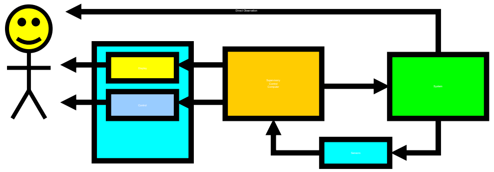
- **Simple Sensor Based Collision Alerting Systems Do Not Provide Adequate Alert Performance due to Kinematics**
  - SOC Curve Analysis
    - P(FA), P(MD) Performance
- **Enhanced Collision Alerting Systems Require Inference or Measurement of Higher Order Intent States**
  - Automatic Dependent Surveillance (Broadcast)
  - Environment Inferencing
    - Observed States



SURVEILLANCE STATE VECTOR

- **Aircraft Surveillance State Vector, $X(t)$ containing uncertainty & errors $\delta X(t)$ is given by:**
  - ☐ Traditional dynamic states
  - ☐ Intent and goal states

$$X(t) = \left\{ \begin{array}{c} Position, R(t) \\ Velocity, V(t) \\ Acceleration, A(t) \\ \hdashline Intent, I(t) \\ Goals, G(t) \\ \vdots \end{array} \right\}, \qquad \delta X(t) = \left\{ \begin{array}{c} \delta R(t) \\ \delta V(t) \\ \delta A(t) \\ \hdashline \delta I(t) \\ \delta G(t) \\ \vdots \end{array} \right\}$$



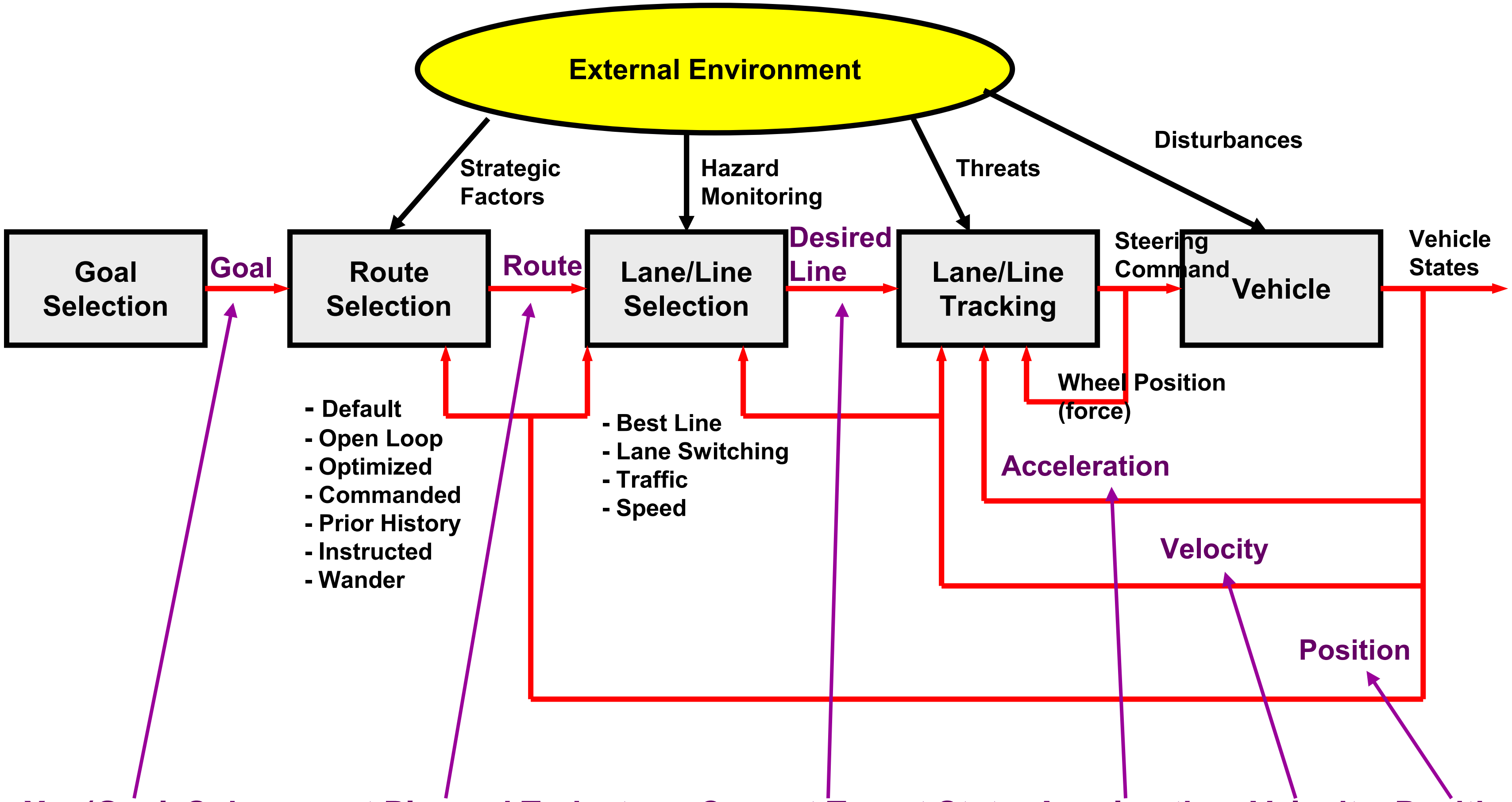
INTENT STATE VECTOR

- **Intent State Vector can be separated into current target states and subsequent states**

$$I(t) = \begin{Bmatrix} \textit{Current target states} \\ \textit{Subsequent planned trajectory} \end{Bmatrix} \qquad \text{(Eqn. 3)}$$

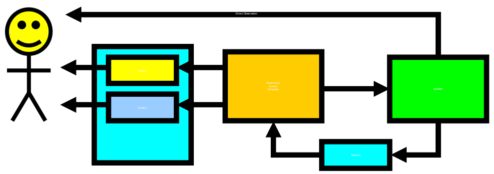
(Courtesy of James Kuchar. Used with permission.)

Automobile Lateral Tracking Loop



X = (Goal, Subsequent Planned Trajectory, Current Target State, Acceleration, Velocity, Position)

(Courtesy of James Kuchar. Used with permission.)



Intent Observability States

- **Roadway**
- **Indicator Lights**
  - Break Lights
  - Turn Signals
  - Stop Lights
- **Acceleration States**
- **GPS Routing**
- **Head Position**
- **Dynamic History**
- **Tracking Behavior**