

1) Road Vehicle Engineering:

You are the VP of powertrain engineering at BMW and are considering the development of an electric version of the BMW 3-series. In preparation for a board meeting, you do some back-of-the-envelope calculations to figure out what this powertrain might look like. How large of a battery pack is required? How about the motor and controller? Can you get away with a single-speed transmission or do the customer's expectations of the performance of a sports sedan require a multi-speed transmission? As a person with a technical background, you'd like to have an idea of the ballpark size and specifications of such a system to determine its feasibility before requesting further development.

- A) Continuous power: How much power is required to push this car down the road at 60mph? This is important in determining the size of the battery pack, which will become an important factor in the overall weight of the vehicle and the sizing of the motor/transmission needed for acceleration.

Consider the most significant losses of a car moving at constant velocity:

Air Drag:

$$\mathbf{F}_D = -\frac{1}{2}\rho AC_d (\mathbf{v} \cdot \mathbf{v}) \frac{\mathbf{v}}{||\mathbf{v}||}$$

See derivation

Where

\mathbf{F}_D is the force vector of drag,

ρ is the density of the fluid,^[3]

\mathbf{v} is the velocity of the object relative to the fluid,

A is the reference area,

C_d is the drag coefficient (a dimensionless parameter, e.g. 0.25 to 0.45 for a car)

[Source](#)

To estimate the cross-sectional area of the vehicle, use the drawing from their product brochure for a reference.

(all measurements in millimeters). Reference [source](#)

Tire rolling resistance:

“Overcoming inertia no longer plays an appreciable role in the vehicle's resistance during steady speed highway driving. For those conditions it is estimated that driveline

friction is about 15%; air drag is about 60% and tire rolling resistance represent about 25%. [Source](#)

Motor and controller efficiency:

This varies based on the type of motor used. For EV's, three common types of motors are: Brushed DC, Brushless DC and AC Induction. Under PS3 there are three datasheets for motors large enough to power a car – while their absolute power ratings may be outside of the range of what you need for this vehicle (i.e. you may decide that the motor you need for this car may need to be more/less powerful than the ones given here), take their efficiency specs and torque curves to be representative of all of their respective motor types. Note that the efficiency rating for the brushed DC motor doesn't account for the losses in the controller – assume that a brushed DC motor controller is 95% efficient.

- B) Now that you have the power that's required to move this particular vehicle down the road, say that you want this vehicle to have a 200-mile range on the highway (distance driven between recharges). How big will your battery pack be? How much will it weigh? Will the volume of the cells physically fit in the vehicle without taking up too much cargo and passenger room?

List the weight and volume of the battery cells required to achieve this performance, using middle-ballpark figures for:

- i) Lead-Acid
- ii) Ni-MH (Nickel Metal-Hydride)
- iii) Li-Ion

- iv) Where would you consider fitting this battery pack in the vehicle? Given that it will be a significant factor in the overall weight of the vehicle, what would be an optimum location for weight balance and handling? (assume that the production engineers can alter the stock 3-series chassis to fit your battery wherever you'd like to place it).

- C) The stock chassis weighs 3,428 lbs – the gas engine, transmission, exhaust and fuel tank, which are no longer necessary, weigh about 600lbs. Assuming that your EV drivetrain will push the overall vehicle weight to 4,200 lbs, how much torque at the wheels is required to push this car from 0-60mph in 6.9 seconds (the stock acceleration time for this car with an automatic

transmission)? Do a simple $F=ma$ style calculation and assume that aerodynamic and rolling resistance drag forces are negligible at slow speeds up to 60mph (all power is going into overcoming inertia). Assume that the OD of the wheel (including the tire) is 25".

- D) The acceleration figure in part C is assuming that you can get constant power to the ground (an internal combustion engine only produces its rated power over a narrow RPM range, so the transmission has to shift gears on its way to 60mph to keep the engine operating in its optimal range). Although electric motors have a wider optimum RPM range for torque, power and efficiency, the gearing will need to be selected so that you can accelerate quickly while still being able to achieve the top vehicle speed within the motor's possible RPM range. Because of the broader torque and RPM curve, it is possible to use a transmission with fewer gear ratios (i.e. 2 speeds as opposed to 5 or 6).

