16.540 Spring 2006

PRESSURE FIELDS AND UPSTREAM INFLUENCE

PLAN OF THE LECTURE

- Pressure fields and streamline curvature
  - Streamwise and normal pressure gradients
  - One-dimensional versus multi-dimensional flows

- Upstream influence and component coupling
  - How does the pressure field vary upstream of a fluid component and when does this matter?

- Pressure fields and the asymmetry of real fluid motions

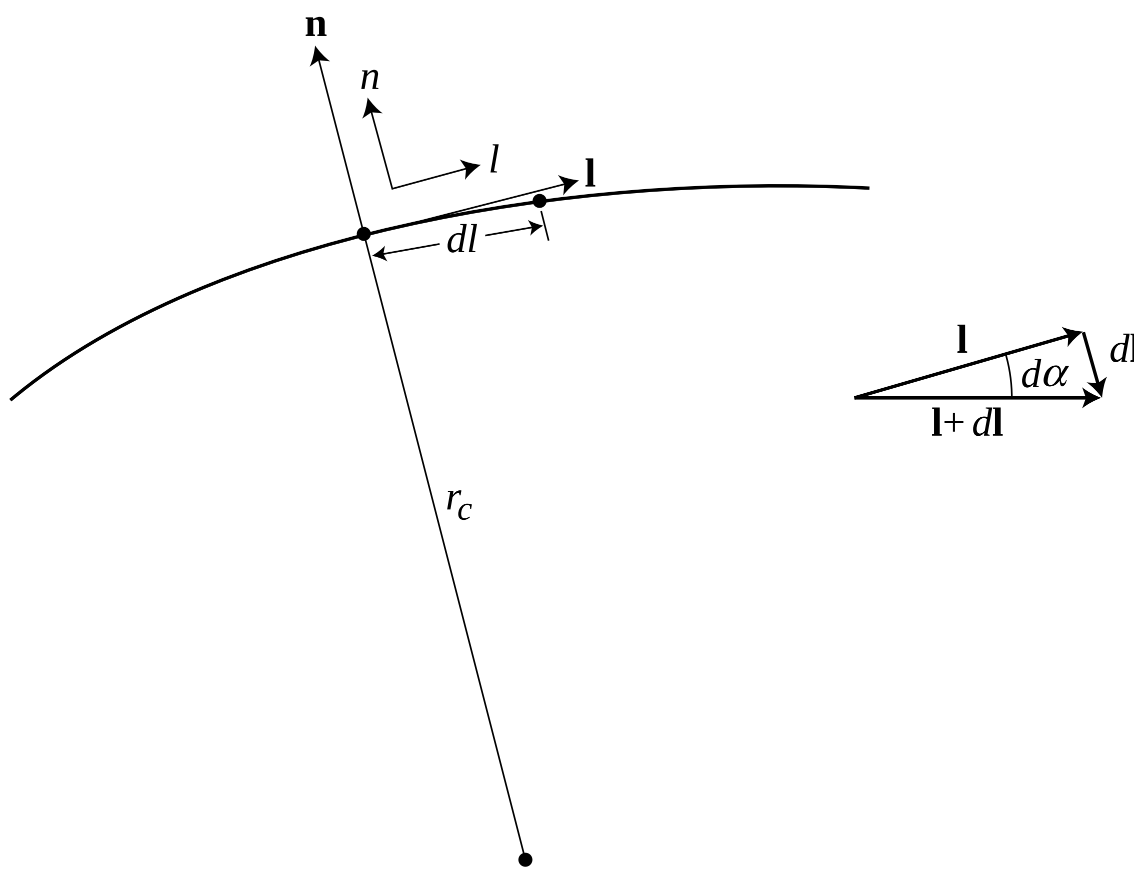
NORMAL AND STREAMWISE PRESSURE GRADIENTS

- Inviscid flow
- Streamwise:

$$udu = -dp/\rho$$

$or,$

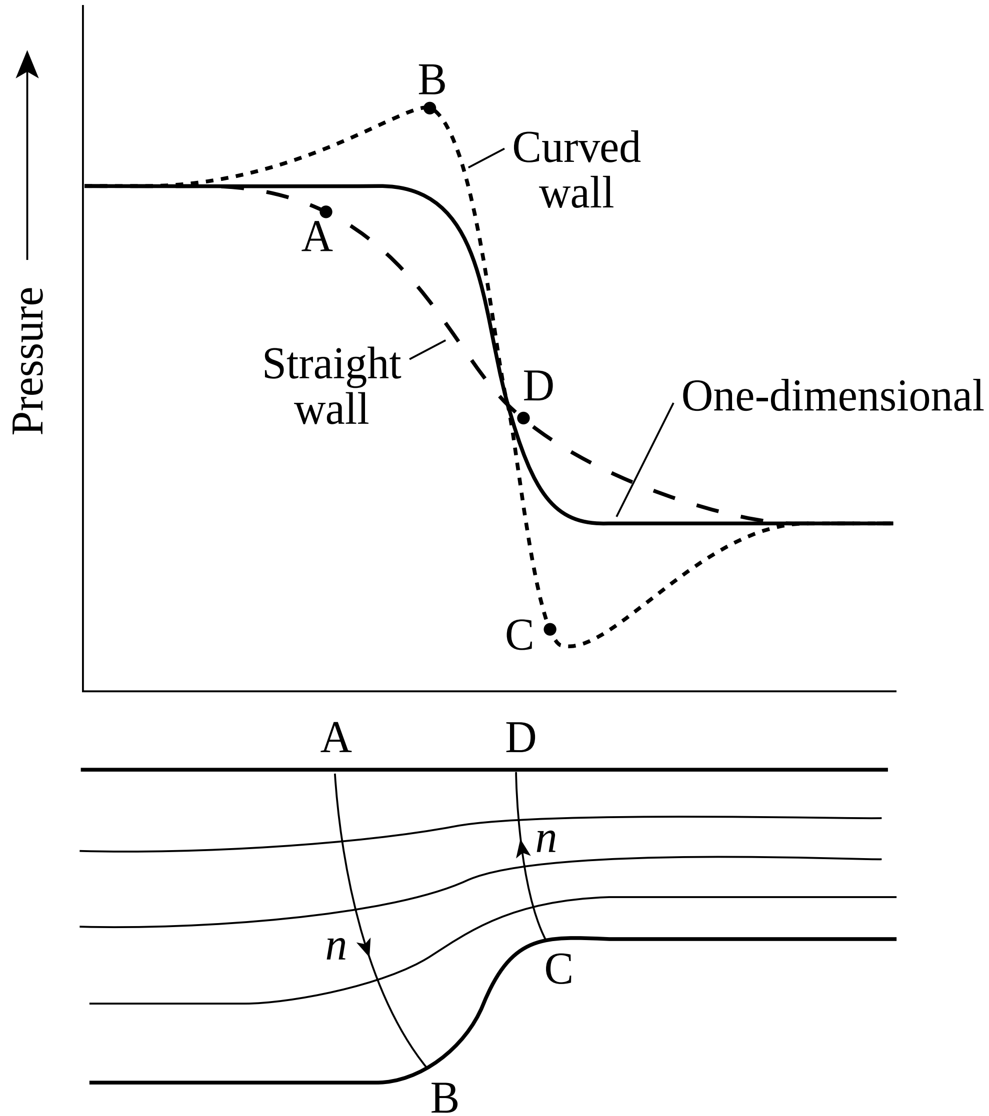
$$u\frac{\partial u}{\partial l} = \frac{1}{\rho}\frac{\partial p}{\partial l}$$



- Normal

$$\frac{u^2}{r_c} = \frac{1}{\rho}\frac{\partial p}{\partial n}$$

STREAMLINES AND WALL STATIC PRESSURES



ONE-DIMENSIONAL AND TWO-DIMENSIONAL DESCRIPTIONS

- In a one-dimensional representation of a contraction, the pressure gradient is always negative (or zero)

- If adopt a higher fidelity (2D) description, this is not true

- Is it *possible* to have a contraction in which there is *no location* that has an adverse (non-favorable) pressure gradient?

- Why is this important?

UPSTREAM INFLUENCE OF FLUID COMPONENTS

- Approximate equation for the static pressure field
- 2-D, inviscid, steady flow, constant density
- Velocity viewed as a uniform mean flow, $\overline{u}_X$, plus "small" non-uniformities, u'_x, u'_y :

$$u_x = \overline{u}_x + u'_x$$

$$u_y = u'_y$$

- Neglect products of small quantities in momentum equation

$$\text{x-momentum:} \quad \overline{u}\frac{\partial u'_x}{\partial x} + u'_y\frac{\partial u'_x}{\partial y} = -\frac{1}{\rho}\frac{\partial p'}{\partial x}$$

where p' is the departure from uniform static pressure

There is no term $u'_x \dfrac{\partial \overline{u}}{\partial x}$ because $\overline{u}$ is uniform

So,

$$\bar{u}_x \frac{\partial u_x'}{\partial x} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} \qquad \text{(a)}$$

and

$$\bar{u}_x \frac{\partial u_y'}{\partial x} = -\frac{1}{\rho}\frac{\partial p'}{\partial y} \qquad \text{(b)}$$

Take $\frac{\partial (a)}{\partial x} + \frac{\partial (b)}{\partial y}$, yielding, using continuity

$$0 = \bar{u}\frac{\partial}{\partial x}\left(\frac{\partial u_x'}{\partial x} + \frac{\partial u_y'}{\partial y}\right) = -\frac{1}{\rho}\left(\frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial y^2}\right)$$

$$\boxed{\nabla^2 p' = 0}$$ Laplace' s Equation

- Famous equation with neat properties
- We will apply this to see the upstream influence

UPSTREAM INFLUENCE

- Important question in internal flow systems-
  - When are components coupled aerodynamcially
  - When can they be considered independent?

- Laplace's equation gives direct and simple qualitative answer
- Laplace's equation gives direct and simple quantitative answer
- First part:

- $\nabla^2 p'$ has no intrinsic length scale
- If pick a y - length => x - length is set
- Consider an "unrolled" annular flow field

A DIGRESSION: WHAT DO I MEAN BY "LENGTH SCALE"?

- What is an example of an equation ***with*** a length scale?

- How about the momentum equation for viscous, constant-pressure flow?
  - This is an idealized example but it does make the point

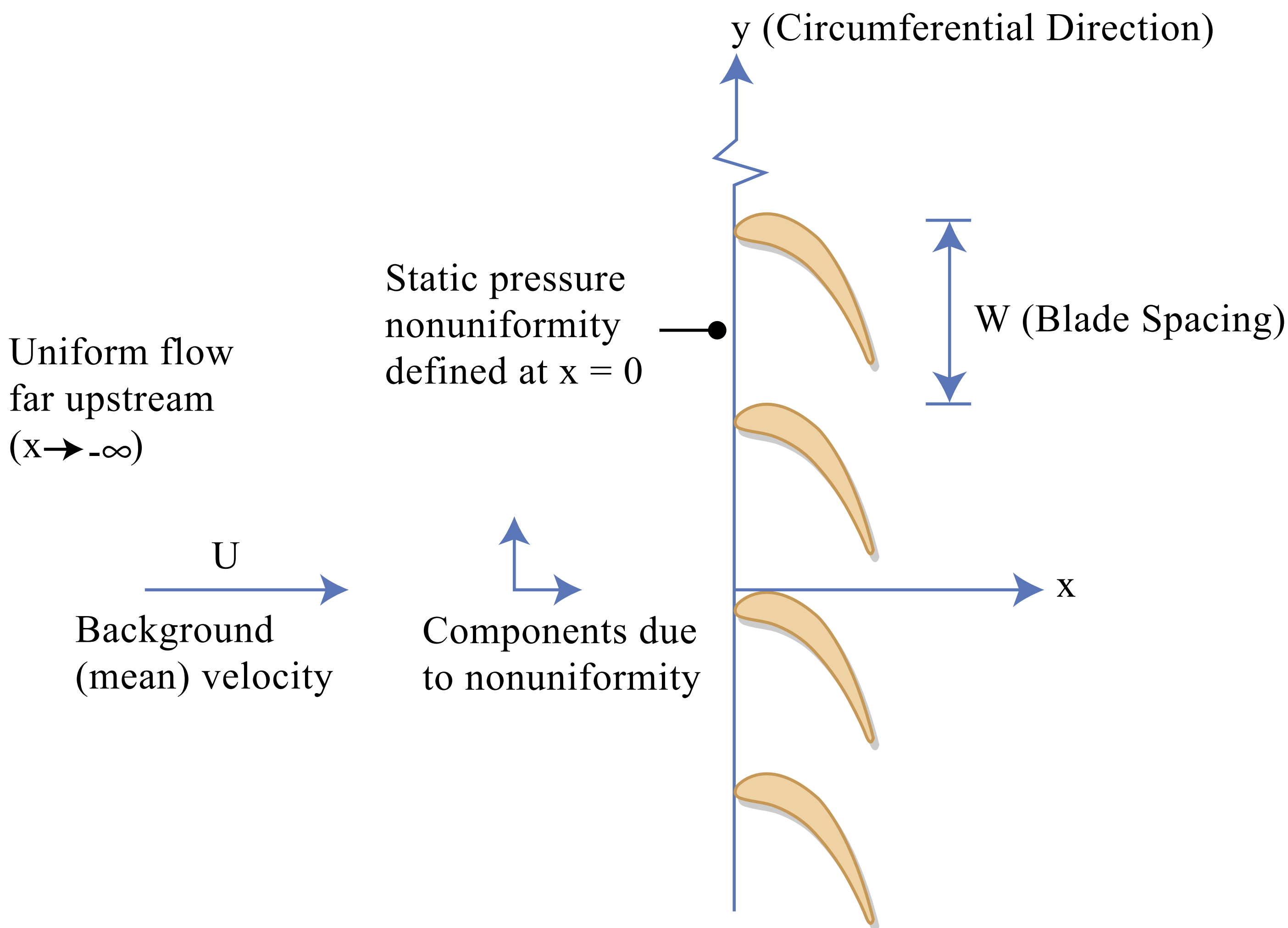
- Does this equation lead to some length scale, i.e. does a length scale naturally arise out of the structure of the equation?

