Session 16: **Hall Thrusters**

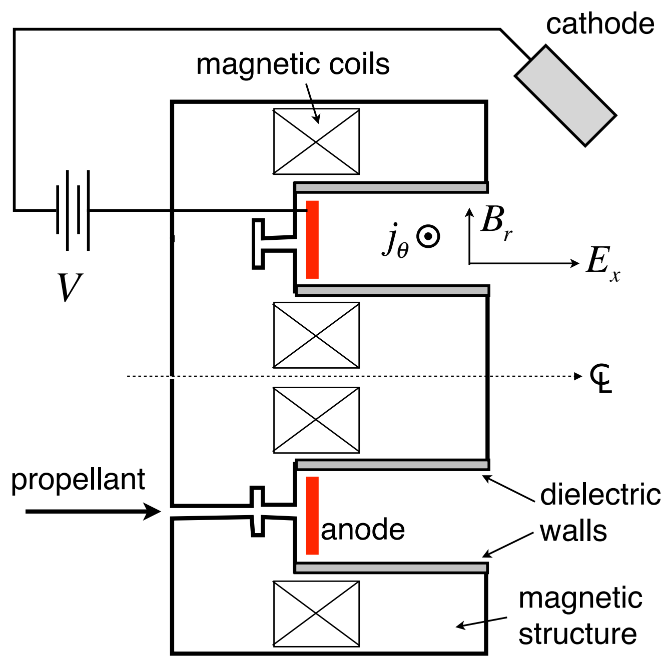
Hall thrusters are electrostatic ion accelerators in which the grid system (which serves in classical ion engines to anchor the negative charges used to accelerate the ions) is replaced with a relatively strong magnetic field perpendicular to the flow. This magnetic field impedes the counterflow of electrons in the accelerating field, and, as will be shown, does away with the space-charge limitation which restricts the flow and thrust of ion engines.

Hall thrusters have had a curious and long history. Discovered in the early 1960s in the U.S. [1], and possibly independently in the USSR [2], they were later abandoned in the West when it became apparent that there were strong instabilities which could not be completely eliminated, and when some additional work in Germany [3] indicated higher effective plasma collisionality than had been expected. Contributing to the demise was the simultaneous successful development of gridded ion engines of the Kaufman type, which appeared to satisfy most high I_{sp} mission requirements then envisioned, and the promise of efficient hydrogen arcjets for the low and intermediate I_{sp} ranges. It later developed that many important missions optimize in the I_{sp} range of $1000 - 2000$ sec, not well covered by either of these types of engines, and this remained an unmet requirement for many years, at least in the West.

Development continued, however, in the USSR, particularly at the Kurchatov Institute in Moscow [2], under A.I. Morozov's leadership. Progressively more efficient configurations evolved there, and it was realized that the instabilities, while present and annoying, did not materially interfere with performance. The Soviet generic name for this type of engine was Thruster with a Closed Electron Drift, and two competing implementations have been developed, known respectively as the Stationary Plasma Thruster (SPT) and the Thruster with an Anode Layer (TAL). The general principles are the same in both, and will be discussed below.

By the early 1980's these engines had achieved operational status in the USSR, and have since flown in many (> 50) missions, which, however, were until recently limited to relatively small total impulses. Starting in 1991, with the complete removal of the earlier communications barrier, development re-started in the West, in the form of collaborative efforts with Russian teams aimed at improved life, lighter electronics and flight qualification to the generally more stringent Western standards. The strong interest shown by the user community stems from the ability of Hall thrusters to operate with fairly good efficiency ($\sim 50\%$) in the hitherto difficult specific impulse range around 1500 sec, and also from their relative simplicity compared to ion engines.

Many different varieties of Hall thrusters exist, created in several laboratories and companies around the world. Academic efforts have been directed mostly towards the understanding of ionization, electron trapping and diffusion, and loss mechanisms. These efforts include exploring the low power regime (< 50 W) with aims in miniaturizing plasma propulsion for applications in small, or power-limited spacecraft. While able to operate, small Hall thrusters suffer from reduced life and efficiency, due for the most part to the violent plasma environment required to maintain a collisionality level similar to larger devices. Recent work has been directed towards the design and development of high-power Hall thrusters in the 10-50 kW range, motivated in good measure by the steady improvement of solar power systems.



