

16.522 Space Propulsion Analysis of a Vacuum Hall Thruster

The Vacuum Hall Thruster is an example of a non-neutral stable discharge that brings out a variety of interesting phenomena. As you will be building one, this document might help you in its design. You could also use this analysis as a theoretical basis of your work, to be presented during your project debriefing.

The device has angular geometry, as described in lecture. The \vec{B} field is created by an auxiliary magnetic circuit, based on permanent magnets. The apparatus is placed in a “vacuum”, meaning in a neutral background with a residual gas pressure of the order $10^{-6} - 10^{-4}$ Torr. Starting with the inevitable random electron, which is always present, any electron near the face of the anode will be forced to execute a combination of Larmor rotations and $\vec{E} \times \vec{B}$ drift, the latter being in the circumferential direction. Thus, except for collisions, the electron is trapped in the magnetized region. When a neutral particle passes by, it has a probability of colliding and becoming ionized. After this, the new electron is trapped, but the ion is accelerated axially and leaves at high speed. One additional effect of the ionizing collision (as well as any other electron collision) is to scatter the electron into a different magnetic line, on the higher potential side, so that electrons actually reside for a finite time in the magnetic field region.

The electron population will build up to an upper limit, determined by space charge limitations. As more electrons accumulate, the local potential in that region becomes lower. If a **potential well** were to develop, this would prevent ions from escaping, and their accumulation would eventually cancel the potential well. So, a reasonable assumption is that in the steady state, the potential distribution would have zero slope at the edge of the region where $B_r \neq 0$. In that case, we can assume ions will not accumulate, and provided the gas density is low enough, their occurrence will be infrequent enough to assume $n_i \ll n_e$.

- (a) Assume Δ is the axial size of the region where the potential changes from the applied value V to zero and that B_r and n_e are constant in $0 < x < \Delta$ and zero elsewhere. Calculate the 1-D potential and field (E_x) distribution in this layer.
- (b) An electron released from rest in a crossed (E_x, B_r) field will move on a cycloid. Show that the maximum displacement along ($-x$) is,

$$-\Delta x = 2 \frac{m_e E_x}{e B_r^2}$$

so that after a collision we may assume an electron will “jump” $\frac{m_e E_x}{e B_r^2}$ in the direction of the anode.

- (c) Take as a representative value of E_x its mean value in the $0 < x < \Delta$ layer (as computed in part (a)), and calculate the number of collisions N_c which a “representative electron” (say, one created at $x = \Delta/2$) will undergo before being collected by the anode.
- (d) Of these collisions, a fraction Q_i/Q_{tot} will be ionizing collisions (Q_i is the ionization cross-section and Q_{tot} is the total cross-section, including excitation, elastic scattering,

etc). In the steady state, this number should be unity, because each electron must “reproduce” itself before escaping. Deduce from this a relationship between E_x , B_r and Δ .

- (e) Substitute in this relationship the E_x found in part (a) (say, the mean value in the layer), and calculate the steady state electron density n_e and the layer voltage V .
- (f) Since each electron only needs to produce one other electron, the energy lost will be no more than a few times the ionization energy of the gas (a few tens of volts). This can be ignored in comparison to the applied voltage (a few kV). Using this, estimate the mean electron velocity \bar{c}_e (in the rms sense), assuming our “representative electron” gains $V/4$ from its birth to the representative point of observation.
- (g) Using this, calculate the overall ion production rate per unit of layer area (assuming a uniform gas density n_n). Express this as an ion current density j_i (A/m²).

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