16.55 Ionized Gases
Problem Set #2

Problem 1

Inertia-less Ideal Fluid Equation Parallel to the $\vec{B}$ field

Derive a fluid momentum equation for the momentum along $\vec{B}$ of a magnetized charged species in a collisionless plasma, ignoring mean inertia forces. The plasma can be anisotropic ($T_\perp \neq T_{||}$), and there can be an electric field $\vec{E}$ with a parallel component $E_{||}$.

For an inhomogeneous $\vec{B}$, a mirror force over a charged particle is implied:

$$F_{particle} = -\frac{mv_\perp^2}{2B}\frac{dB}{dz}$$

In consequence, one expects to see in the parallel momentum equation a term,

$$-\frac{P_\perp}{B}\frac{\partial B}{\partial z}$$

Does your equation recover this term when the plasma is isotropic? Explain.

If one of the terms in your equation involves the pressure anisotropy ($P_{||} - P_\perp$), explain its origin in mechanical terms, i.e., with P meaning force per unit area.

Hints: Start from Eq. (28) of the Notes, that gives the magnetic mirror force density, and add to it an electrostatic force density. Derive the corresponding energy conservation equation along the $\vec{B}$ direction. Manipulate this equation to isolate the axial derivative of $nmw_{||}^2$. You will also see a term containing the axial derivative of $nmw_{||}$; transform it using continuity and conservation of magnetic flux in a flux tube. Finally, average over all particle velocities to get the desired equation.

Problem 2

Consider two particles whose interaction is governed by the following rectangular-well potential:

$$\begin{aligned} V(r) &= 0 \qquad \text{for} \quad r > a \\ V(r) &= -V_0 \qquad \text{for} \quad r \leq a \end{aligned}$$

1. Calculate the differential scattering cross-section $\sigma(\chi)$, and show that it is given (for $b < a$) by,

$$\sigma(\chi) = \frac{p^2 a^2 \left[p\cos(\chi/2) - 1\right]\left[p - \cos(\chi/2)\right]}{4\cos(\chi/2)\left[1 - 2p\cos(\chi/2) + p^2\right]^2}$$

   where,

$$p = \left(1 + \frac{2V_0}{\mu g^2}\right)^{1/2}$$

2. Calculate the total and the momentum-transfer cross-sections. Plot these sections (normalized by πa^2) as functions of p.