Hall Thruster Physics

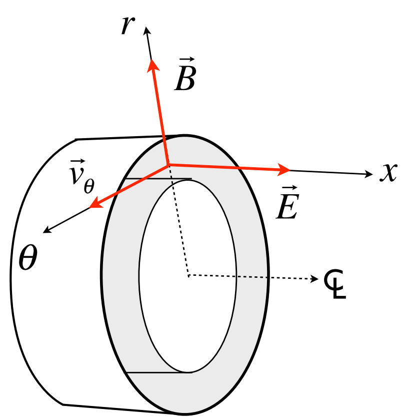
A Hall thruster schematic is shown in the figure above. It consist of a coaxial annular cavity where plasma is created by passing current between the annular anode on the upstream end of an otherwise dielectric cavity and the externally located cathode. The propellant enters this plasma cavity via an annular manifold at the anode. A radial magnetic field is applied, either by ring-shaped permanent magnets, or through coils and soft iron yokes. The magnetic field greatly slows down the axial mean velocity of the electrons, which, due to the low collisionality prevailing, are forced to execute mostly $\vec{E} \times \vec{B}$ drift around the annulus, while being radially confined by sheaths on the insulating walls.

The ions meanwhile, are only weakly affected by the magnetic field, and, if the density is low enough that collisions are rare, are simply accelerated by the electrostatic field to an exit velocity

$$v_i = \sqrt{\frac{2e\phi}{m_i}} \tag{1}$$

where ϕ is the potential at the place where the ion is created (with respect to the outside potential). Because of the quasi-neutrality facilitated by the presence of the electrons, no space-charge limitation arises in this type of thruster (as opposed to gridded ion thrusters), and the acceleration distance can be several cm, compared to the typical $0.5-1$ mm gap used in ion engines. This flexibility is one of the main advantages of the Hall thruster, removing the strong thrust density limitation dictated by the Child-Langmuir law in ion engines.

To the extent that collisions do occur, but, more importantly, because of electron scattering by a combination of plasma electrostatic fluctuations and wall collisions, electrons also travel axially across the magnetic field under the influence of the applied axial electric field. They are then collected by the upstream anode, and pumped by the power supply to an external cathode. The emitted electrons mainly join the accelerated ions to form a neutralized plasma beam, but, inevitably, a fraction also diverts upstream into the accelerator section. This fraction is to be minimized (this is the role of the magnetic field) because their acceleration to the anode potential is one of the devices loss mechanisms. On the other hand, not all of this energy is lost to the anode, because a good part of it is used to produce ionization of the injected neutral gas.



The name "Hall Thruster" arises form the mechanism by which thrust forces are exerted on the solid parts of the engine. As indicated, ions are simply accelerated by the electrostatic field, but since the ions are in a quasineutral plasma throughout, an equal and opposite electrostatic force is exerted on the free electrons in that plasma. However, in the presence of the radial magnetic field, these electrons are not free to accelerate towards the anode; instead, they drift azimuthally (perpendicular to both $\vec{E}$ and $\vec{B}$) at such a velocity as to generate an equal and opposite magnetic force on themselves. If we denote by x the forward axial direction the electrons end up drifting with a velocity,

$$\vec{v}_\theta = \frac{\vec{E} \times \vec{B}}{B^2} \tag{2}$$

The ions have no such azimuthal drift (or have a very small one, because their gyro (Larmor) radius is larger than the device's length), and so a net azimuthal current density arises, called a Hall current,

$$\vec{j}_\theta = -en_e \frac{\vec{E} \times \vec{B}}{B^2} \tag{3}$$

Given this current, the magnetic (Lorentz) force density on it is $\vec{f} = \vec{j}_\theta \times \vec{B}$, and an equal and opposite force is exerted by the plasma currents on the magnetic structure. We then have,

$$\vec{f} = en_e\vec{E} \tag{4}$$

i.e., the same as the forward electrostatic force on the ions, as it should be. The important point is that the structure is not electrostatically acted on (electric fields and electric pressures are too weak here), but magnetically, through the Hall current - hence the name. The device is a *Hall thruster*, but an *electrostatic accelerator*, a duality which has led to some confusion.

Electron diffusion in a magnetic field

The description of Hall thruster physics cannot be complete without some discussion of the diffusion of electrons towards the anode, forced by its higher electric potential. Eventually, as is the case in the ionization chamber of ion engines, all electrons, primaries and secondaries are collected by the anode. The goal is to maximize the lifetime of electrons to increase the probability of ionizing collisions with neutrals.

