16.810

Engineering Design and Rapid Prototyping

Lecture 4

16.810 Computer Aided Design (CAD)

Instructor(s)

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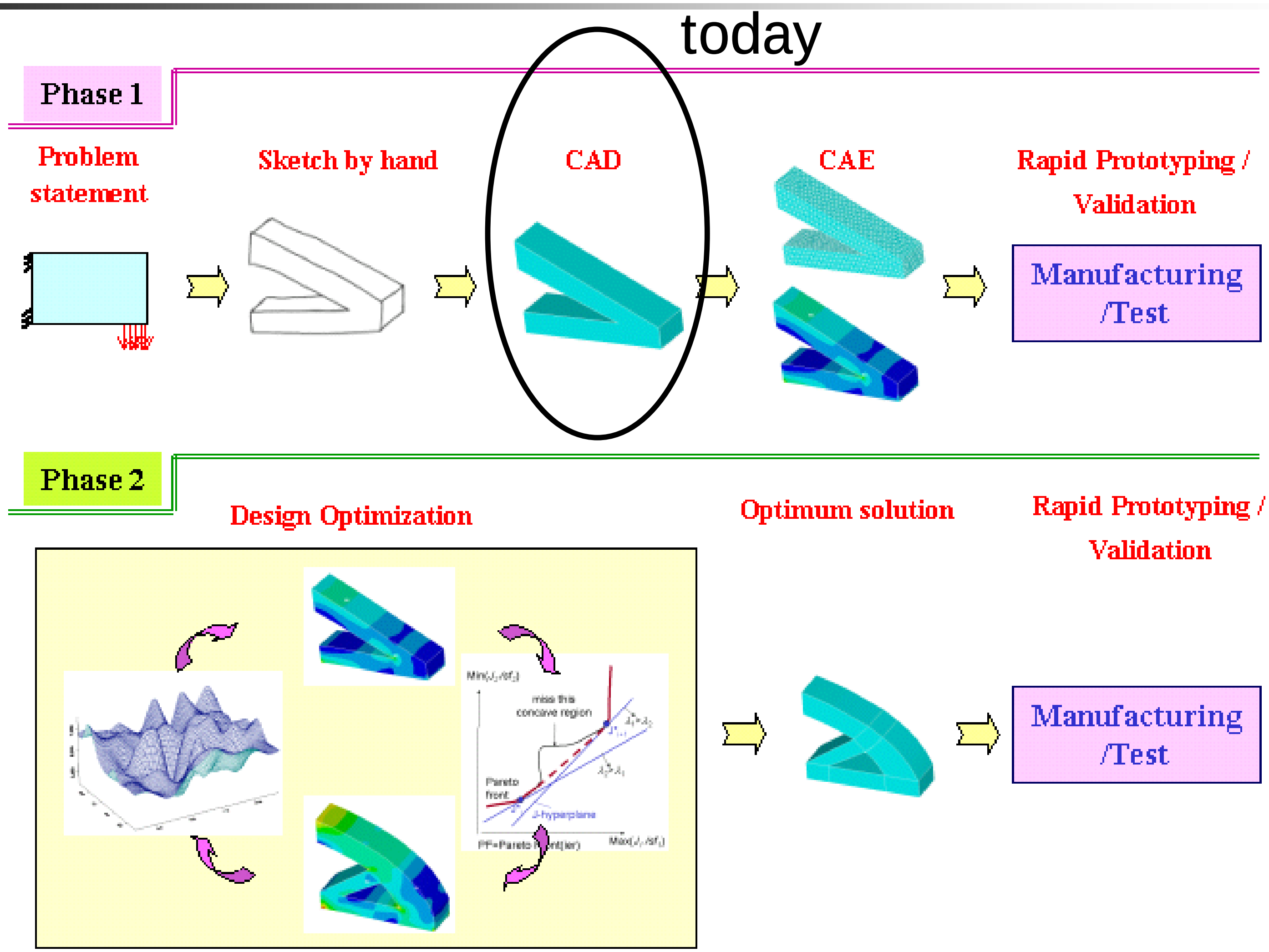
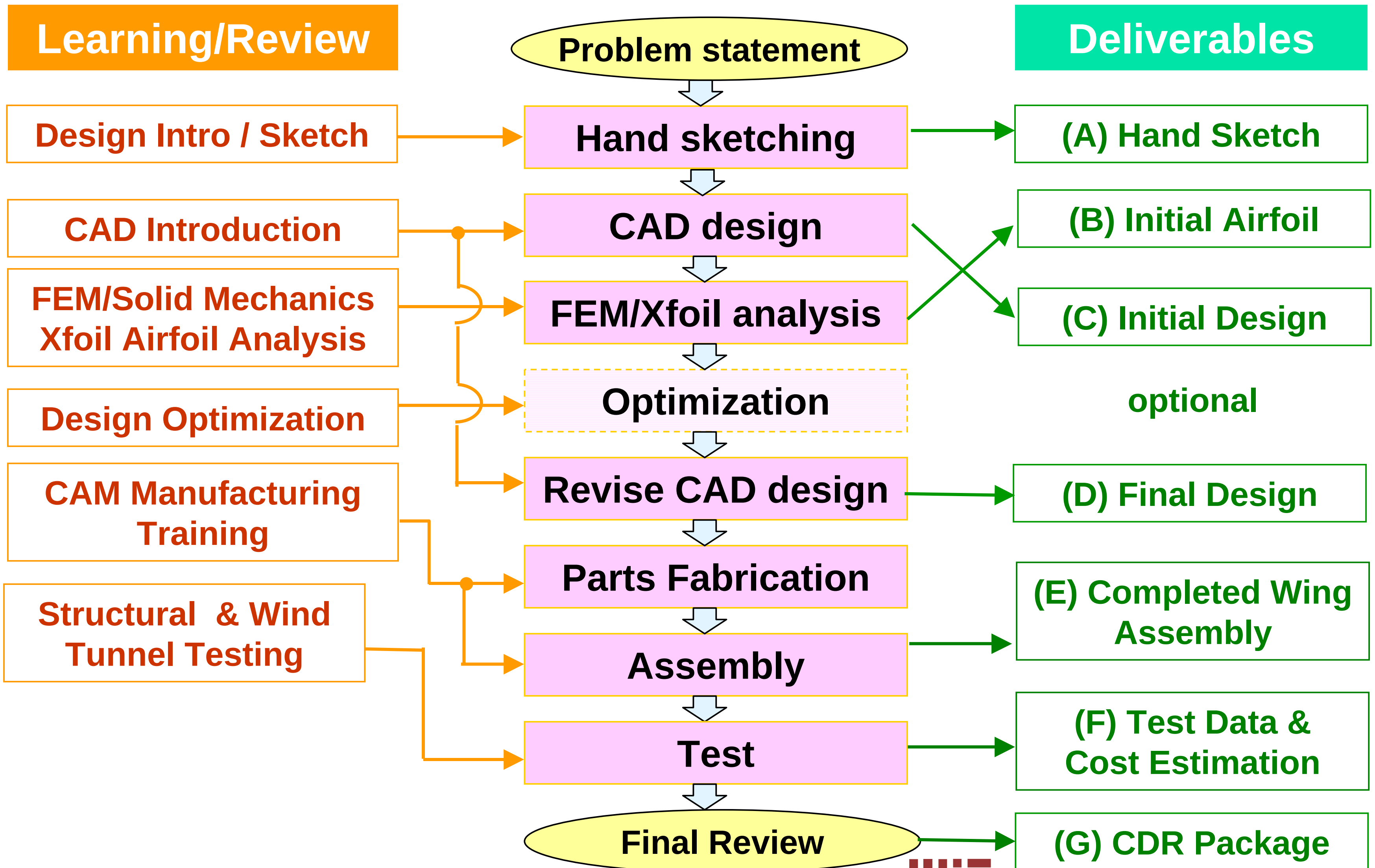
16.810 Plan for Today