This is good news, as you can potentially save much of the weight and complexity (and cost) of a multi-speed transmission. Because the moment of inertia of an electric motor's rotor is much less than that of an engine, you might even be able to get away without a clutch and let the synchros do the job of matching this RPM directly. However, the synchros need to dissipate the heat generated in matching these revs. Before pitching this to the design team, you test the viability of this idea.

Assume that you'd like to try using a two-speed transmission and that you set the vehicle's maximum speed at 130mph. The motor's top speed is 8,000RPM and you shift gears at 65mph (this means that at 65mph you can either be in 1st gear spinning at 8,000RPM or in 2nd gear at 4,000RPM). Assume that the motor's rotor can be modeled as a cylindrical shell: 4" OD, 3" ID and 12" long (typical of an AC induction motor).

- i) With copper's density at 8.94g/cm^3 , what is the moment of inertia of the rotor?
- ii) How much energy will be dissipated by the synchro in slowing the rotor down from 8,000RPM down to 4,000RPM while shifting gears?
- iii) Assume that half of this energy goes into heating up the conical brass synchro ring (the other half into the gear). The synchro ring, for estimating volume can be modeled as a hollow cylinder OD of 4", ID of 3.5" and length of .25". (Brass: $.377\text{kJ/kgK}$, $8,400\text{kg/m}^3$)

Model this interaction as thermally insulated from the rest of the transmission (i.e. the heat doesn't have time to sink into the shaft, gear or lubricating oil during multiple shifts that occur within a short time of each other).

- a) How much will the temperature of the synchro ring rise if the driver shifts up, then down, then back up very quickly (within about 5 seconds – 3 shifts total)?
- b) Why is this heat issue not as drastic with a conventional gas engine and standard multi-speed manual transmission?

- E) Consider a few methods for getting around these issues without shifting to a more complicated and expensive multi-speed transmission that has more than 2 speeds.
- i) How can you avoid the problem of overheating clutches using motor controls?
 - ii) Do you even need a two speed transmission? Consider some changes to the motor/controller system that would still allow you to reach your performance targets with a single gear. Note that the peak (short term, 5-10 second) power output of electric motors can be near twice their continuous rating.

- F) A new trend going around the EV world is that of “hub motors” – motors that are contained within a wheel. A nice advantage is that you eliminate a transmission, with the motor casing driving the wheel directly; this means fewer moving parts and things that can eventually wear out. The maximum torque of an electric motor is proportional to the volume of the rotor (power is not necessarily the same – increasing RPM at this torque can lead to higher power) – a formula for the “maximum electromagnetic shear strength” is below:

$$\Gamma(\text{est Nm}) \cong \frac{D^2(\text{motor housing, m}) * L(\text{motor housing, m})}{6.75 * 10^{-5}}$$

The constant at the bottom of this equation (6.75E-5) is taken by reverse-calculation using the motor of a Tesla Roadster Sport as a control (a very high-performance hand-wound motor). This formula essentially calculates the performance of a similar motor sized to a different scale.

Consider that the hub motor has to fit within a 16” wheel (the inner part of the rim is about 14”) – using the torque value you calculated in part C, how long does this motor need to be to ensure the same acceleration value? Is this reasonable?

- G) Getting a little more involved than just the high-level feasibility calculations – if you were to use a motor with a peak torque output of 200 ft-lbs, check to see if you can use a pinion gear that BMW already manufactures at high volume. If your parts department has a 1.5” OD, 20 tooth gear with a 20degree pressure angle in stock, will it be strong enough to handle this load? Assume it’s 2”long (face width), made from 4130 normalized steel which has a yield strength of 500MPa. A great [resource](#) for these calculations.

2) Train Catenary Wires:

France's TGV high-speed railroad operates at 25kV and most of its trains are rated to 9,000kW (that's 12,000 horsepower!). How thick must the catenary wires be to keep the catenary wire transmission losses at less than 5% over a transmission distance of 500miles? Assume copper wire with a resistivity of $1.68E-8$ ohm*meter.