Let us begin by analyzing the electron diffusion in the absence of magnetic fields. In the electron momentum balance, main forces are the pressure gradient, the electric field and collisional retardation. Neglecting electron inertia and ignoring the electric field (since linear acceleration does not affect the conclusion), this reduces to,

$$\nabla p_e \;\approx\; -n_e m_e \nu_e \vec{v}_e \tag{5}$$

Where the collision frequency is $\nu_e = \sum n_j \bar{c}_e Q_{ej}$. We also have $p_e = n_e k T_e$, and for near-isothermal conditions $\nabla p_e = k T_e \nabla n_e$. Solving for the electron flux,

$$n_e \vec{v}_e \;\approx\; -\frac{\nabla p_e}{m_e \nu_e} \;\approx\; -\frac{k T_e}{m_e \nu_e} \nabla n_e \tag{6}$$

This is Fick's law of diffusion, with a diffusivity,

$$D_e \;=\; \frac{k T_e}{m_e \nu_e} \tag{7}$$

It is clear then that collisions impede the diffusion of electrons. In this situation, large collisions frequencies would result in the longest lifetimes before electrons are collected by the anode.

Assume now that a magnetic field (perpendicular to ∇p_e) is applied. We add the magnetic force to the momentum equation,

$$k T_e \nabla n_e \;\approx\; -n_e m_e \nu_e \vec{v}_e - e n_e \vec{v}_e \times \vec{B} \tag{8}$$

To solve for for the flux, take the cross product with $\vec{B}$ and write,

$$k T_e \nabla n_e \times \vec{B} \;\approx\; -n_e m_e \nu_e \vec{v}_e \times \vec{B} - e n_e \vec{v}_e \times \vec{B} \times \vec{B}$$

Let us use our tensorial notation to find a vectorial identity for $\vec{v} \times B \times B$,

$$\left[\vec{v} \times \vec{B} \times \vec{B}\right]_i = \epsilon_{ijk} \epsilon_{jlm} v_{el} B_m B_k = -(\delta_{il}\delta_{km} - \delta_{im}\delta_{kl}) v_{el} B_m B_k = v_{ek} B_i B_k - v_{ei} B_k B_k = -v_{ei} B^2$$

since $\vec{v}_e \perp \vec{B}$. We then have,

$$k T_e \nabla n_e \times \vec{B} \;\approx\; -n_e m_e \nu_e \vec{v}_e \times \vec{B} + e n_e \vec{v}_e B^2 \tag{9}$$

Eliminating $\vec{v}_e \times \vec{B}$ between Eqs. (8-9) leads to,

$$n_e \vec{v}_e \;\approx\; \frac{-\dfrac{k T_e}{m_e \nu_e} \nabla n_e + \dfrac{e k T_e}{m_e^2 \nu_e^2} \nabla n_e \times \vec{B}}{1 + \left(\dfrac{e B}{m_e \nu_e}\right)^2} \tag{10}$$

Recall that this formulation leaves the electric field out. To include it, just replace ∇n_e by $\nabla n_e + \frac{en_e}{kT_e}\vec{E}$. Including this term in Eq. (6) leads to the usual definition of electric conductivity. Define the non-dimensional factor,

$$\beta = \frac{eB}{m_e \nu_e} = \frac{\omega_{ce}}{\nu_e} \tag{11}$$

where ω_{ce} is the cyclotron frequency for electrons. Then,

$$n_e \vec{v}_e \approx \frac{-D_e \nabla n_e - \beta \times D_e \nabla n_e}{1+\beta^2} \tag{12}$$

Of these two terms, the second is perpendicular to both $\vec{B}$ and ∇n_e, and is called the $\nabla p \times \vec{B}$ drift. The main interest is on the first term, which is along $-\nabla n_e$ and $-\vec{E}$, as in regular diffusion and conduction (when there is an electric field).

We see that this cross-field diffusion is governed by $n_e \vec{v}_e \approx -D_\perp \nabla n_e$, with,

$$D_\perp = \frac{D_e}{1+\beta^2} \tag{13}$$

So, a high Hall parameter β can greatly reduce diffusion, compared to that in the absence of a magnetic field. High β means both, high magnetic field and/or low collision frequency.