- CAD Lecture (ca. 50 min)
  - CAD History, Background
  - Some theory of geometrical representation
- SolidWorks Introduction (ca. 40 min)
  - Led by TA
  - Follow along step-by-step
- Start creating your own CAD model of your part (ca. 30 min)
  - Work in teams of two
  - Use hand sketch as starting point

Course Concept



Course Flow Diagram (2005)



16.810 What is CAD?

- Computer Aided Design (CAD)
  - A set of methods and tools to assist product designers in
    - Creating a geometrical representation of the artifacts they are designing
    - Dimensioning, Tolerancing
    - Configuration Management (Changes)
    - Archiving
    - Exchanging part and assembly information between teams, organizations
    - Feeding subsequent design steps
      - Analysis (CAE)
      - Manufacturing (CAM)
  - ...by means of a computer system.

16.810 Basic Elements of a CAD System

Input Devices

Keyboard
Mouse

CAD keyboard
Templates
Space Ball

Main System

Computer
CAD Software
Database

Output Devices

Hard Disk
Network
Printer
Plotter

Human Designer

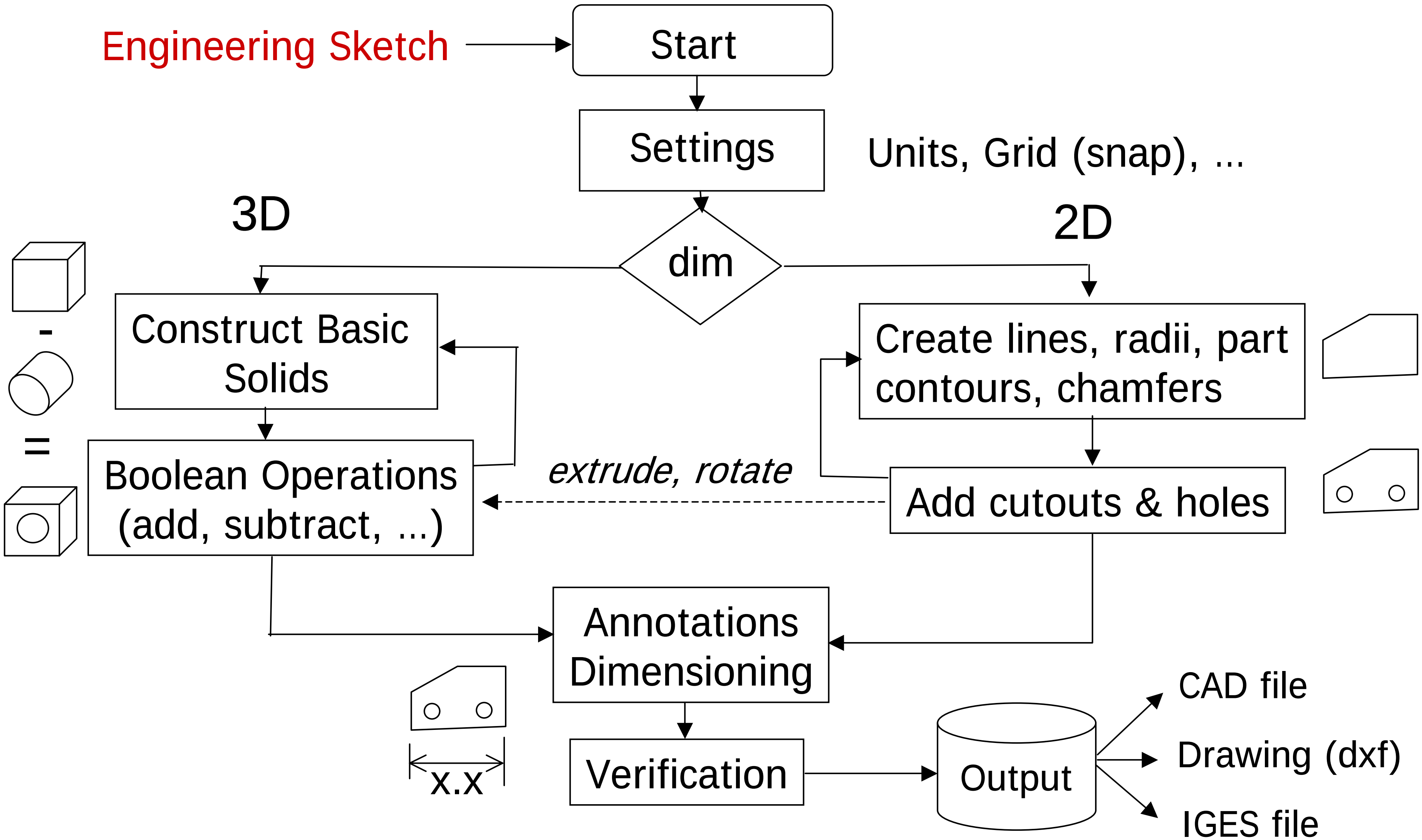
16.810 Brief History of CAD

- 1957 PRONTO (Dr. Hanratty) – first commercial numerical-control programming system
- 1960 SKETCHPAD (MIT Lincoln Labs)
- Early 1960's industrial developments