Ohms = (resistivity*length)/(cross-sectional area)

3. Hybrid Vehicles:

A) Hybrid electric vehicles in production today come in many different types, with advantages and disadvantages for each design. Although most production hybrids don't fit perfectly into these neat divisions, the following three hybrid architectures are representative of the major classifications.

Match the diagrams to specific hybrid architectures:

- (A) Series Hybrid (B) Parallel Hybrid (C) Series-Parallel Hybrid

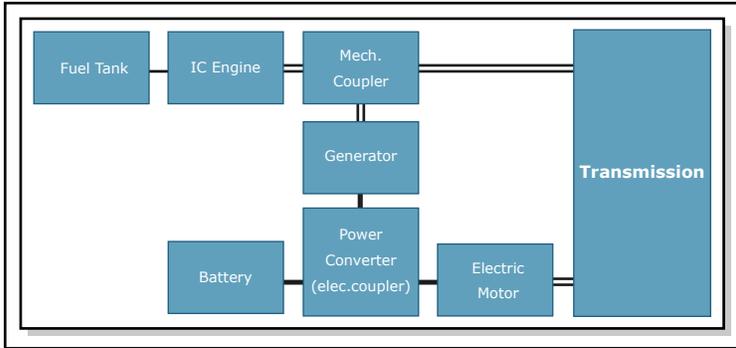


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Answer: **(C)**

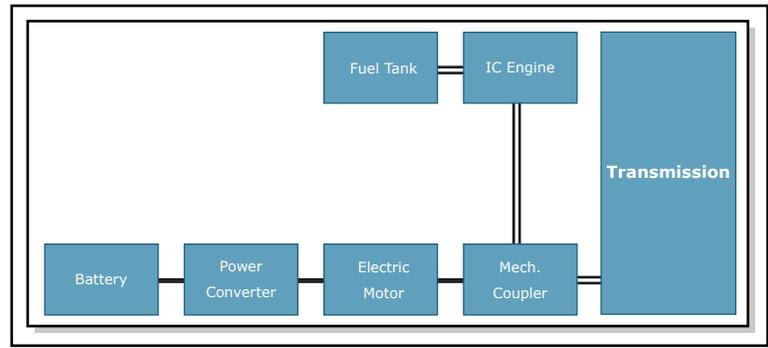


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Answer: **(B)**

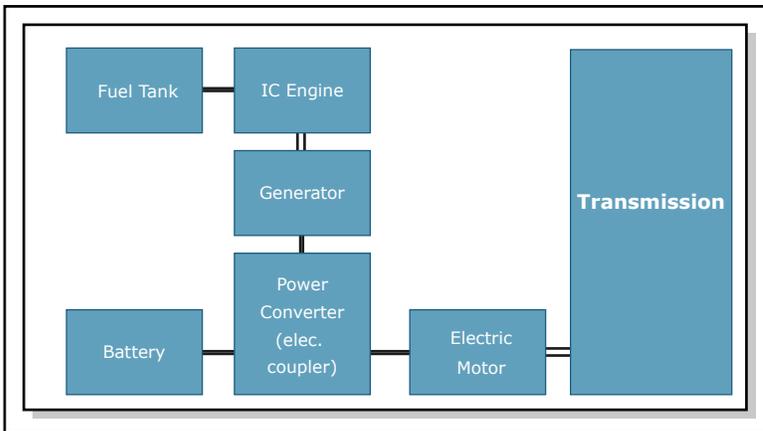


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Answer: **(A)**

B.) Another name for a series hybrid is a “range extended electric vehicle” (for plug-in versions with large battery packs), which is how a car like the Chevrolet Volt claims to be an electric vehicle based on its 40 mile all-electric mode. Put yourself in the position of a controls engineer for a series hybrid vehicle. There are at least seven “operating modes” (such as all-electric traction) that you will have to design the controller to cover. List as many as you can think of (a mode is basically where the power is coming from and where it is going to).