- If so, what does it mean physically?

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = \nu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} \right)$$

BACK TO LAPLACE: RELATION OF CIRCUMFERENTIAL AND AXIAL LENGTH SCALES

- Consider an “unrolled” annular flow field, mean radius r_m
- Suppose we have a circumferential length scale r_m/n, then the axial length scale (extent of upstream influence) is also r_m/n
- Circumferential length scale sets the region of upstream influence
- Will show this explicitly in two examples:
- Upstream influence of a *circumferentially* periodic non-uniformity
- Upstream influence of a *radially* non-uniform flow



Flow domain used in estimation of upstream influence region for periodic array (turbine blading); region of interest is $x < 0$.

Figure by MIT OCW.

UPSTREAM INFLUENCE OF A CIRCUMFERENTIALLY NON-UNIFORM FLOW

- Blade row with blade-to-blade spacing W
- Whatever the loading distribution (compressor, turbine, pump) the static pressure distribution along x=0 can be written as
  - $p'(0,y) = \sum_{n=-\infty}^{\infty}\left(a_n e^{2\pi i n y/W}\right)$; Fourier series
  - To match this boundary condition, $p'(x,y)$ must also be of this y-dependence
  - Also p' (static pressure non-uniformity) must be bounded far upstream
  - Thus

$$p'(x,y) = \sum_{n=-\infty}^{\infty} f_n(x)\left[a_n e^{2\pi i n y/W}\right]$$

  - Plugging in to Laplace, $f_n(x)$ is found to have the form of exponentials: $e^{2\pi n x/W}$ and $e^{-2\pi n x/W}$

- Physical solutions must be bounded far upstream
- Thus, only positive exponentials allowed

$$p'(x,y) = \sum_{n=-\infty}^{\infty} e^{2\pi|n|x/W}\left[a_n e^{2\pi i n y/W}\right]$$

- Lowest harmonic (often, but not always, n=1) has the largest upstream influence

$$p'(x,y) \propto e^{2\pi|n|x/W}\left[a_1 e^{2\pi i n y/W} + a_{-1} e^{-2\pi i n y/W}\right]$$

$$p'(x,y) = |p'(0,y)| e^{2\pi|n|x/W} \qquad \text{Exponential decay}$$

- Different phenomena have different upstream extents of influence

FEATURES OF THE SOLUTION

- We neglected nonlinear terms: are they important over the domain of interest or, more precisely, *for the problem of interest?*
- What else did we neglect?
- At a distance W/2 upstream the non-uniformity is 0.04 its value at x=0
- Blade spacing length scale is W
- Inlet distortion length scale is radius of the compressor
  - Upstream effects are much stronger
- What can we state about applicability and limits of the conclusions from this simple analysis?

INSTRUMENTATION FOR TF30 ENGINE TEST

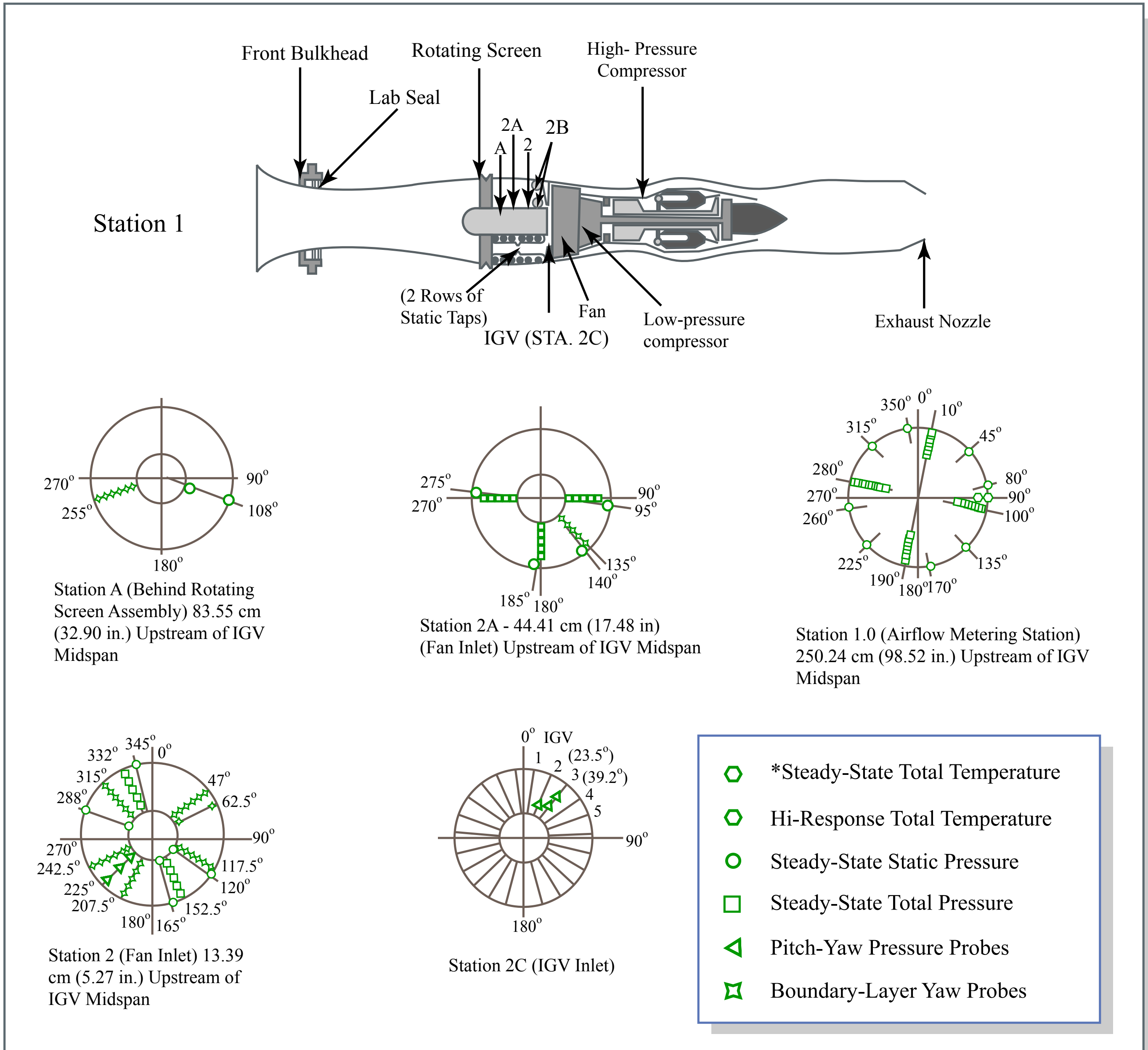


Figure by MIT OCW.

BULLET NOSE EXTENSION WITH PRESSURE TAPS

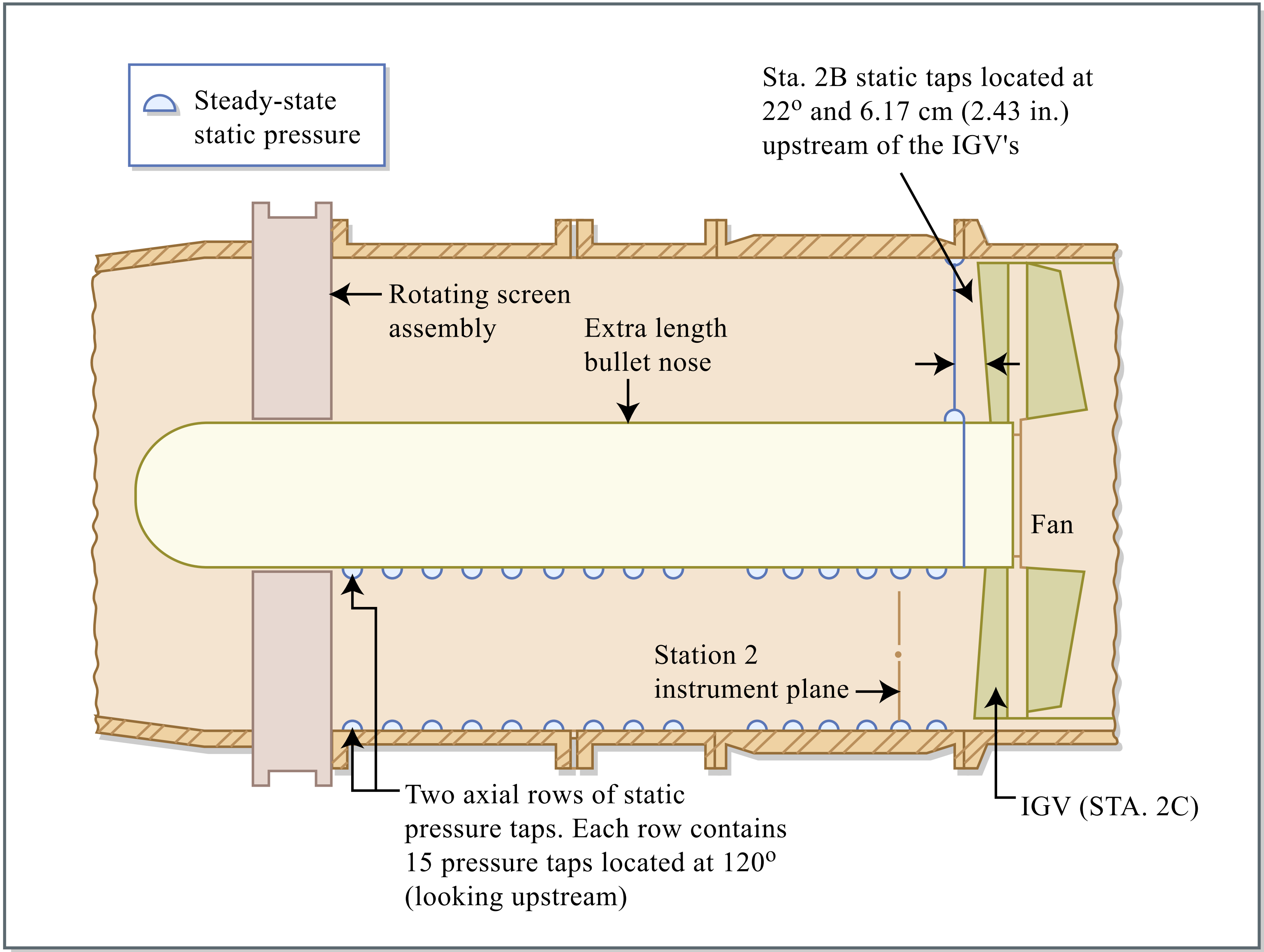
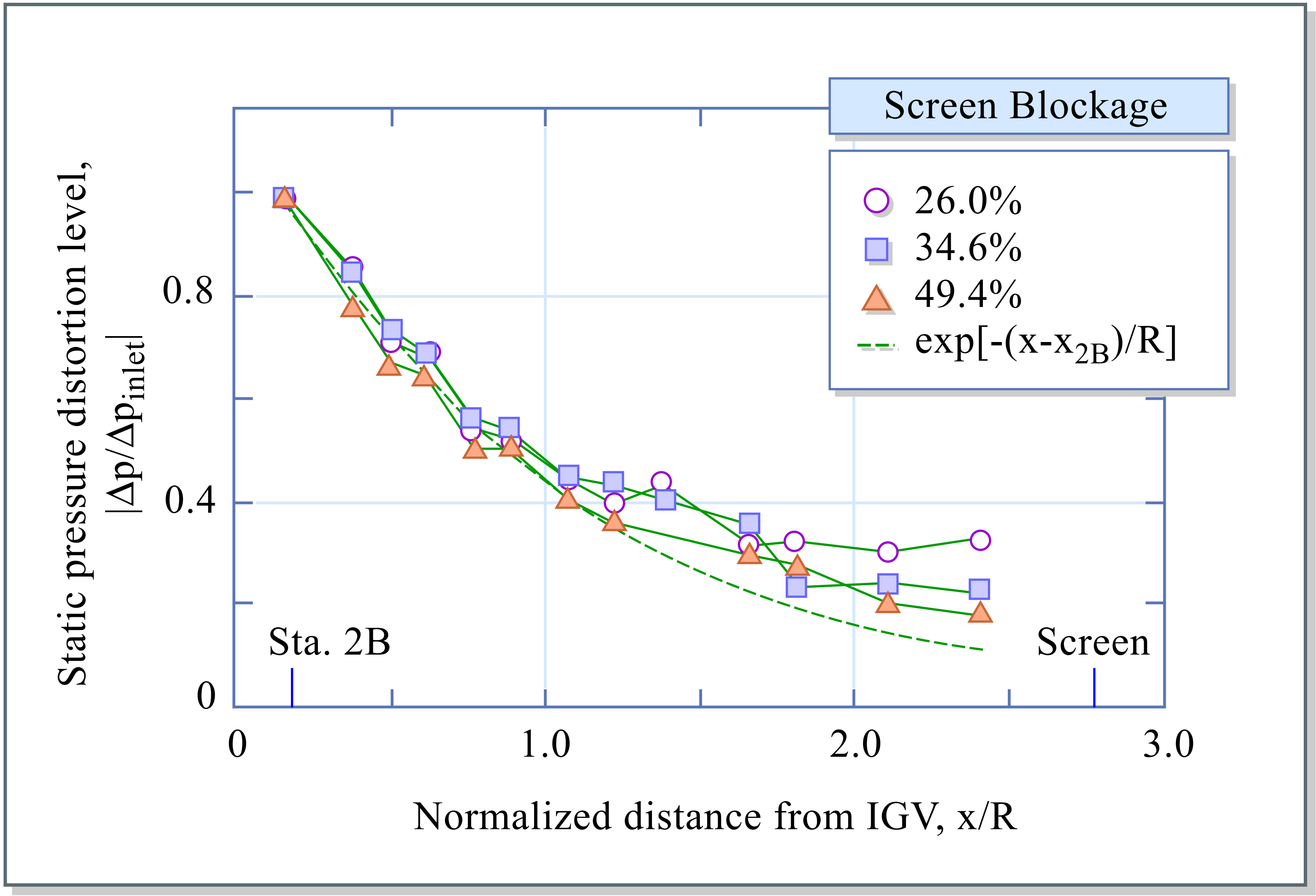