As an example, in an ion engine or Hall thruster, with $T_e = 4$ eV $= 46,400$ K the *en* and *ei* cross-sections are roughly,

$$Q_{en} \sim 10^{-19}\ \mathrm{m}^2 \qquad \text{and} \qquad Q_{ei} \sim 4 \times 10^{-18}\ \mathrm{m}^2$$

and,

$$\bar{c}_e = \sqrt{\frac{8kT_e}{\pi m_e}} \sim 1.34 \times 10^6\ \mathrm{m/s}$$

For particle densities,

$$n_e \sim 2.8 \times 10^{17}\ \mathrm{m}^{-3} \qquad \text{and} \qquad n_n \sim 7.4 \times 10^{18}\ \mathrm{m}^{-3}$$

then,

$$\nu_{en} \sim 9.9 \times 10^5\ \mathrm{s}^{-1} \quad \text{and} \quad \nu_{ei} \sim 1.5 \times 10^6\ \mathrm{s}^{-1} \quad \text{so that} \quad \nu_e \sim 2.49 \times 10^6\ \mathrm{s}^{-1}$$

At a point in the engine where $B = 100$ gauss $= 0.01$ Tesla,

$$\omega_{ce} \sim 1.76 \times 10^9\ \mathrm{s}^{-1}$$

which means that the Hall parameter is,

$$\beta = \frac{\omega_{ce}}{\nu_e} = 706 \gg 1$$

Under these conditions, Eq. (13) reduces to,

$$D_\perp \approx \frac{D_e}{\beta^2} = \frac{kT_e}{m_e}\frac{\nu_e}{\omega_{ce}^2} \tag{14}$$

This last form shows that collisions favor diffusion, in contrast with Eq. (7), with no magnetic field, or $\beta \ll 1$. Eq. (14) also shows that $D_\perp$ scales as B^{-2}:

$$D_\perp \approx \frac{D_e}{\beta^2} = \frac{m_e kT_e}{e^2}\frac{\nu_e}{B^2}$$

and so, increasing B should provide very strong confinement of electrons. With the given numbers, we find,

$$D_e = 2.83 \times 10^5 \text{ m}^2/\text{s} \qquad \text{and} \qquad D_\perp = 0.57 \text{ m}^2/\text{s}$$

A diffusing substance spreads (in 1-D) roughly as $x \sim 2\sqrt{Dt}$. So, to spread by 1 cm, electrons would require a time $t \sim 4.5 \times 10^{-5}$ s.

It turns out, however, that electrons can diffuse faster than this in most cases. The physical reasons are apparently related to the *equivalent collisionality* produced by scattering of the electrons by small-scale plasma density fluctuations which are almost always present. This is the same situation that has kept tokamaks from delivering fusion power (only in that case it is the H^+ ions that leak through the confining $\vec{B}$ field).

Bohm obtained an empirical expression (with some theoretical guidance) for this so-called *anomalous* or *Bohm* diffusion,

$$D_{\text{Bohm}} = \frac{kT_e}{16eB} \tag{15}$$

and experiments in ion engines and Hall thrusters appear to confirm the B^{-1} dependence, but also seem to indicate a somewhat smaller diffusivity magnitude.

An often used expression is,

$$D_{\text{anomalous}} = \frac{kT_e}{c_B eB} \qquad (c_B \sim 16 \text{ to } 100) \tag{16}$$

It is of some interest to see what collision frequency would produce the same diffusivity as these fluctuations,

$$\frac{kT_e}{c_B eB} = \frac{m_e kT_e}{e^2}\frac{\nu_{\text{anomalous}}}{B^2} \quad \rightarrow \quad \nu_{\text{anomalous}} = \frac{\omega_{ce}}{c_B} \tag{17}$$

and so c_B can be thought of as an *anomalous Hall parameter.* For modeling purposes, one often adds together the regular and anomalous collision frequencies $\nu_e + \nu_{\text{anomalous}}$ in calculating diffusivity $D_\perp$ by Eq. (14).

Thrust Capability

Unlike the case of a gridded ion engine, there is not a clear-cut upper limit to the obtainable thrust density in a Hall thruster. In principle, increasing the mass flow rate through a given engine would increase thrust by the same factor, provided the same degree of ionization can be maintained. The voltage is presumed to be kept the same, since this controls specific impulse fairly directly. The ion beam current and the electron current leaking into the accelerating channel would increase as the mass flow rate, so that the energetic ability of these electrons to ionize the increased neutral flow would be maintained. Since no space charge develops these increases in flow, current and power could go on indefinitely.