Pure electric traction mode, Pure engine traction mode, Hybrid traction mode, Engine traction with battery charging mode, Regenerative braking mode, Battery charging mode, Hybrid Battery Charging Mode

C) Design a series hybrid powertrain with the following vehicle properties:

Vehicle Specs

Total Vehicle Mass: 1500kg **Coefficient of Rolling Resistance:** 0.01 **Coefficient of aerodynamic drag:** 0.3
Front Area: 2.0 m² **Transmission efficiency (single gear):** 0.9

Performance Specs

Acceleration Time (0 to 100 km/hr): 10 +/- 1 s

Max Speed: 160 km/hr

i) Use the following equation to give a first order estimate for the power requirements of the electric motor as the vehicle accelerates from 0 km/hr to 100 km/hr.

$$P_t = \frac{\delta M}{2t_a} (V_f^2 + V_b^2) + \frac{2}{3} M g f_r V_f + \frac{1}{5} \rho_a C_D A_f V_f^3,$$

Where δ =rotational inertia (assume 1.06), M = vehicle mass, t_a = acceleration time, V_f = final speed, V_b = motor base speed (assume motor speed ratio of 4... $V_b=V_f/4$), ρ_a = density of air = 1.202 kg/m³, and the other terms should be evident.

Average Power Req 0-100 km/hr in 10 s: 71.0045 kW (see Matlab code)

ii) In designing the engine and generator, you must consider two operating extremes: long steady state driving on a highway as well as constant stop-and-go traffic in a city. In the first case, the vehicle should not have to rely on any peaking power from the battery pack, otherwise the batteries will be quickly depleted. In the city driving case, the engine needs to deliver enough power to maintain a constant battery SOC over time so that enough power can be drawn for hill climbing and acceleration (regen braking will make up for some energy loss).

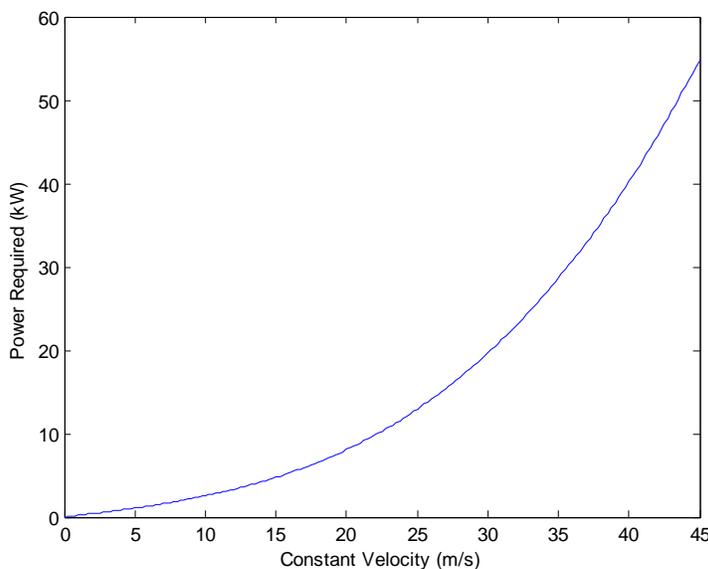
Use the following equation to estimate the power output of the engine on a flat road at a constant velocity:

$$P_{e/g} = \frac{V}{1000\eta_t\eta_m} \left(Mg f_r + \frac{1}{2} \rho_a C_D A_f V^2 \right) \text{ (kW)}.$$

where $\eta_t = 0.9$ and $\eta_m = 0.8$ are the efficiencies of the transmission and traction motor, respectively.

Sketch a graph of load power vs constant vehicle speed. What is the engine power requirement at the vehicle's max speed, and how does this compare to the power required to accelerate the vehicle from 0-100 km/hr?

Power Constant Max Speed: 53.0381 kW (see Matlab code)



iii) Use the following equation to estimate the average load power of the vehicle in city driving (use an aggressive deceleration value, perhaps 0.3 g, for dV/dt , and a more modest acceleration value). Assume one full start and stop cycle every 45 seconds with a max speed of 25 m/s. This calculation also assumes full regenerative braking. However, in real life, this is not possible. Assume 50% regenerative braking efficiency in your calculation...which term below should be adjusted to reflect this? Note: piecewise discrete integration in matlab is an easy way to solve this.

$$P_{ave} = \frac{1}{T} \int_0^T \left(Mg f_r + \frac{1}{2} \rho_a C_D A_f V^2 \right) V dt + \frac{1}{T} \int_0^T \delta M \frac{dV}{dt} V dt$$

How does your answer compare to the constant speed load in the previous part?