Figure by MIT OCW.

VARIATION OF STATIC PRESSURE WITH DISTANCE UPSTREAM OF AXIAL COMPRESSOR

[Soeder and Bobula]



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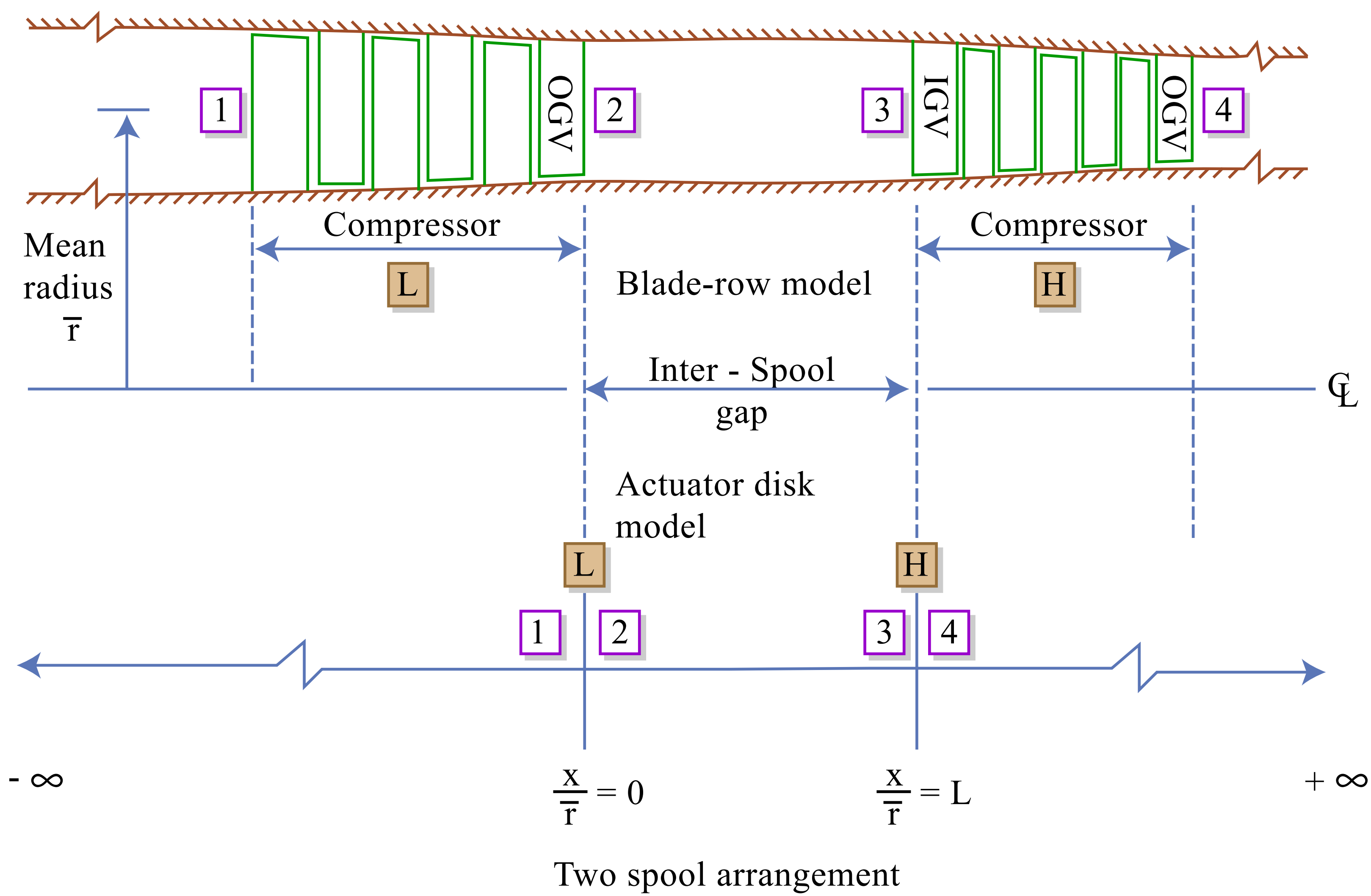
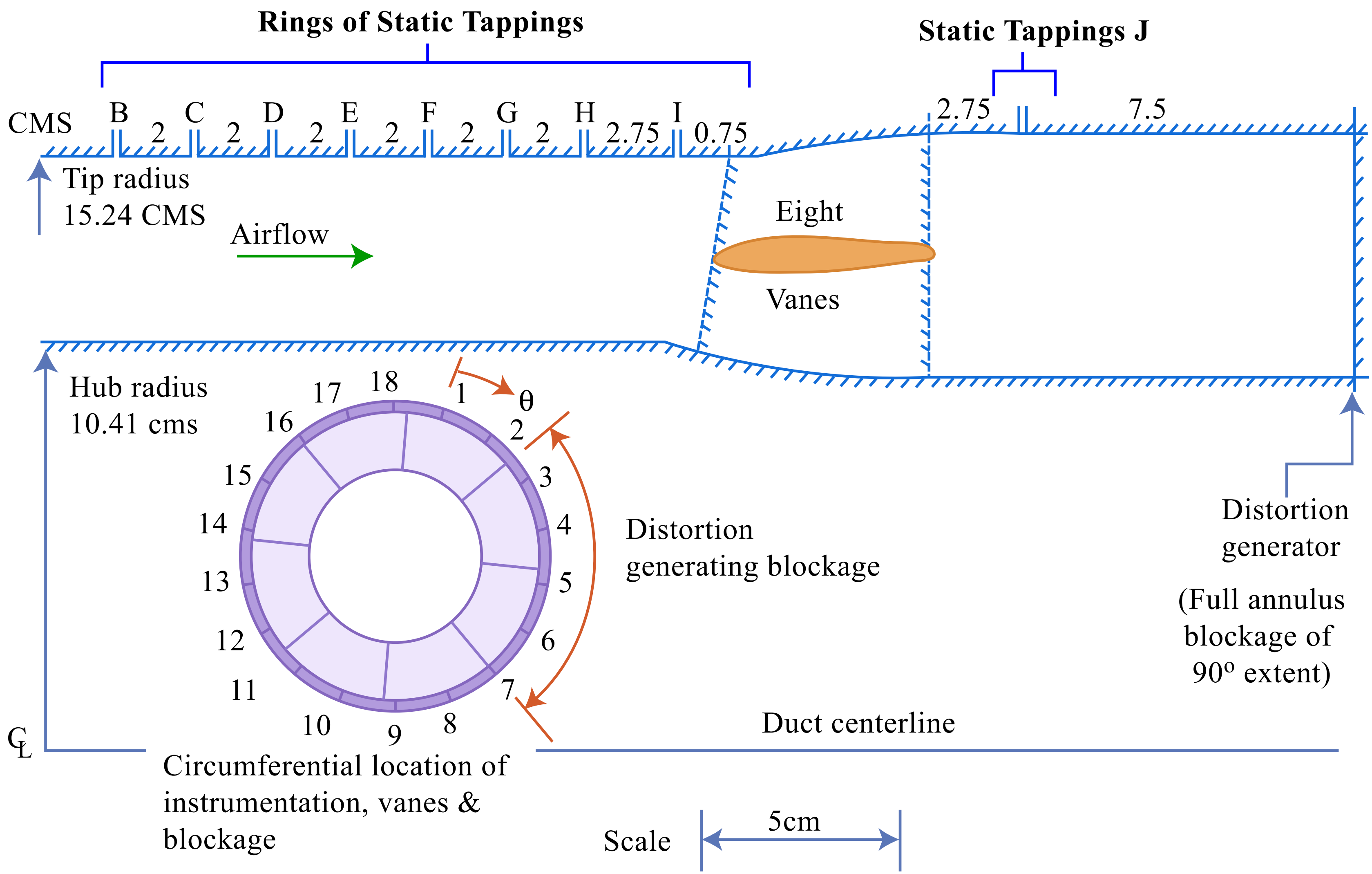


Compressor Coupling in Two Shaft Engine

Figure by MIT OCW.

[Williams and Ham]



Model Test Geometry and Instrumentation

Figure by MIT OCW.

[Williams and Ham]

UPSTREAM DECAY OF STATIC-PRESSURE DISTORTION

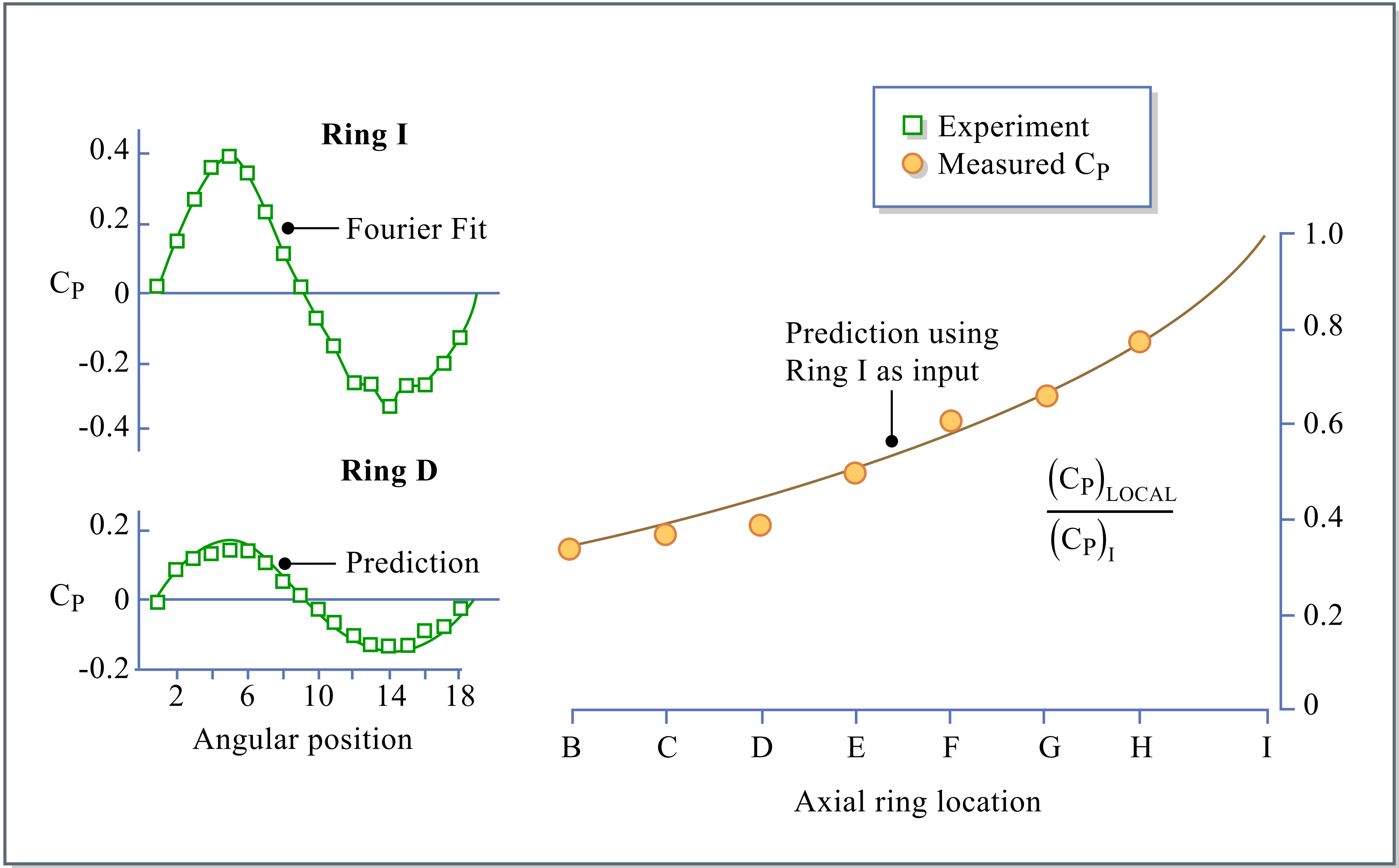


Figure by MIT OCW.

