

16.522 Space Propulsion Problem Set 1

The traditional low thrust optimization methods consider situations with a power system that is dedicated solely to propulsion and is inaccessible to the payload. But many missions are themselves power-intensive and it becomes possible to use the mission power supply for electric propulsion during the spacecraft deployment phase, and also for orbital corrections later on. An example of this is communications satellites.

In particular, it is interesting to observe that Google, SpaceX and Virgin Galactic are proposing ambitious constellations of relatively small LEO satellites, each with a mass of about 150 kg and carrying more than 500 W of mission power. The objective is to provide global coverage, therefore hundreds of satellites would be needed. It would be extraordinarily expensive to deliver each satellite in its service orbit with dedicated launchers. Instead, a single launch could position many satellites in a parking orbit and electric propulsion could then be used to raise the satellites' altitude to their final service orbit.

These small satellites could be quite constrained in mass and volume. For example, deviations from a baseline mass could be severely penalized (for instance, when requiring more propellant) by increasing the mission cost. In addition, we need to figure out how to select the operation time, with some consideration for the penalties incurred by this time being too long. One simple model for this is the optimization of the **cost-penalized transportation rate** instead of payload size (i.e., payload mass divided by operation time and by penalty costs). This homework is devoted to the formulation of this problem.

Let the payload mass m_{pay} be related to power through $m_{pay} = \beta P$, where β ($\sim 0.1 - 0.2$ kg/W) is determined by the payload design. Here the payload includes its power system (solar arrays, converters, batteries, etc), as it would exist in the absence of an electric propulsion system. In addition, the propulsion system (thruster and power converters) has a mass $m_{ps} = \alpha P$, where α ($\sim 0.02 - 0.1$ kg/W) is determined by the propulsion system design.

The thruster efficiency will be allowed to depend on the specific impulse using the model introduced in class,

$$\eta = \frac{\eta_0}{1 + (v_L/c)^2}$$

where the loss velocity v_L is a function of the thruster type and design, while η_0 (the high-specific impulse limit) is the maximum thruster efficiency.

- (a) Let m_0 be the initial mass, m_s the structural mass (a fixed fraction ϵ of m_0), and show that the mass balance reads,

$$1 - \epsilon = \left[1 + \frac{\alpha + \beta}{2\eta_0 t} (c^2 + v_L^2) \right] \left[1 - \exp\left(-\frac{\Delta v}{c}\right) \right]$$

where t is the thrusting time and Δv is the total velocity increment. Introduce the dimensionless quantities,

$$x = \frac{c}{\Delta v}, \quad z = \frac{v_L}{\Delta v} \quad \text{and} \quad \tau = \frac{2\eta_0 t}{(\alpha + \beta)\Delta v^2}$$

and, after substituting into the mass balance, solve for τ as a function of x , z and ϵ .

- (b) Define the payload fraction as $\phi = m_{\text{pay}}/m_0$ and show that,

$$\phi = \frac{\beta}{\alpha + \beta} (e^{-1/x} - \epsilon)$$

- (c) We assume that each kg of propellant mass will increase the mission cost by ξ_p (in \$/kg) and that each day of thruster firing will incur an additional penalty of ξ_t (in \$/day), such that the penalty cost is,

$$C = \xi_p m_p + \xi_t t$$

Every kg of payload mass has a cost given by f , such that the payload cost is $C_{\text{pay}} = f m_{\text{pay}}$. Normalizing the penalty cost by the payload cost, show that,

$$C' = \frac{C}{C_{\text{pay}}} = \frac{k_1}{\phi} (1 - e^{-1/x}) + \frac{k_2}{\phi} \tau$$

and write down expressions for k_1 and k_2 in terms of the problem variables. Finally, define the non-dimensional cost-penalized transportation rate,

$$\psi = \frac{\phi}{\tau C'} = \frac{\beta}{\alpha + \beta} \left(\frac{e^{-1/x} - \epsilon}{\tau C'} \right)$$

- (d) Once z and ϵ are fixed, one can use $x = c/\Delta v$ as a sweep parameter, and construct plots of $\tau(x)$, $\phi(x)$ and $\psi(x)$ that will allow selection of the optimal time (maximum ψ) and the corresponding specific impulse, propellant fraction, etc. Do this for $\epsilon = 0.1$, $z = 1$. For ϕ and ψ , assume $\alpha = 0.05$ kg/W, $\beta = 0.15$ kg/W. Identify the minimum non-dimensional time possible, τ_{min} .
- (e) As a specific example, consider a spacecraft with an initial mass $m_0 = 150$ kg in its parking orbit, a Δv for low-thrust climb to its operational orbit of 450 m/s, and an additional Δv for orbit corrections over 4 years and de-orbit of 500 m/s. The thrusters are electrospray devices with $\eta_0 = 0.8$, and the loss velocity implied by $z = 1$. Calculate the total thrusting time and apportion it to climb-out and corrections/de-orbit. Calculate the specific impulse, engine efficiency, total power, payload mass, propellant mass, power supply mass, and payload and penalty costs (compare propellant vs time penalty costs). If the mean molecular mass of ions in the propellant is 200 g/mol, calculate the ion cost in volts. Assume $f = \$20,000/\text{kg}$, $\xi_p = \$30,000/\text{kg}$ and $\xi_t = \$1,150/\text{day}$.

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