Average Power City: 13.15 kW (see Matlab code)

---->Accel: 32.62 kW Constant: 12.93 kW Decel: -27.67 kW

iv)Based on the calculations in the last two parts, there are several approaches you could use to size the engine. How would you choose a power rating of your engine, and are there any disadvantages (in terms of vehicle design or engine operation) to your selection?

The engine should meet the average power requirements of the vehicle at a constant max velocity ($\geq 53\text{kW}$). This is the worst case steady state behavior. The motor should meet the max power demanded. In this case, that would be about 71kW (actually should be a bit larger if you don't take the average acceleration power but rather the max value during the acceleration)

v)Briefly describe how this engine and motor sizing procedure would be different for a parallel hybrid electric vehicle (a couple of sentences).

In a parallel hybrid electric vehicle, mechanical coupling enables tractive power to be drawn from both the engine and the electric motor directly to the wheels. So, the max output power of the engine and the electric motor combined needs to equal the max output power required, enabling us to select a smaller motor size.

D) Power Management: Various power management strategies are used in hybrid vehicles depending on the drivetrain architecture as well as the energy storage capacity of the vehicle. The following graphs illustrates a “thermostat” style control scheme for a parallel hybrid electric vehicle. SOC stands for the state of charge of the batteries and PPS stands for Power Peaking Source, which in this case is the battery pack.

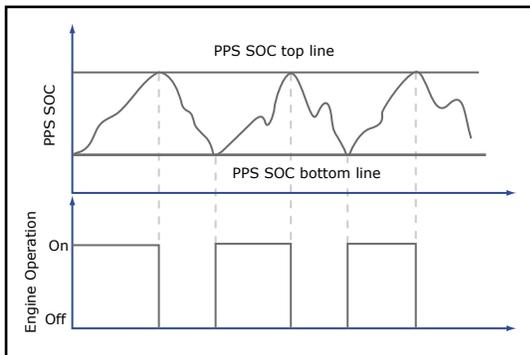
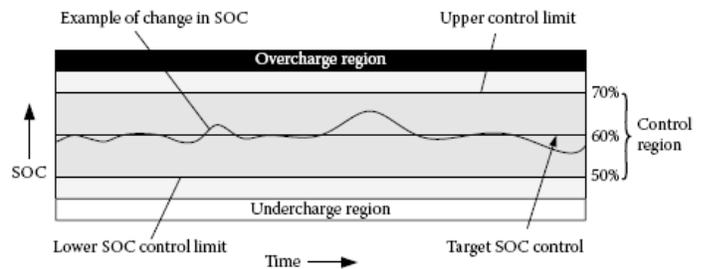