[Williams and Ham]

FEATURES OF THE UPSTREAM FLOW FIELD (Low Mach Numbers)

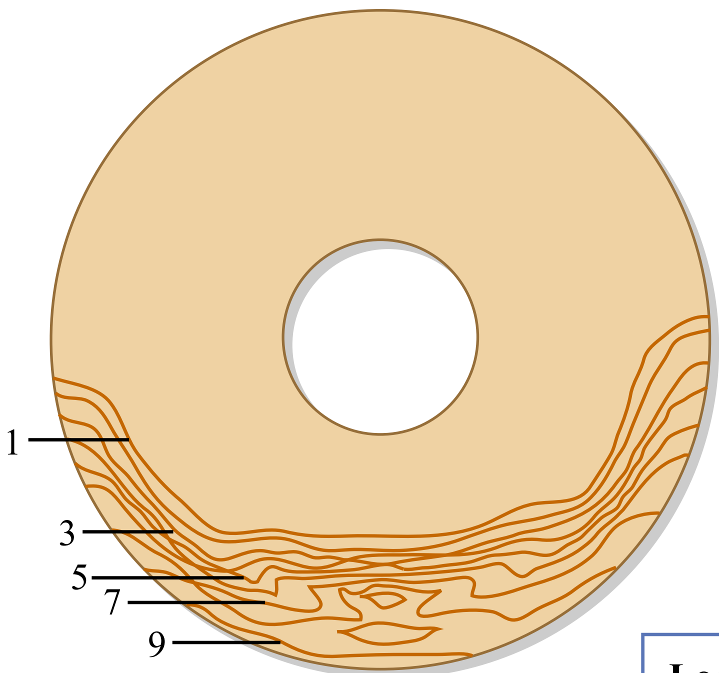
- Inlet distortion (non-uniformity with length scale R)
- Total pressure constant along streamlines (Bernoulli)
- Static pressure obeys Laplace's equation: $\nabla^2 p' = 0$

$$\frac{1}{r_m^{\ 2}}\frac{\partial^2 p'}{\partial \theta^2} + \frac{\partial^2 p'}{\partial x^2} = 0 \quad ; \quad p' \text{ is static pressure disturbance}$$

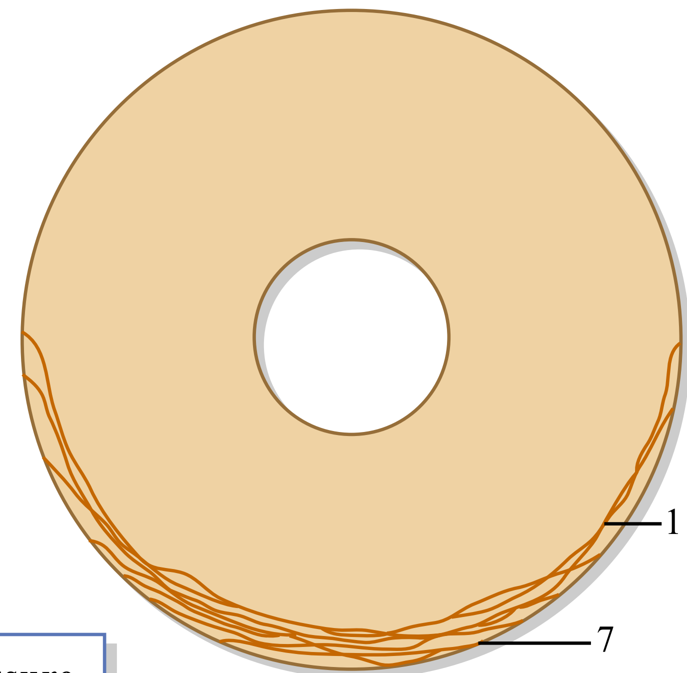
- No inherent length scale in equation
- Length scale set by boundary conditions
- If θ - length scale is ~ L then upstream decay of static pressure is like $e^{-2\pi|x/L|}$
- Region of influence ~ diameter

Tunnel Speed = 147 knots

Engine mass flow = 36.1 lb/s/ft^2



Incidence = +30^o
without engine interaction



Incidence = +35^o
with engine interaction

Local total pressure recovery

1 = 0.99	6 = 0.90
2 = 0.98	7 = 0.86
3 = 0.96	8 = 0.82
4 = 0.94	9 = 0.78
5 = 0.92	0 = 0.74

Short Pitot Intake Effect of Presence of an Engine

Figure by MIT OCW.

[Hodder]

UPSTREAM INFLUENCE OF A RADIALLY NON-UNIFORM ANNULAR FLOW

- Again have uniform background flow in x-direction, $u'_x, u'_r << \bar{u}_x$

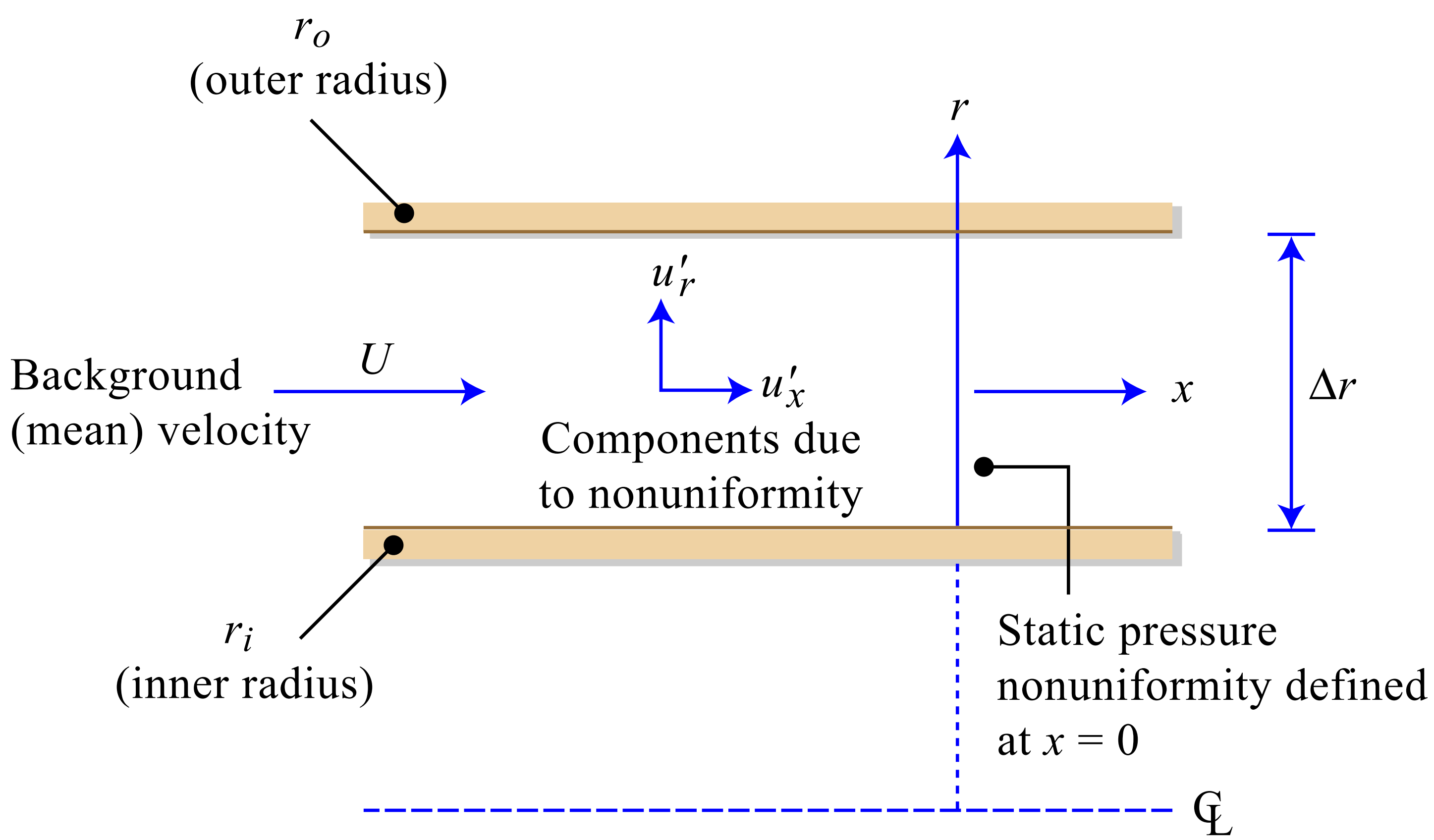
- Pressure varies with radius at x=0 (boundary condition)

$$\frac{\partial}{\partial \theta} = 0 \quad ; \text{ axisymmetric variation}$$

$$\bar{u}_x \frac{\partial u'_x}{\partial x} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} \quad ; \text{ take } \frac{\partial(\ )}{\partial x}$$

$$\bar{u}_x \frac{\partial u'_r}{\partial x} = -\frac{1}{\rho}\frac{\partial p'}{\partial r} \quad ; \text{ take } \frac{1(\ )}{r} + \frac{\partial(\ )}{\partial r}$$

$$\frac{\partial^2 p'}{\partial r^2} + \frac{1}{r}\frac{\partial p'}{\partial r} + \frac{\partial^2 p'}{\partial x^2} = 0 \quad ; \quad \nabla^2 p' = 0$$



Annular flow geometry used in estimation of upstream influence region for axisymmetric flow; region of interest is $x < 0$.

Figure by MIT OCW.

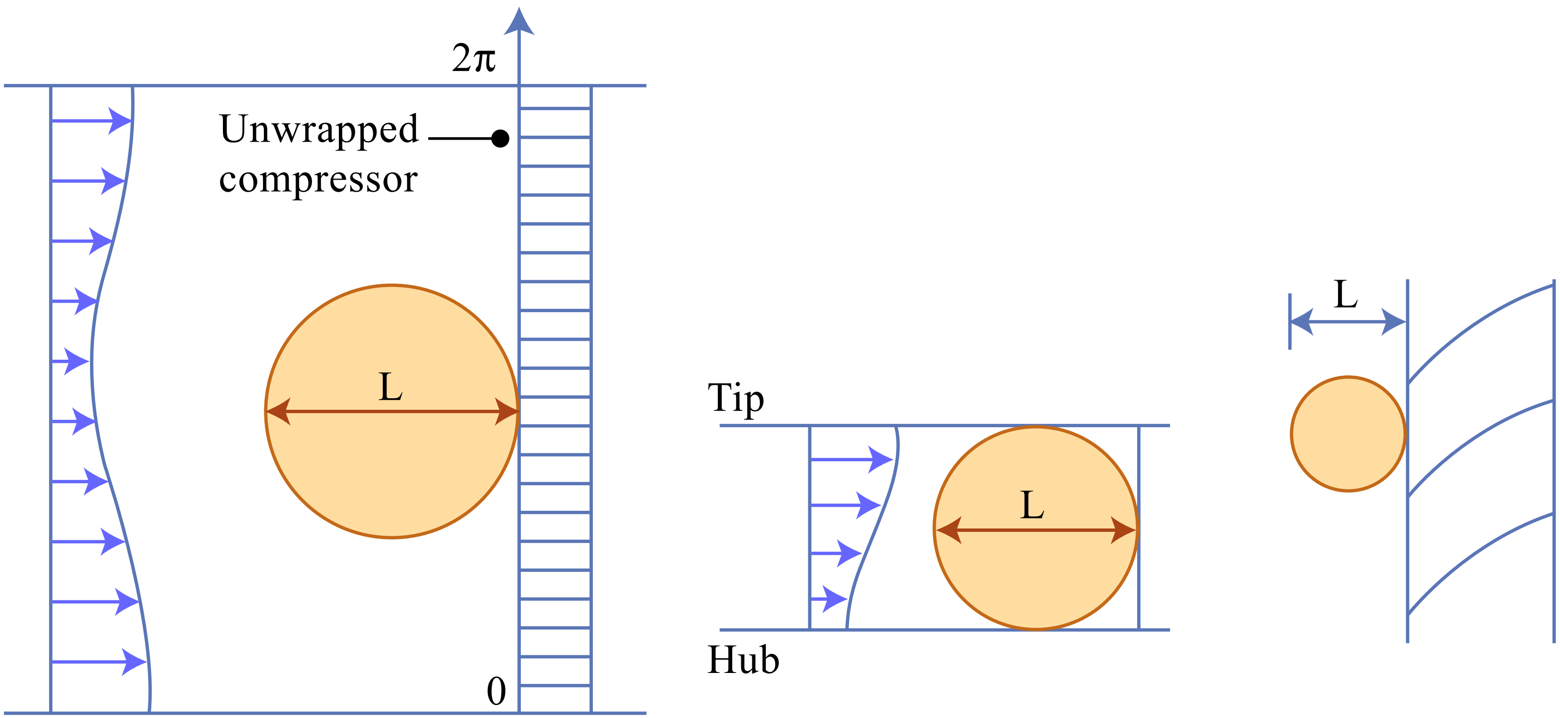
- Suppose annulus has a high hub/tip radius ratio
- Length scale of non-uniformities is $\Delta r = r_o - r_i$
- Ratio of $\frac{1}{r}\frac{\partial p'}{\partial r}$ to $\frac{\partial^2 p'}{\partial r^2}$ is $\frac{\Delta r}{r_m} << 1$