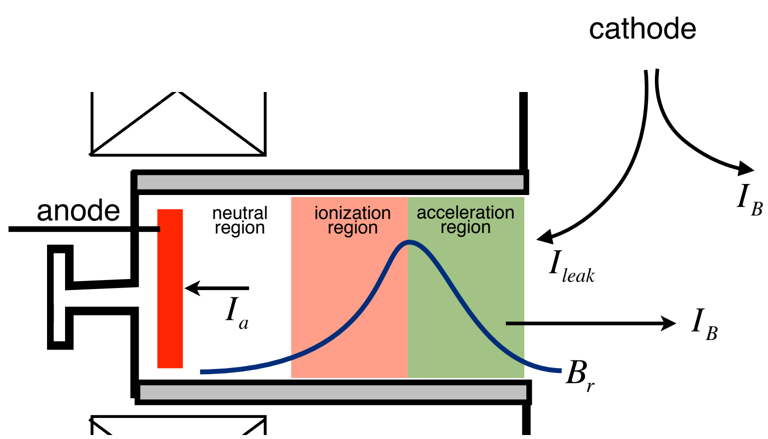
The above reasoning has ignored the various effects of collisions. As the flow rate, and hence the gas density increases, so does the scattering collision frequency of electrons with heavy particles, $\nu_{eH} = n_H \bar{c}_e Q_{eH}$. When this frequency becomes comparable to the gyro or cyclotron frequency ω_{ce}, electrons can no longer maintain pure azimuthal drift, because each collision will allow the electric field to nudge them in the direction of the anode by about one Larmor radius, correspondingly to a velocity similar to the azimuthal drift velocity. Thus, electrons are now able to cross the magnetic barrier at a velocity,

$$v_{e_x} \approx \frac{E}{B}\frac{\nu_{eH}}{\omega_{ce}} \tag{18}$$

which increases with ν_{eH}, hence with n_H and ultimately with $\dot{m}$. The electron density n_e in the acceleration region is governed by the flow of ions from the upstream ionization region, as shown in the schematic below, and increases generally in the same proportion as $\dot{m}$ or I_B. This means that the electron leakage current,

$$I_{\text{leak}} = e n_e \frac{E}{B}\frac{\nu_{eH}}{\omega_{ce}} \tag{19}$$

will increase faster than I_B, due to the n_H factor in ν_{eH}.



As will be discussed later, one of the factors determining the overall efficiency η of the device is the anode efficiency,

$$\eta_a = \frac{I_B}{I_a}$$

and from the schematic, ignoring wall losses, $I_B = I_a - I_{\text{leak}}$, so,

$$\eta_a \; = \; 1 - \frac{I_{\text{leak}}}{I_a} \; = \; \frac{1}{1 + I_{\text{leak}}/I_B}$$

which shows how the higher electron collisionality would hurt efficiency. By the way, Eq. (19) also shows that, if this were the only problem with higher flow rates, increasing the magnetic field strength (if it were feasible) would be a way to counteract it.

Quantitatively, we would like to keep a low leak ratio, say $I_{\text{leak}}/I_B \leq 0.2$. Assuming the ions constituting I_B cross the magnetic barrier at about 1/2 of their eventual speed v_i, this yields,

$$\frac{I_{\text{leak}}}{I_B} \; = \; \frac{v_{e_x}}{v_i/2} \; = \; \frac{2}{v_i}\frac{E}{B}\frac{\nu_{eH}}{\omega_{ce}} \; \leq \; 0.2 \quad \rightarrow \quad \nu_{eH} \; \leq \; 0.1\frac{v_i B \omega_{ce}}{E} \tag{20}$$

For the SPT-100 thruster, $v_i = 16{,}000$ m/s, $B \sim 0.02$ T (corresponding to an electron cyclotron frequency $\omega_{ce} \sim 3.5 \times 10^9$ rad/s). The axial electric field E is about 5000 V/m in the high B region. Eq. (20) then requires, $\nu_{eH} \leq 2.2 \times 10^7$ s^{-1}. At $T_e \sim 10$ eV, we have $\bar{c}_e \sim 2.1 \times 10^6$ m/s, and the scattering cross-section for *en* collisions is about 3×10^{-19} m^2. We then obtain a limiting neutral density in this region,

$$n_H \; = \; \frac{\nu_{eH}}{\bar{c}_e Q_{eH}} \; \leq \; 3.5 \times 10^{19} \text{ m}^{-3}$$

Measured neutral densities in this type of thruster are still significantly lower than this value.

Operation of Hall thrusters in such *high-flow* regime has recently been considered as an alternative to MPD propulsion when high power is available (upwards of 50 kW). The physics of this regime could be, at least in principle, explored at flow rates several times above the nominal for a given thruster type. Interestingly, no systematic study of this operation mode has been carried out, in part because very high powers have been associated with the development of nuclear reactors for space applications, as always a contentious issue. The limited availability of facilities capable to handle high powers has also played a role in the lack of data in this regime.

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16.522 Space Propulsion
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