  - General Motors – DAC (Design Automated by Computer)
  - McDonnell Douglas – CADD
- Early technological developments
  - Vector-display technology
  - Light-pens for input
  - Patterns of lines rendering (first 2D only)
- 1967 Dr. Jason R Lemon founds SDRC in Cincinnati
- 1979 Boeing, General Electric and NIST develop IGES (Initial Graphic Exchange Standards), e.g. for transfer of NURBS curves
- Since 1981: numerous commercial programs
  - Source: http://mbinfo.mbdesign.net/CAD-History.htm

16.810 Major Benefits of CAD

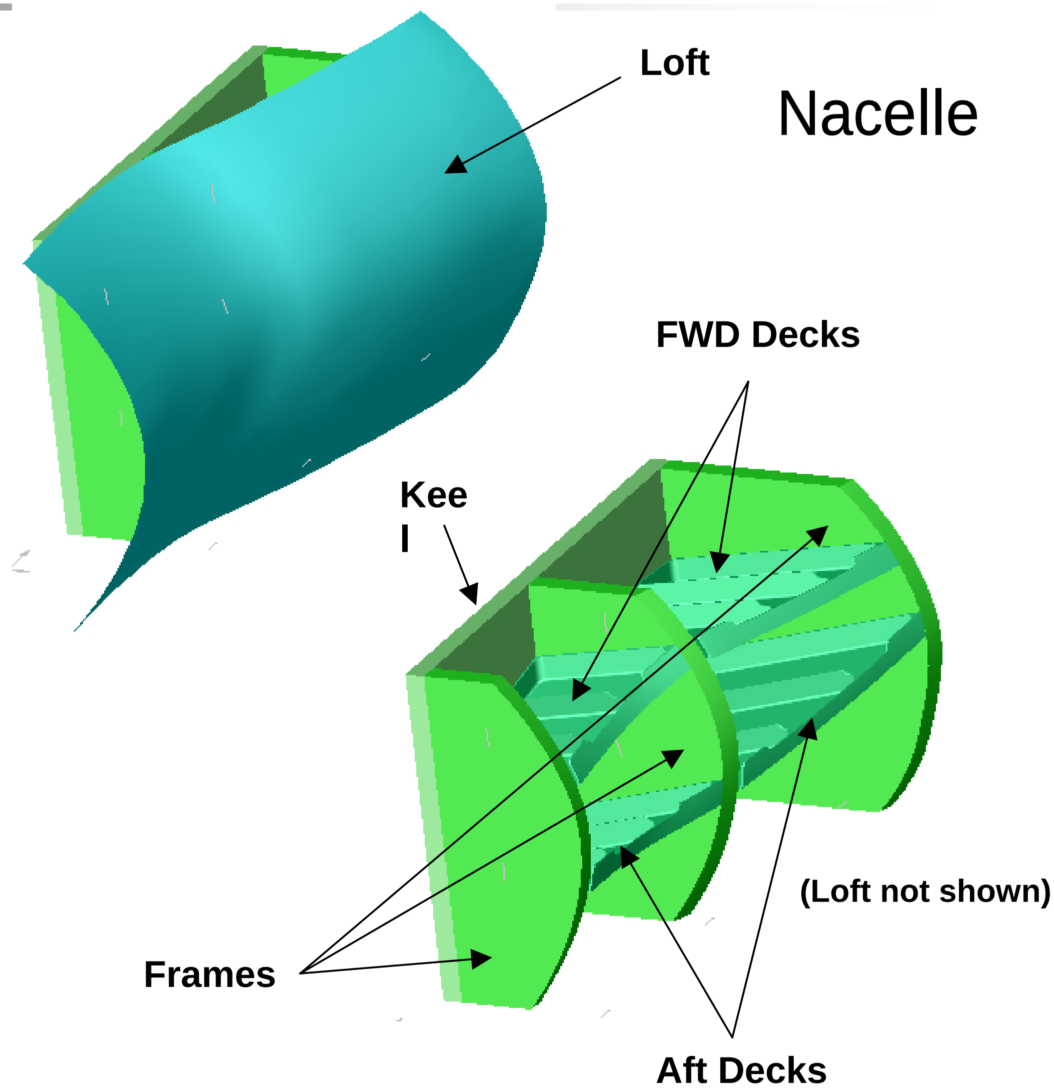
- Productivity (=Speed) Increase
  - Automation of repeated tasks
    - Doesn't necessarily increase creativity!
  - Insert standard parts (e.g. fasteners) from database
- Supports Changeability
  - Don't have to redo entire drawing with each change
    - EO – "Engineering Orders"
  - Keep track of previous design iterations
- Communication
  - With other teams/engineers, e.g. manufacturing, suppliers
  - With other applications (CAE/FEM, CAM)
  - Marketing, realistic product rendering
  - Accurate, high quality drawings
    - Caution: CAD Systems produce errors with hidden lines etc...
- Some limited Analysis
  - Mass Properties (Mass, Inertia)
  - Collisions between parts, clearances