Reduces to $\frac{\partial^2 p'}{\partial r^2} + \frac{\partial^2 p'}{\partial x^2} \cong 0$

Solution is $p' \propto exp\left(-\pi |x| / \Delta r\right)$

Can extend to compresible flow using Prandtl-Glauert transformation: $x \rightarrow x\sqrt{1-\overline{M}^2}$

INTERACTION LENGTH, L, FOR FLOW NONUNIFORMITIES



Circumferential distortion ~ Diameter

Radial Distortion ~ Span

Blade-to-Blade Nonuniformities ~ Gap (Chord)

Figure by MIT OCW.

INTERACTION BETWEEN COMPONENTS SCREEN AND CONTRACTION

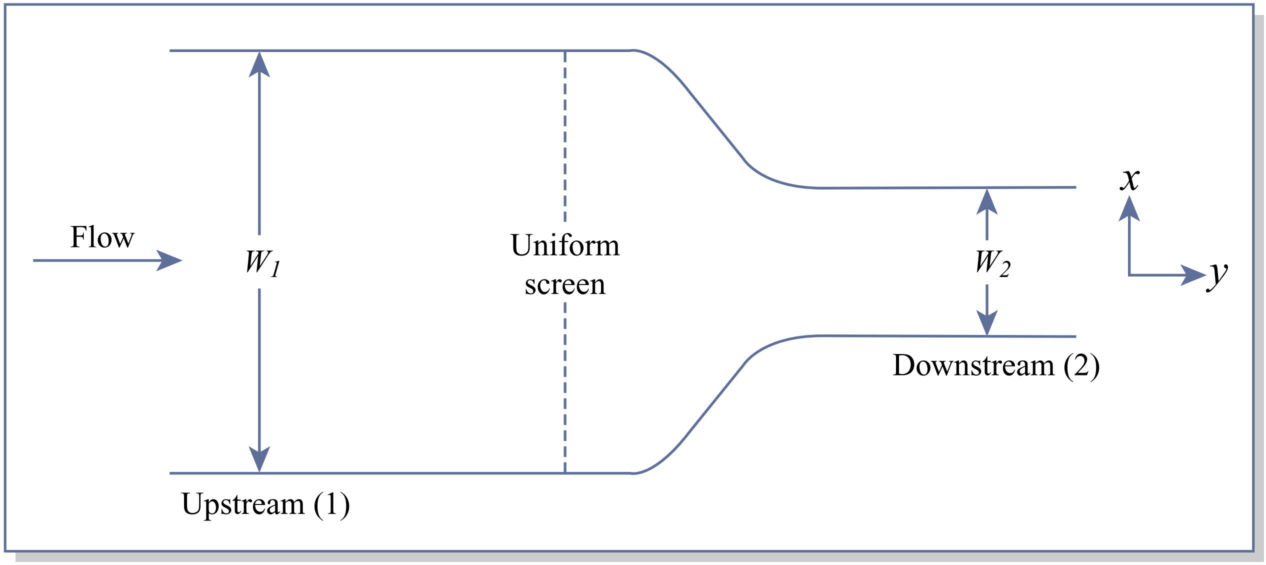


Figure by MIT OCW.

PRESSURE FIELD AT DIFFERENT AXIAL STATIONS

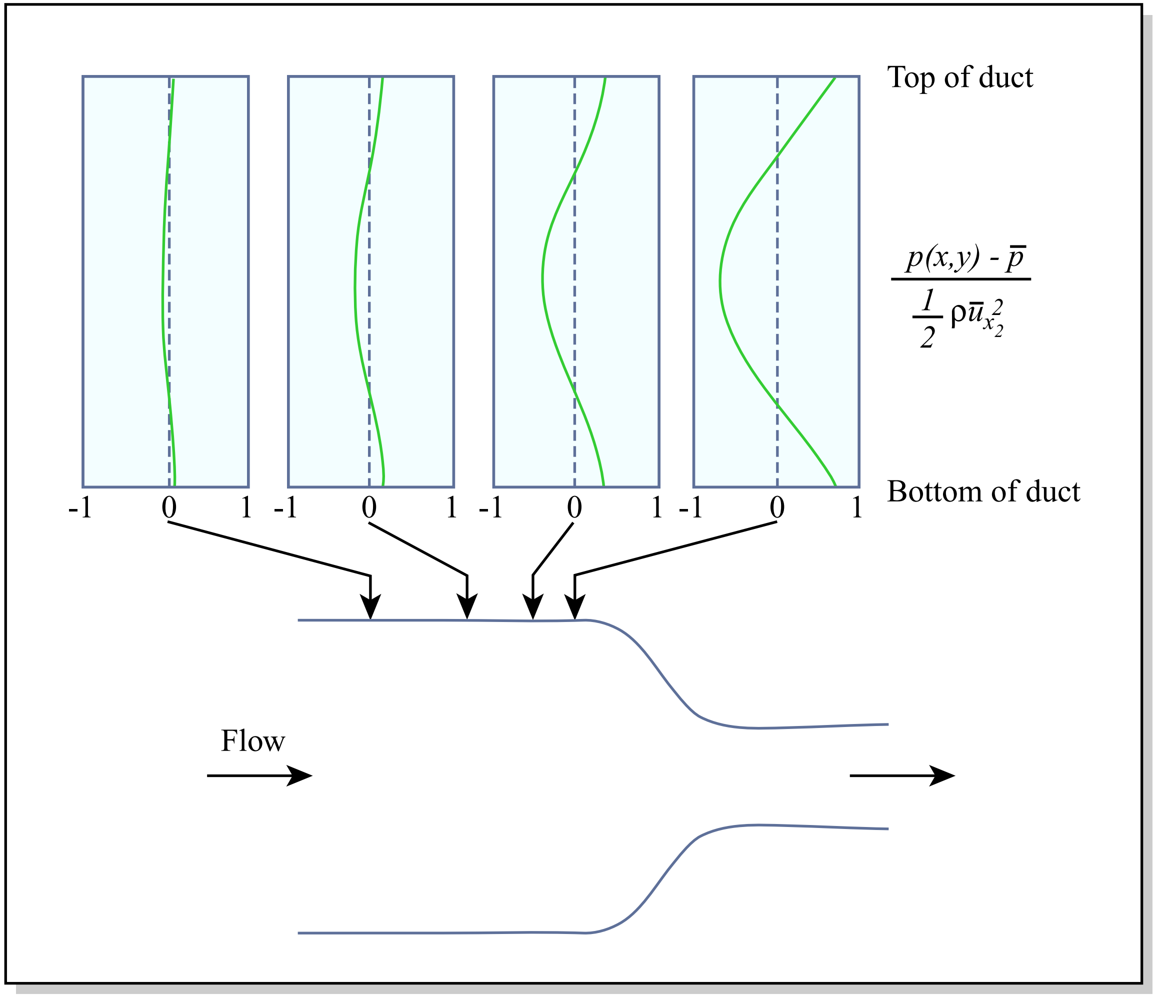


Figure by MIT OCW.

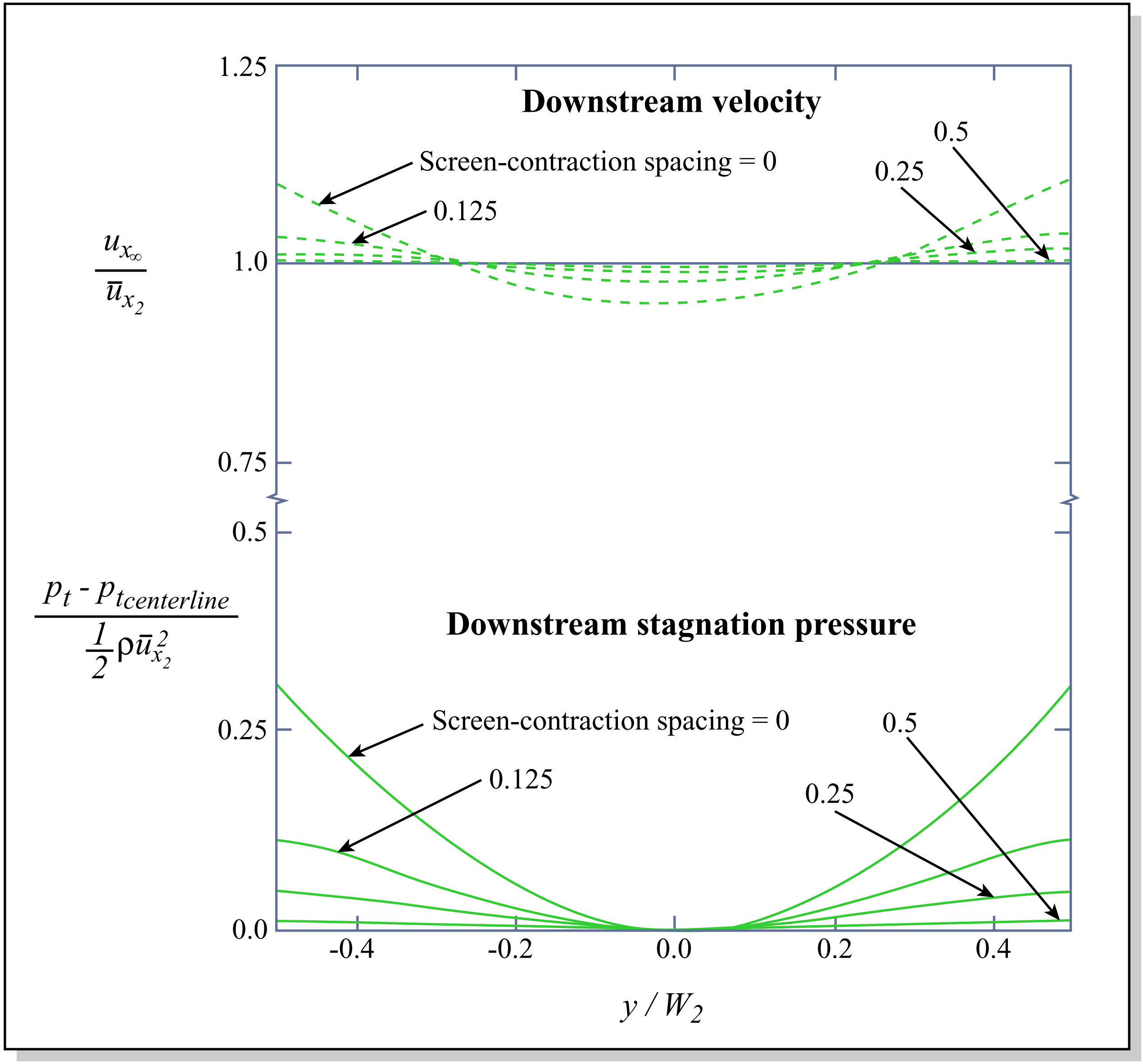
WHAT DO WE EXPECT THE INTERACTION TO DO?

- What effect does a screen have on a non-uniform flow?
- How would you characterize the attributes of a screen?

WHAT DO WE EXPECT THE INTERACTION TO DO?

- What effect does a screen have on a non-uniform flow?
- How would you characterize the attributes of a screen?
- In regions in which the velocity is high what is the local pressure drop through the screen?
- Where are the regions (across the channel) in which the velocity is high?

IMPACT ON DOWNSTREAM CONDITIONS



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WHAT IS THE EFFECT OF A SCREEN ON A NON-UNIFORM UPSTREAM FLOW?