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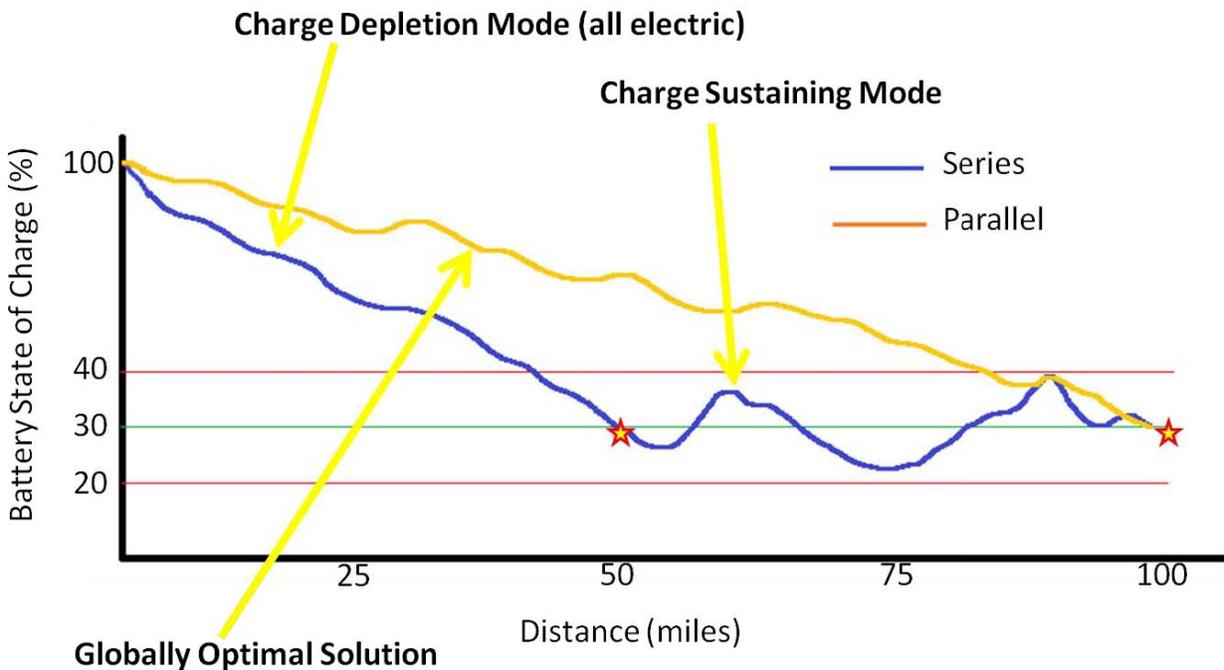


Now, you are converting two hybrid vehicles, one with a series drivetrain architecture and one with a parallel drivetrain architecture, to be plug-in hybrid electric vehicles by installing a 10 kWh LiFePO₄ battery pack as well as the necessary charging hardware in each. You will also have to implement a new power management strategy in the controller.

Assume each vehicle has a potential all-electric range of 50 miles. First, for the series hybrid vehicle, sketch the optimal power management strategy for a 100 mile trip. Is the answer the same for the parallel hybrid? If not, sketch what an optimal power management strategy might look like.

This question is a bit tricky. The key insight is that while the series hybrid only has one energy path to traction (everything must flow through the electric motor), the parallel hybrid has two. So, for power management strategies in the plug-in case, the following strategies will be optimal: for a series plug-in

hybrid, rely on all-electric mode until the batteries are discharged to their minimum steady state value and then operate in “charge sustaining” or “thermostat” mode from then until the next charge. You can see this type of regime in the Chevy Volt, which is a plug-in series hybrid electric vehicle (aka range extended electric vehicle). For a parallel plug-in hybrid electric vehicle, on the other hand, there is one case in which the previous power management strategy would not be “globally optimal,” but it takes some out of the box thinking. If the vehicle or driven have a priori knowledge about future driving conditions on a known route, and if the route exceeds the length of road on which “all electric” mode could be used for the entire trip, it makes sense to use the internal combustion engine for traction at times when it will maximize its efficiency and use the electric motor for all other times as much as possible. For instance, this might mean you would use the ICE during a highway portion of the trip and save battery for a city driving or high congestion portion of the trip. This is a somewhat theoretical result, though, since it depends upon accurate a priori knowledge of road conditions.



E) Regenerative Braking:

i) How much energy is dissipated by braking a 1500 kg vehicle from 100 km/h to 0 km/hr using only the mechanical brakes?

$$E = \frac{1}{2} * m * v^2 = \frac{1}{2} * (1500 \text{ kg}) * (27.78 \text{ m/s})^2 = 0.579 \text{ MJ}$$

ii) If a vehicle is allowed to coast to rest from this speed, it will travel about 2 km before stopping. Braking events usual occur over a few tens of meters, though. Calculate the average power required to stop the vehicle at an “aggressive” and “mild” braking acceleration.