Generic CAD Process



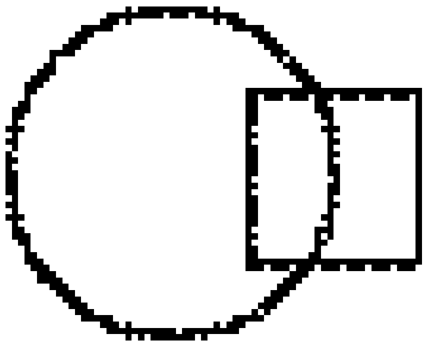
Example CAD A/C Assembly

- Boeing (sample) parts
  - A/C structural assembly
    - 2 decks
    - 3 frames
    - Keel
  - Loft included to show interface/stayout zone to A/C
  - All Boeing parts in Catia file format
    - Files imported into SolidWorks by converting to IGES format



16.810 Vector versus Raster Graphics

Raster Graphics



- Grid of pixels
  - No relationships between pixels
  - Resolution, e.g. 72 dpi (dots per inch)
  - Each pixel has color, e.g. 8-bit image has 256 colors

.bmp - raw data format

42 4D BC 02 00 00 00 00 00 00 3E 00 00 00 28 00 00 00 42 00 00 00 35 00 00 00 01
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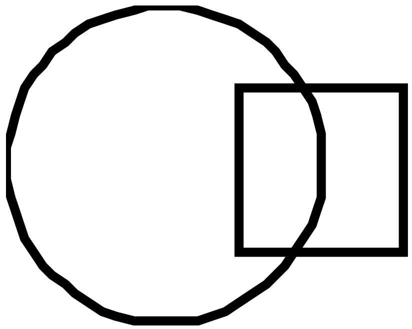
Vector Graphics

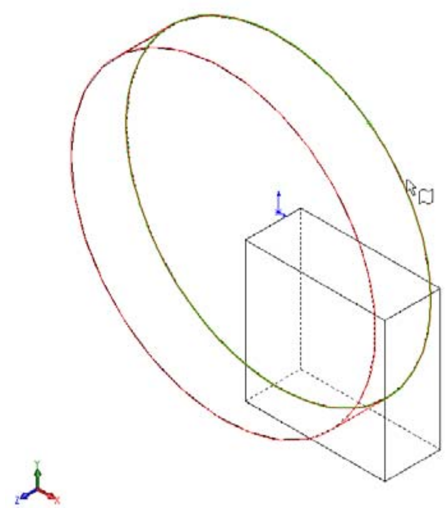


.emf format

CAD Systems use vector graphics

- Object Oriented
  - relationship between pixels captured
  - describes both (anchor/control) points and lines between them
  - Easier scaling & editing

Most common interface file: IGES



16.810 Major CAD Software Products

- AutoCAD (Autodesk) → mainly for PC
- Pro Engineer (PTC)
- SolidWorks (Dassault Systems)
- CATIA (IBM/Dassault Systems)
- Unigraphics (UGS)
- I-DEAS (SDRC)

Some CAD-Theory

Geometrical representation

(1) Parametric Curve Equation vs.