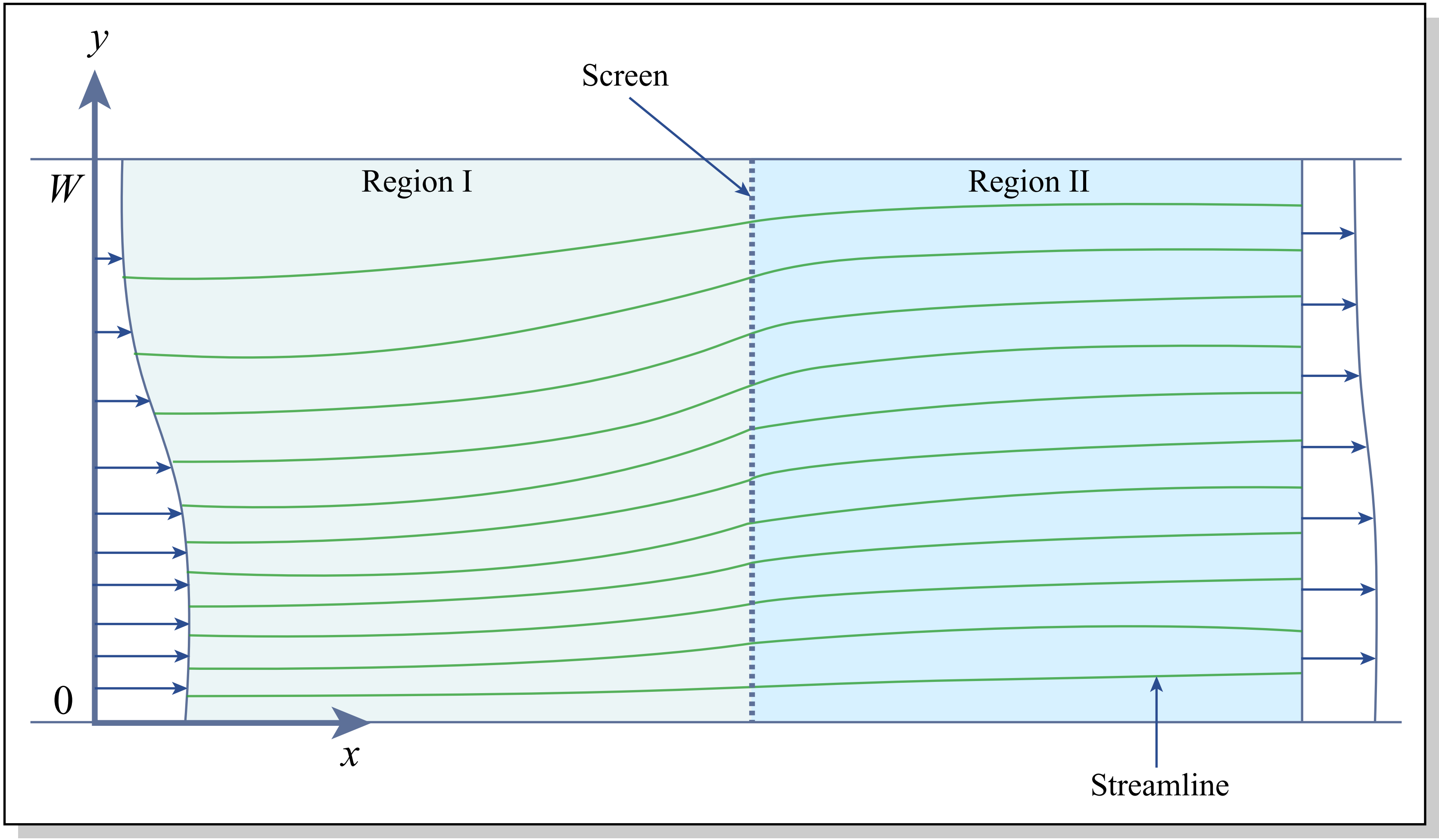
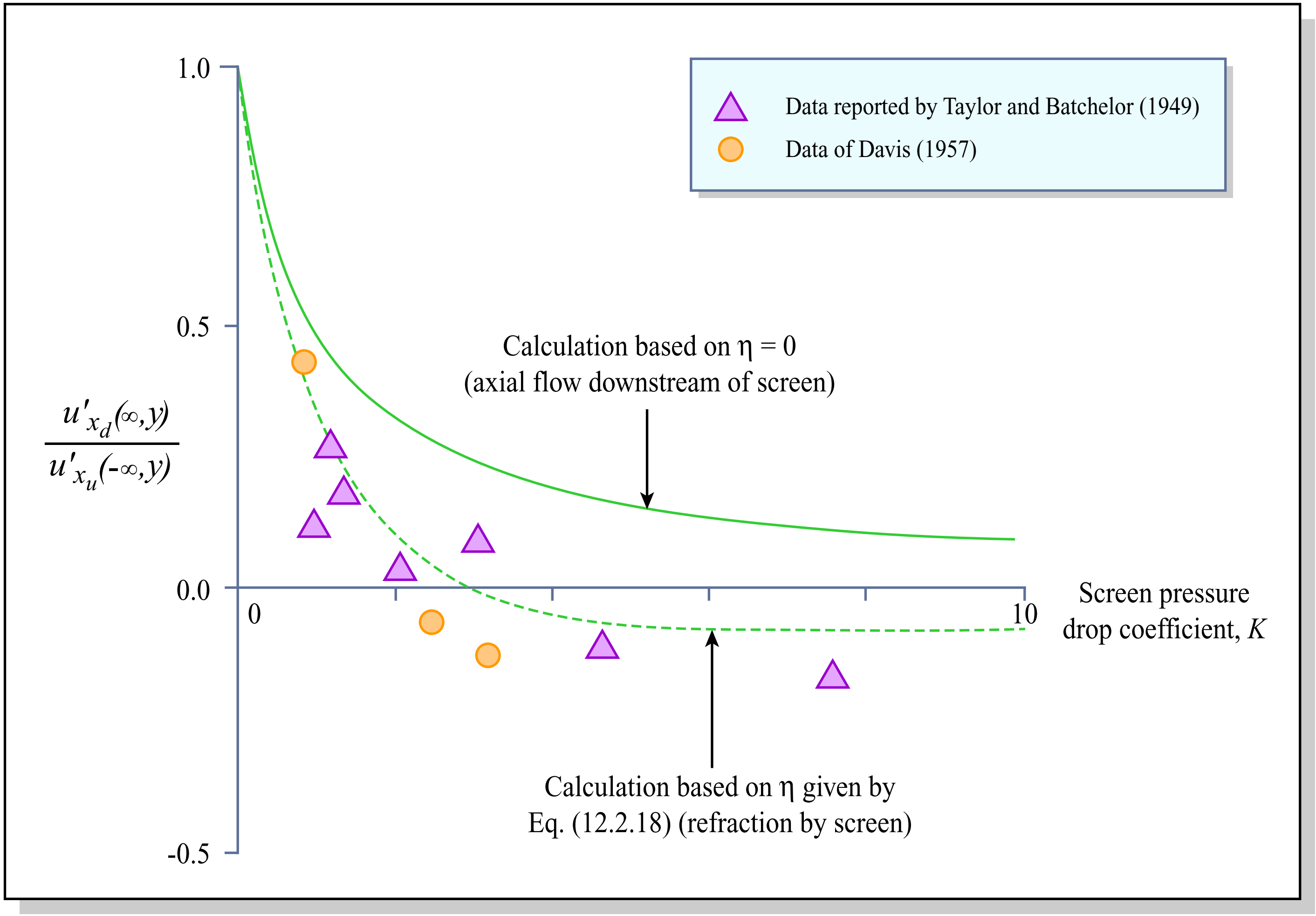


Figure by MIT OCW.

DOES THIS EFFECT DEPEND ON THE SCREEN CHARACTERISTICS? HOW?

- Does the pressure field upstream of the screen depend on the pressure drop through the screen?
- What are the features of the upstream pressure field
- For a given far upstream flow non-uniformity, would we get a more uniform velocity downstream if the pressure drop increased?
- How do we connect the pressure field to the velocity non-uniformity?
  - Upstream?
  - Downstream?
  - Upstream to downstream?

EFFECT OF SCREEN PRESSURE DROP ON DOWNSTREAM VELOCITY (I)



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EFFECT OF SCREEN PRESSURE DROP ON DOWNSTREAM VELOCITY (II)

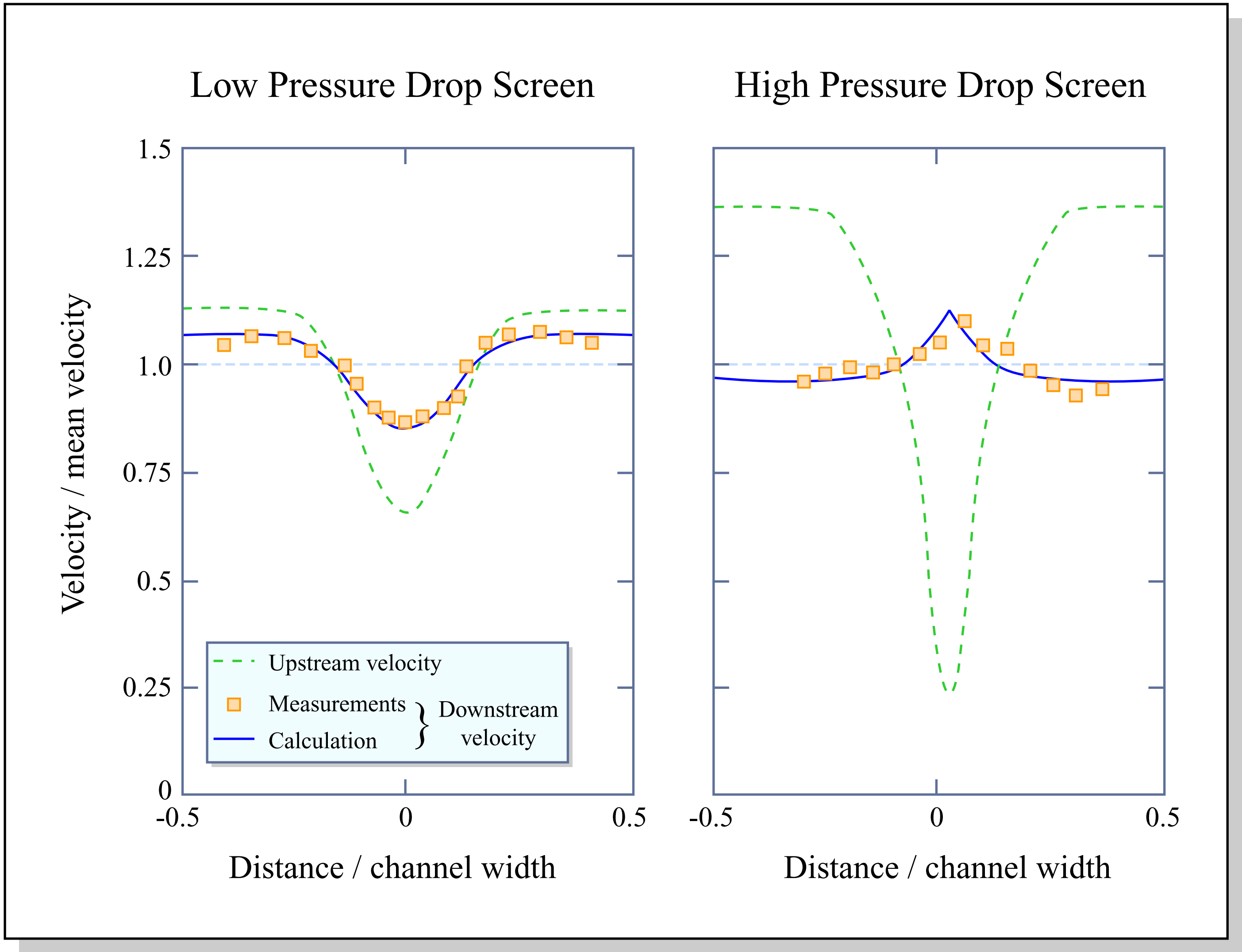


Figure by MIT OCW.

