Session 15: **Arcjet Thrusters**

In a conventional Resistojet, the amount of energy added to the flow (and in consequence the specific impulse) is limited by the maximum working temperature of the materials used in its construction: the walls of the device are directly heated by an electric source and is through contact with these walls that the flow increases its enthalpy. Better performance could be achieved if the gas is heated beyond the limit temperature for the material containing the flow. This is achieved by an Arcjet Thruster, a propulsion device in which a steady electric discharge is generated in the center of the channel. This discharge (or arc) provides the required energy increase that, on average, raises the temperature of the gas beyond what is achievable by a Resistojet.

The details of the discharge are somewhat complicated and in fact form a very rich field of study. As seen in the previous lecture, the electric conductivity of the gas increases with temperature and at some point it reaches a value in which the equilibrium of heating and dissipation provides the steady situation required for the formation of an arc filament. In this lecture we explore some of the generalities of these processes.

Ohmic Dissipation - Stability, constriction

The conductivity increases rapidly with T_e in the fully Coulomb-dominated range,

$$\sigma = \frac{0.0153\, T^{\frac{3}{2}}}{\ln \Lambda} \qquad \text{([Si/m] with in °K)}$$

Notice also how, in this limit (which occurs at high temperature, as α approaches 1), the conductivity becomes independent of the kind of gas in question, except for small influences hidden in $\ln \Lambda$.

One important consequence of $\sigma = \sigma(T)$ is the tendency for current to concentrate into "filaments", or "arcs". To understand this, consider the amount of work done by electric forces to overcome the "friction" on the electrons due to collisions. The force on the n_e electrons in a unit volume is $-en_e\vec{E}$, and these electrons reach a terminal velocity $\vec{u}_e$ as they slide against friction. Hence the power dissipated per unit volume is $D_\Omega = -en_e\vec{E} \cdot \vec{u}_e = -(en_e\vec{u}_e) \cdot \vec{E}$,

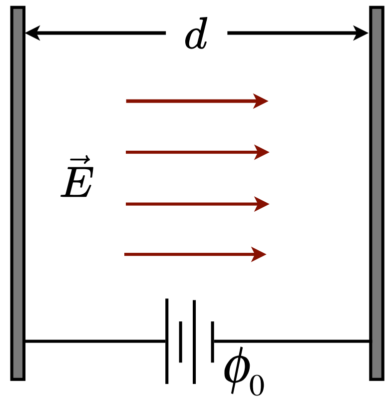
$$D_\Omega = \vec{j} \cdot \vec{E}$$

Since Ohm's law gives $\vec{j} = \sigma\vec{E}$, we can put,

$$D_\Omega = \sigma E^2 = \frac{j^2}{\sigma}$$

The simplest situation is one with an initially uniform plasma subject to a constant applied field $\vec{E}$, such as would occur between the plates of a plane capacitor.

Regardless of the path taken by the current, if the plates are large and the gap is small, the field $E = V/d$ remains unchanged. If we now look at $D_\Omega = \sigma E^2$, we see that the dissipation becomes large wherever the conductivity (hence the temperature) is large. Starting from uniform temperature, if a small non-uniformity arises such that T is higher along a certain

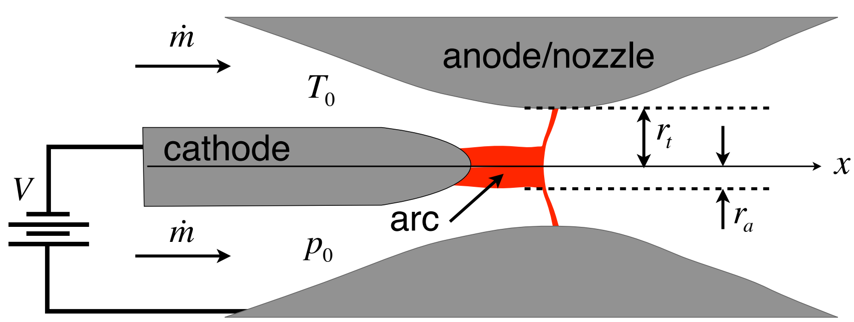


path, that path becomes more conductive, heats up due to extra Ohmic dissipation, and this reinforces the initial non-uniformity. The result is a *constriction* of the current into a filament or "arc".