Assume “aggressive braking” = 0.5 g and “mild braking” = 0.15 g
 Stopping Time Aggressive = $(27.78 \text{ m/s}) / (9.81 \text{ m/s}^2 * 0.5) = 5.66 \text{ s}$
 Stopping Time Mild = $(27.78 \text{ m/s}) / (9.81 \text{ m/s}^2 * 0.25) = 18.9 \text{ s}$

Average Power of Stopping (mild) = 0.579 MJ / 18.9 s = 30.6 kW

Average Power of Stopping (Aggressive) = 0.579 MJ/5.66 s = 102.3 kW

iii) In the following figure, for a New York City driving cycle, the bars represent average braking power when a vehicle comes to rest from a particular top speed. The line represents an electric motor’s speed-power profile. Based on this chart, do you think that any additional energy conversion components would need to be added beyond the electric traction motor in order to implement regen braking? Why or why not? At what speeds is regen braking most effective?

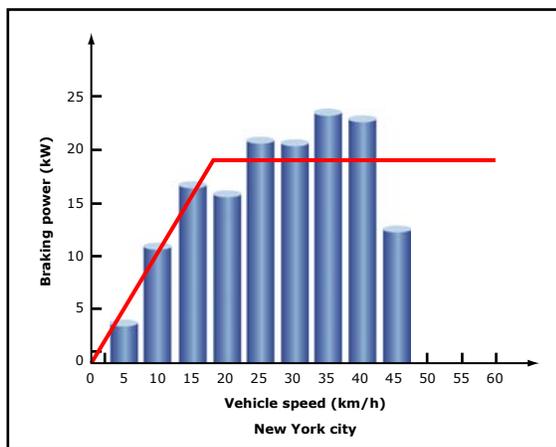
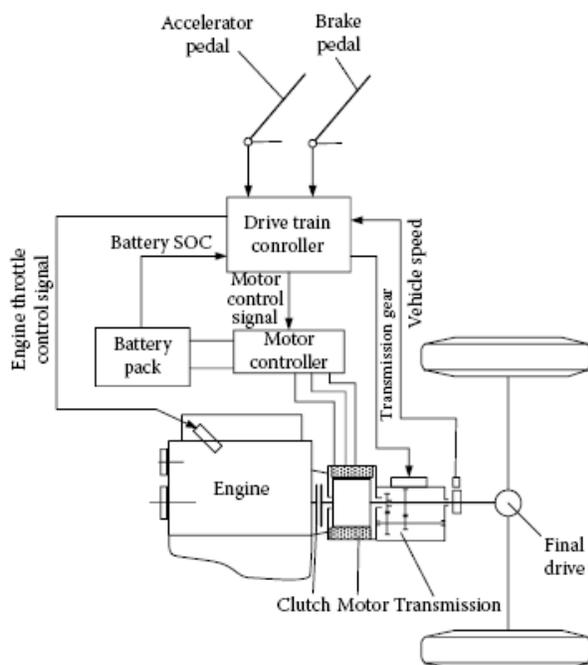


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No, as this chart demonstrates, the power dissipation profile matches pretty well with the electric motor’s speed-power profile. This is convenient, since we don’t need to add a separate generator in most cases to make use of regen braking. Rather, we can make use of the traction motor itself. The chart of braking power above is experimental (hence the unusual result that braking power at the max speed actually decreases). However, it is clear that regen braking is most effective when the electric motor can absorb the maximum amount of power from the deceleration in order to avoid waste heat in the brakes. So, the motor is very effective up until some speed (looks like above 35 or 40 km/hr) at which much of the vehicle’s kinetic energy is not recaptured by regen braking but rather by the mechanical brakes.

F) One compromise for auto manufacturers that do not have the time or resources to implement a completely ground-up hybrid redesign of a powertrain is the so-called “mild hybrid”. Draw a block diagram of a simple “parallel” implementation of a mild hybrid. What component(s) are eliminated and what vehicle capability is added (as compared to a traditional ICE vehicle) that improves efficiency at low speeds?



The electric motor is added in parallel to the engine. In the transmission, the torque converter is eliminated, leading to better efficiencies at low speeds. The engine is also turned off at low speeds and during idling...the electric motor is used to restart the engine or to drive the vehicle at very low speeds.

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