Low pressure drop screen High pressure drop screen

REGION OF INFLUENCE OF SCREEN
[*K* is screen pressure drop]

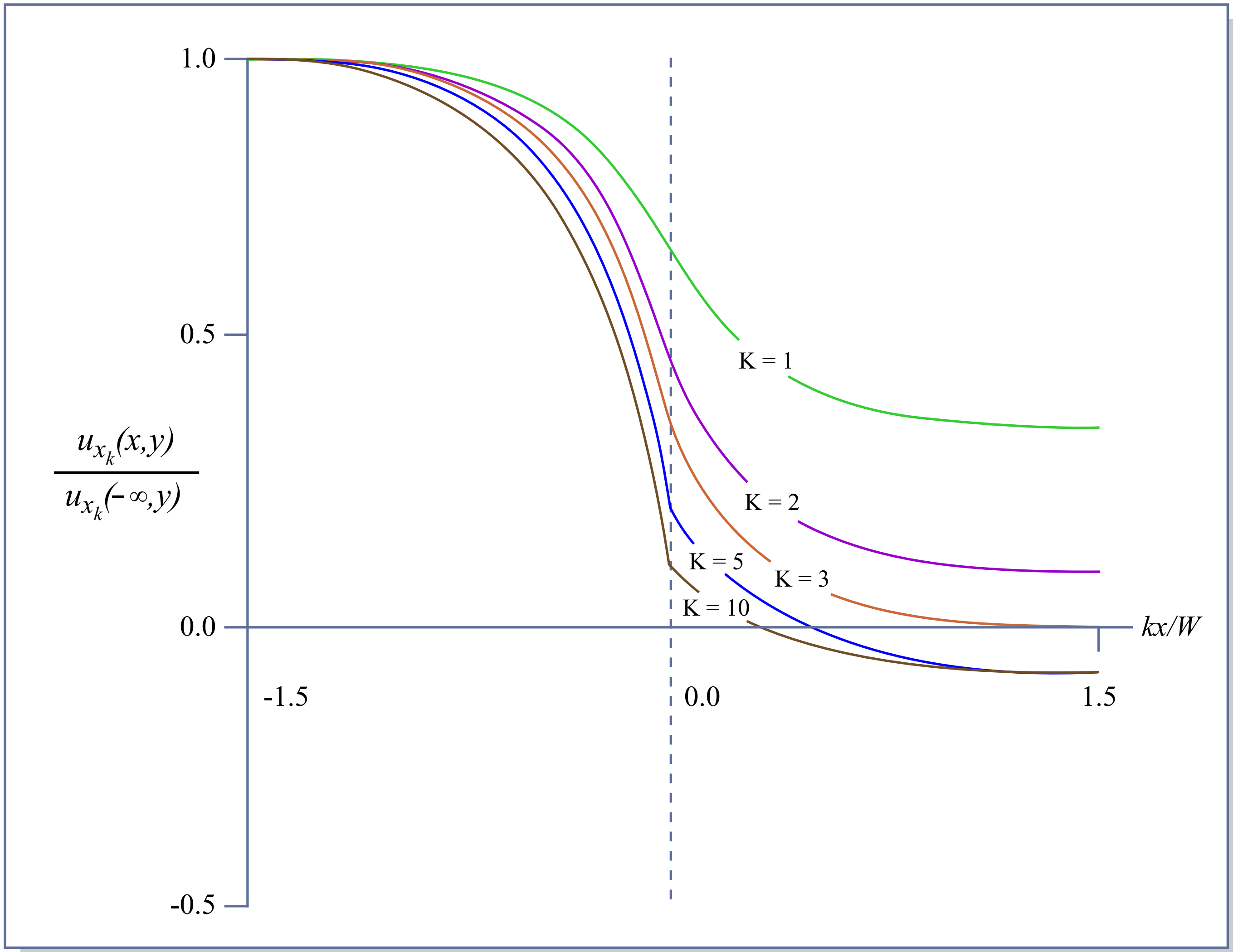


Figure by MIT OCW.

A MORE COMPLICATED (OR IS IT?) EXAMPLE: STATIC PRESSURE FIELD UPSTREAM OF A COMPRESSOR STATOR/STRUT GEOMETRY

[The figure shows only one section (one "wavelength") of a periodic geometry]

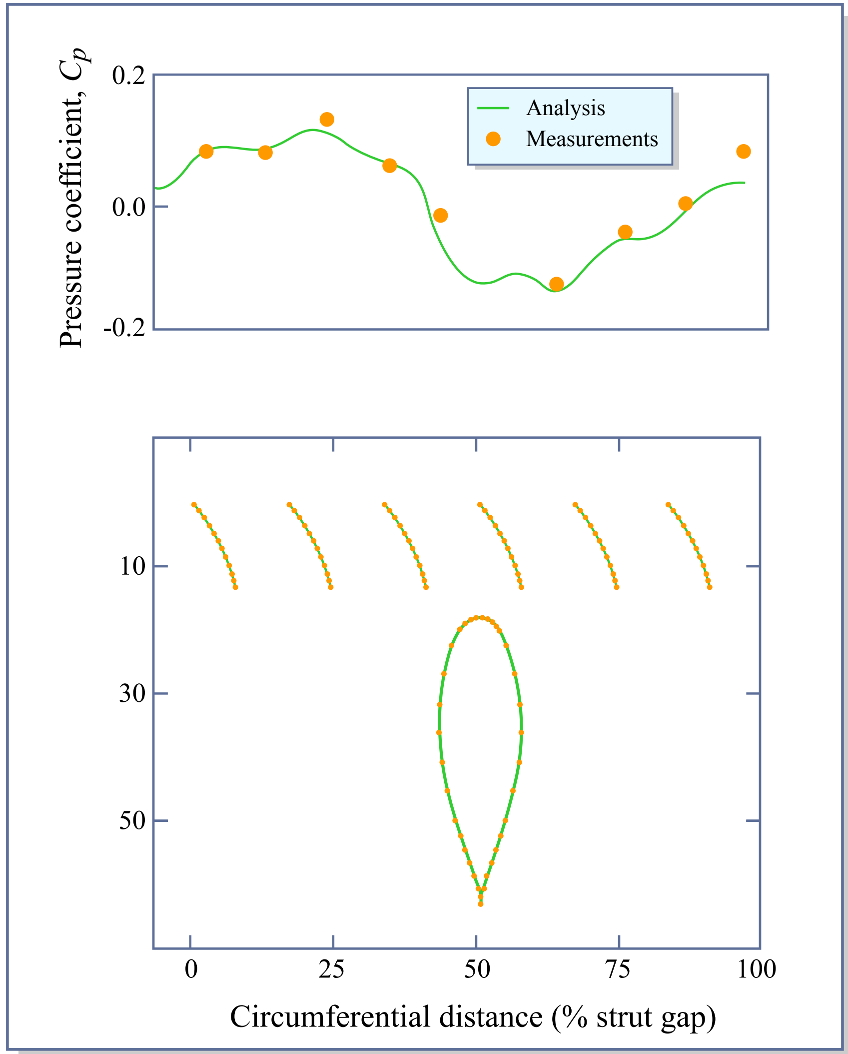


Figure by MIT OCW.

WHAT ARE THE FEATURES OF THIS PRESSURE FIELD?
(Location is 0.5 of a stator chord upstream of the stator leading edge)

- The figure shows a section of a periodic geometry. The geometry is repeated (on both sides) to simulate a full annulus with, say, twelve struts and 72 blades
- There is a large length-scale non-uniformity
- There are smaller length scale “bumps” on this
- How would we explain the features of this pressure field?

Inflow & Outflow to Fluid Devices - Asymmetry of Real Fluid Motion

- **Examples we looked at had fundamental <u>asymmetry</u>**

  (inflow to inlet: streamlines entered from all directions - however outflow of ejector: parallel jet exited in direction of exit nozzle)

- **Different streamlines configuration associated with inflow or exit flow!**

  Note: asymmetry was implicit in control volume treatment

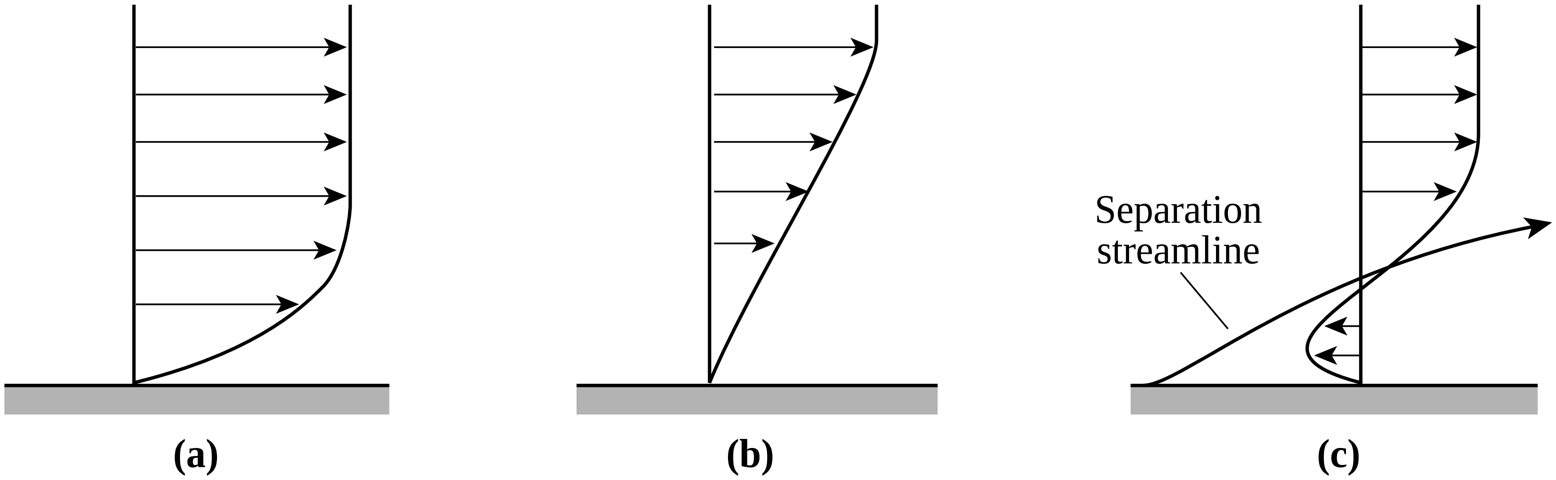
- **Cause for asymmetry $\Longrightarrow$ no-slip condition at solid surface, feature of all real fluids!**

- **For high Re-flow:** $$u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \Longrightarrow du = -\frac{dp}{\rho u}$$

so for same Δp, higher u yields smaller Δu

Boundary Layer Subjected to Pressure Rise

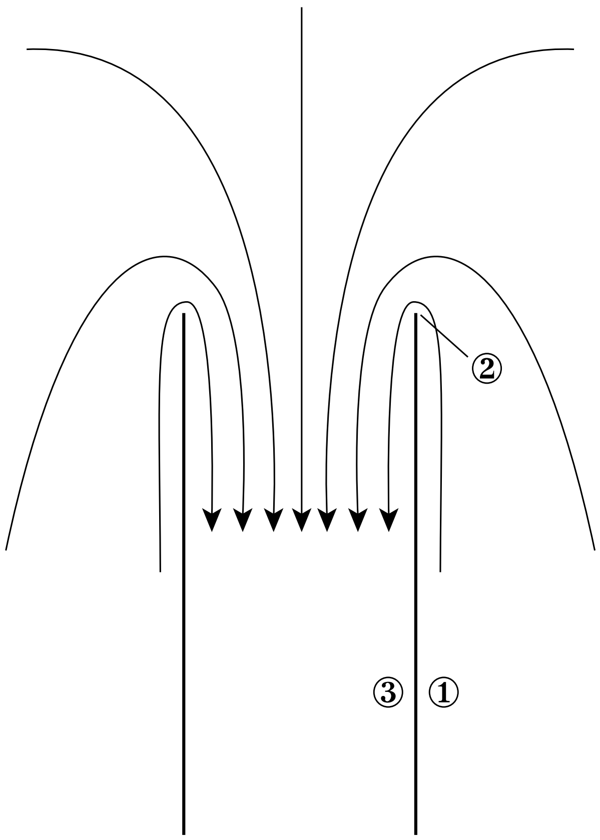
- **Same Δp in free-stream as in BL**
- **Fluid in BL retarded by viscous forces (a) to (b) to (c)**
- **For same pressure rise, BL suffers larger drop in velocity than fluid outside BL ⇨ flow will eventually separate**



- **The Δp at which Δu is driven to 0 in BL is less than that which free-stream can attain**

Contrast Between Inflow to, Outflow from, Pipe

- **Inflow:**
  - **favorable pressure gradient from 1 to 2 (acceleration of fluid in BL)**
  - **at 2 have region of low pressure (streamline curvature)**
  - **from 2 to 3 static pressure rises again, BUT outside boundary layer (BL) some streamline convergence, so severity of adverse pressure gradient lessened**
  - **adverse pressure gradient mild enough to avoid separation**



For high Re, BL thin ⟹ streamlines for flow into pipe will follow geometry

Contrast Between Inflow & Outflow of Pipe

- How about outflow of pipe:
  - will outflow have also this streamline configuration?
  - why or why not? What are pressure gradients driving the outflow pattern?