In principle, the constriction process would continue indefinitely and lead to arcs of zero radius and infinite current density. But as the temperature profile steepens, heat will increasingly diffuse away from the hot core to the cooler surroundings, and, provided it can be removed efficiently from there, an equilibrium is eventually reached at some finite arc radius and arc core temperature. Clearly, the detailed end result will depend on the details of the thermal management of the gas: the more efficient the cooling of the background, the more the constriction can progress, and the hotter the eventual arc. This counter-intuitive result (more cooling leads to hotter arcs) is one of several paradoxical properties of arcs, all of them related to their being the result of a statically unstable situation.

This analysis neglects the additional constriction or "pinching" that could be produced magnetically via a magnetic force $\vec{f} = \vec{j} \times \vec{B}$ that arises by the self-field induced by the arc current $\nabla \times \vec{H} = \vec{j}$.

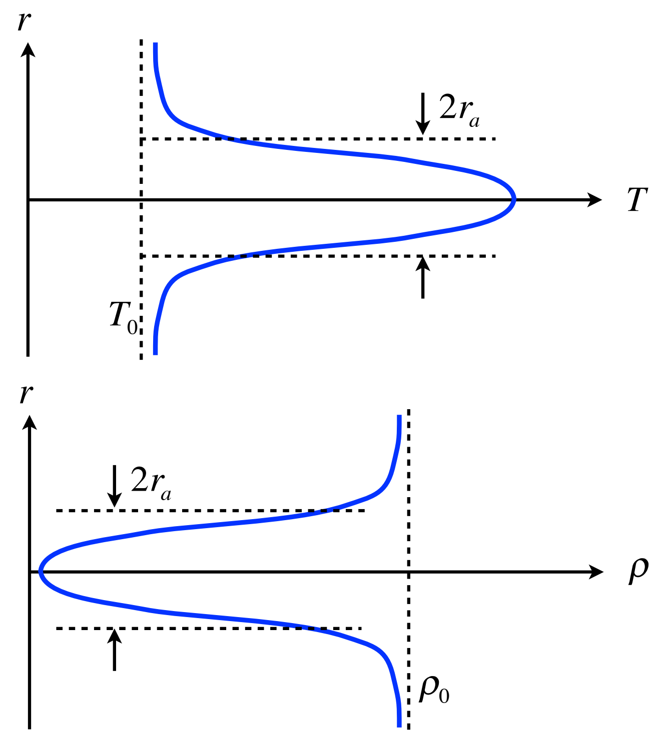
A schematic of an arcjet thruster is shown below:



These devices take advantage of the channel constriction created by an arc to improve the thruster performance. To see how this is possible, consider the radial distributions of temperature and density at some cross section of the arc.

It is clear that the arc core can reach very large temperatures, but in this region the density becomes very small. The effect is not heating the flow, but changing the flow field.

From the regular definitions of thrust coefficient and characteristic velocity,



$$c_F \;=\; \frac{F}{p_0 A_t} \qquad \text{and} \qquad c^* \;=\; \frac{p_0 A_t}{\dot{m}} \qquad \text{so that} \qquad c \;=\; c_F c^*$$

However, a channel constriction due to the arc will modify the flow rate through an area variation $A_t' = \pi\,(r_t^2 - r_a^2)$, while c^* remains constant since the upstream properties do not appreciably change,

$$c^* \;=\; \frac{\sqrt{RT_0}}{\Gamma(\gamma)}$$

The thrust coefficient is a measure of how effective is a nozzle to expand the hot gases generated upstream, and depends on the physical size of the throat. Therefore, it also remains unchanged in this situation.

The specific impulse then becomes,

$$c \;=\; c_F c^* \frac{r_t^2}{r_t^2 - r_a^2} \;=\; c_F c^* \phi$$

where $\phi > 1$. For instance, if $r_a = r_t/2$, then $\phi = 1.3$, which means 30% more Isp.

A more detailed model of the temperature distribution is needed to calculate the actual performance of arcjets. In a fundamental way, one would use an energy balance as the basis for this type of modeling. For instance, taking the energy source to be only ohmic dissipation $P_{in} = D_\Omega = \sigma E^2$ and assuming that an axially symmetric, cylindrical constricted arc is not heated by radiation P_r (although the cool gas is, from radiation produced by the arc) and is in local thermal equilibrium, then the gas temperature radial profiles could be found by a power balance,

$$\sigma E^2 \;=\; P_r - \frac{1}{r}\frac{d}{dr}\left(r\kappa\frac{dT}{dr}\right)$$

Given models for the electrical σ and thermal κ conductivities as a function of gas temperature, one should then be able to model heat addition and give first estimates of thruster performance. Better models require the introduction of terms at the arc anode/cathode contacts plus terms accounting for heat convection.

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