- Asymmetry in streamline configuration due to viscosity

- Motions are not reversible in thermodynamic and kinematic sense

External Flow Example:
thin wing: Kutta-Joukowski condition

Internal Flow Example:
flow leaving straight nozzle, parallel to nozzle axis

⟹ Good assumptions for describing real flow

Example: Flow Through a Bent Tube

- Freely rotating bent tube, constant area A, volume rate of flow Q

- Flow entering at center 0 and exiting through bent part

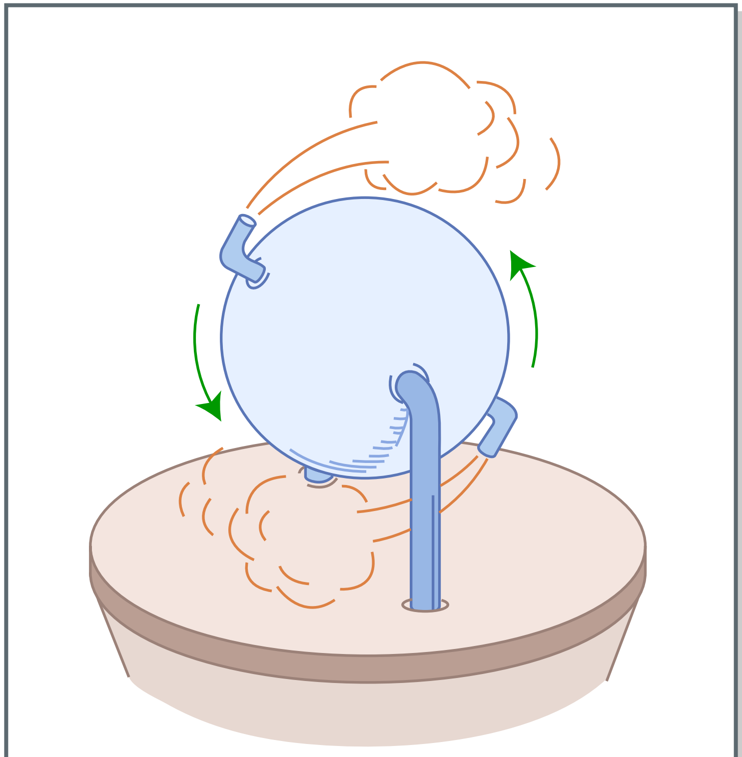


Figure by MIT OCW.

- What is rotation rate Ω ?

- What happens if there is inflow instead of outflow through bent tube?

- Does it rotate? Why or why not?

FLOW INTO (a) AND OUT OF (b) A PIPE IN A QUIESCENT FLUID

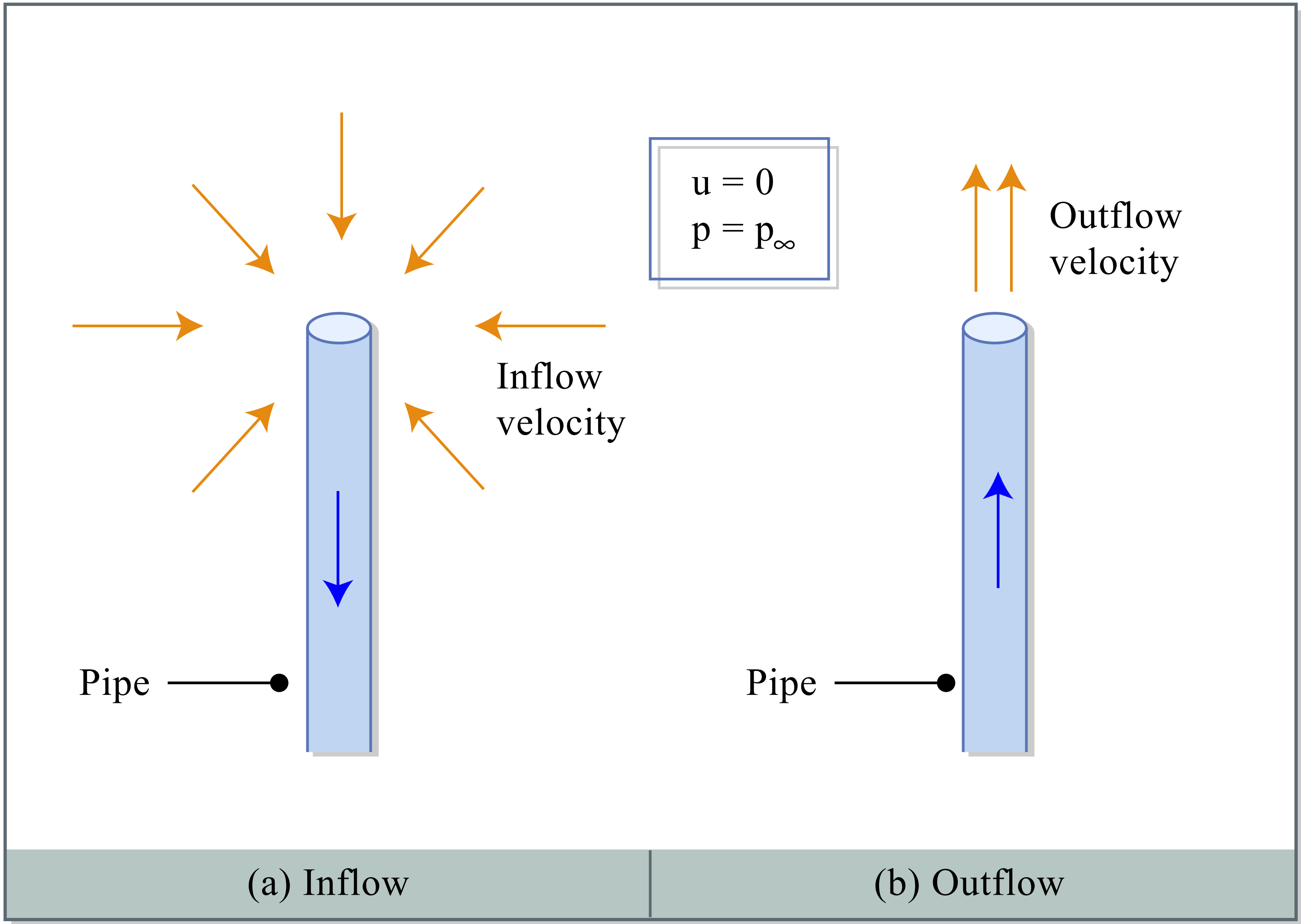
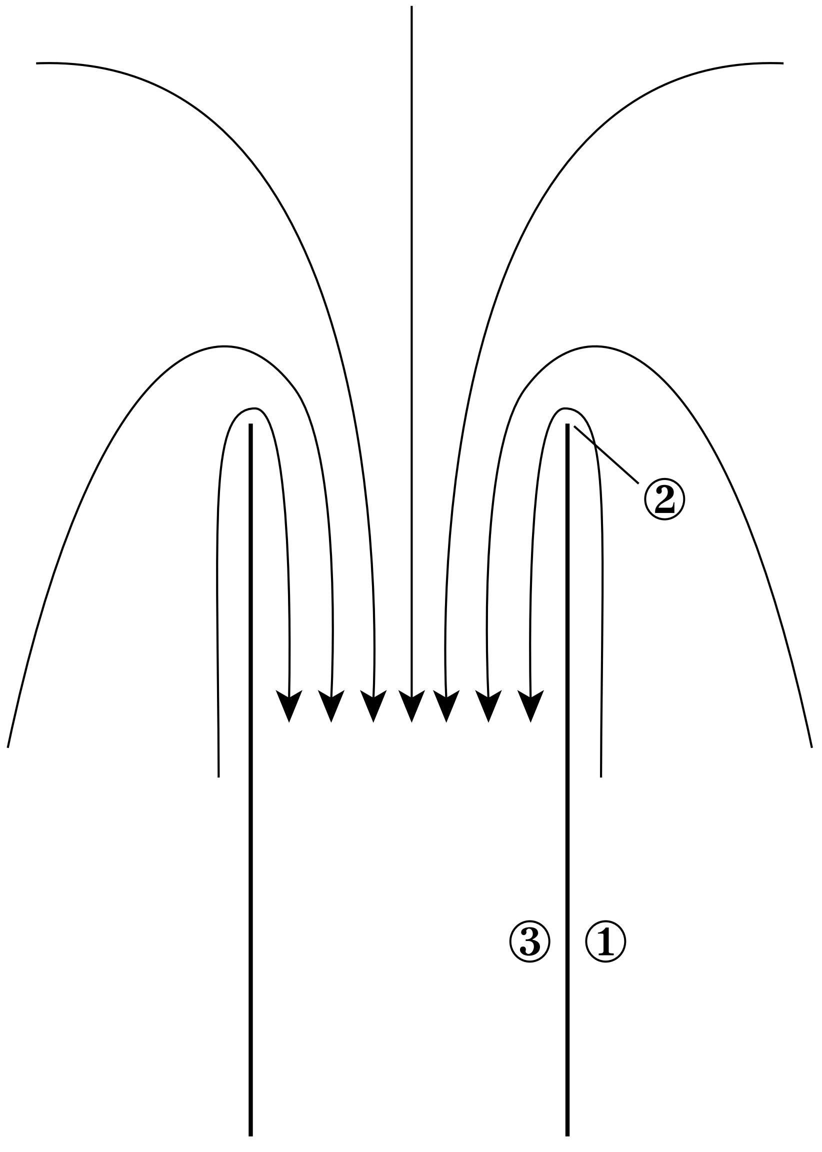
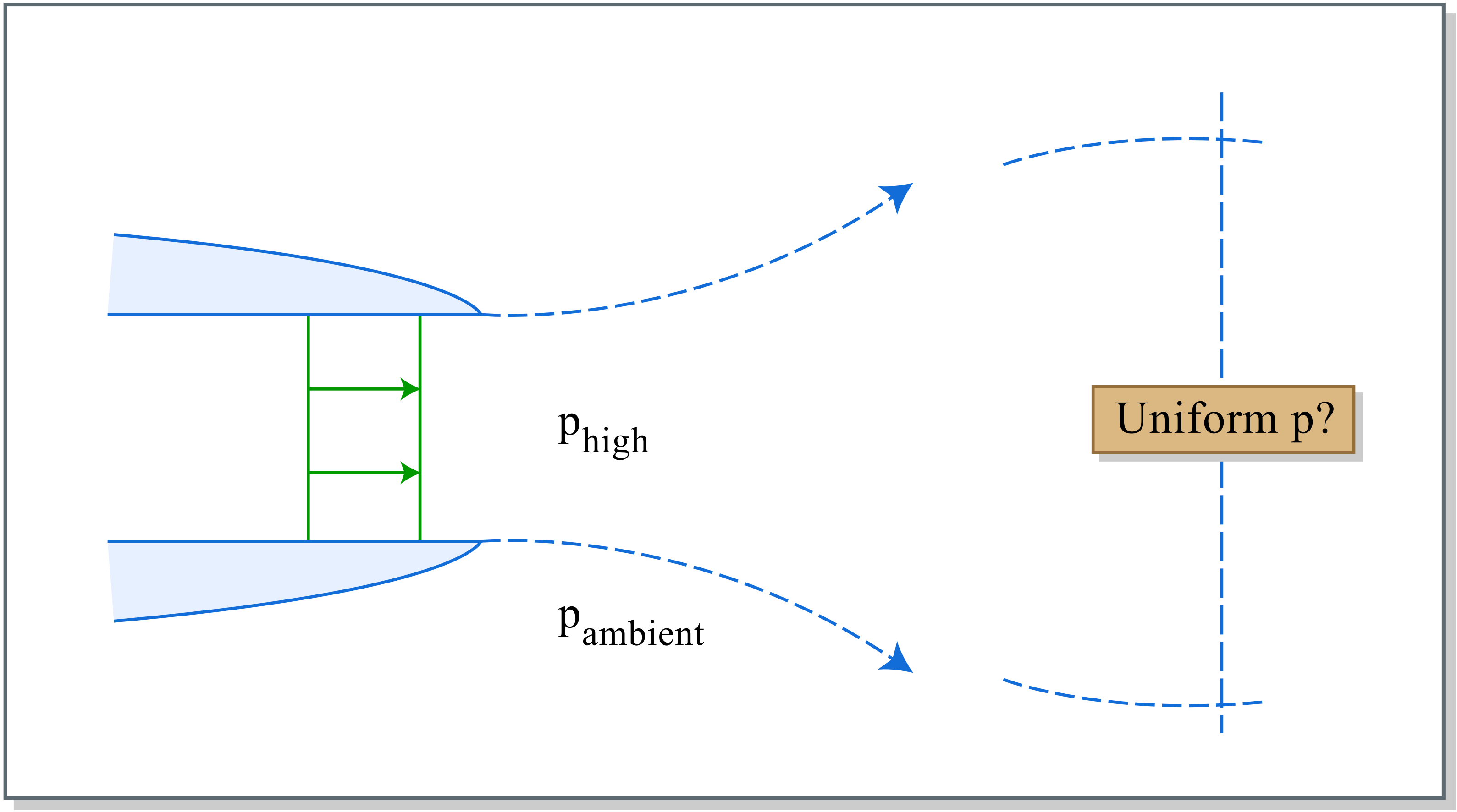


Figure by MIT OCW.

INFLOW FROM A QUIESCENT FLUID INTO A PIPE: FLOW NEAR THE PIPE ENTRANCE



EXIT FLOW FROM A SUBSONIC NOZZLE WITH PRESSURE INSIDE THE JET HIGHER THAN $p_{ambient}$



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