Nonparametric Curve Equation

(2) Various curves (*some mathematics !*)

- Hermite Curve

- Bezier Curve

- B-Spline Curve

- NURBS (Nonuniform Rational B-Spline) Curves

Applications: CAD, FEM, Design Optimization

Curve Equations

Two types of equations for curve representation

(1) Parametric equation

x, y, z coordinates are related by a parametric variable $(u \text{ or } \theta)$

(2) Nonparametric equation

x, y, z coordinates are related by a function

Example: Circle (2-D)

Parametric equation

$$x = R\cos\theta, \quad y = R\sin\theta \quad (0 \le \theta \le 2\pi)$$

Nonparametric equation

$$x^2 + y^2 - R^2 = 0$$ **(Implicit nonparametric form)**

$$y = \pm\sqrt{R^2 - x^2}$$ **(Explicit nonparametric form)**

Curve Equations

Two types of curve equations

(1) Parametric equation

Point on 2-D curve: $\mathbf{p} = [x(u) \quad y(u)]$

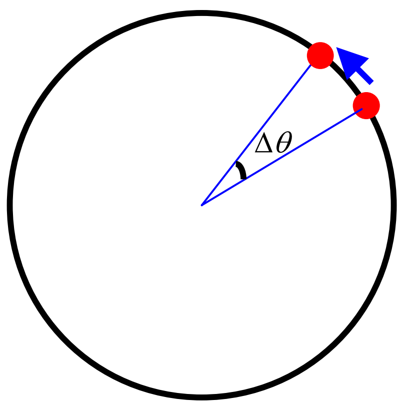
Point on 3-D surface: $\mathbf{p} = [x(u) \quad y(u) \quad z(u)]$

u : parametric variable and independent variable

(2) Nonparametric equation $y = f(x)$: 2-D , $z = f(x, y)$: 3-D

Which is better for CAD/CAE?

: Parametric equation



$$x = R\cos\theta, \quad y = R\sin\theta \quad (0 \le \theta \le 2\pi)$$

It also is good for calculating the points at a certain interval along a curve

$$x^2 + y^2 - R^2 = 0$$

$$y = \pm\sqrt{R^2 - x^2}$$

Parametric Equations – Advantages over nonparametric forms

1. Parametric equations usually offer more degrees of freedom for controlling the shape of curves and surfaces than do nonparametric forms.

e.g. Cubic curve

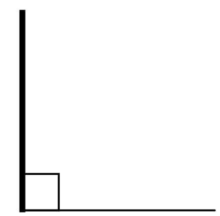
Parametric curve: $x = au^3 + bu^2 + cu + d$

$$y = eu^3 + fu^2 + gx + h$$

Nonparametric curve: $y = ax^3 + bx^2 + cx + d$

2. Parametric forms readily handle infinite slopes

$$\frac{dy}{dx} = \frac{dy/du}{dx/du} \quad \Rightarrow \; dx/du = 0 \;\text{ indicates }\; dy/dx = \infty$$



3. Transformation can be performed directly on parametric equations

e.g. Translation in x-dir.

Parametric curve: $x = au^3 + bu^2 + cu + d + x_0$

$$y = eu^3 + fu^2 + gx + h$$

Nonparametric curve: $y = a(x - x_0)^3 + b(x - x_0)^2 + c(x - x_0) + d$

Hermite Curves

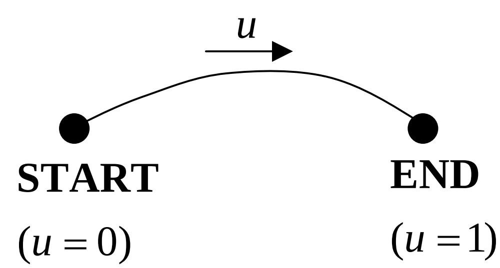
* Most of the equations for curves used in CAD software are of degree 3, because two curves of degree 3 guarantees 2nd derivative continuity at the connection point
→ The two curves appear to one.

* Use of a higher degree causes small oscillations in curve and requires heavy computation.

* Simplest parametric equation of degree 3

$$\mathbf{P}(u) = [x(u) \;\; y(u) \;\; z(u)]$$

$$= \mathbf{a}_0 + \mathbf{a}_1 u + \mathbf{a}_2 u^2 + \mathbf{a}_3 u^3 \qquad (0 \le u \le 1)$$



$\mathbf{a}_0,\ \mathbf{a}_1,\ \mathbf{a}_2,\ \mathbf{a}_3$: Algebraic vector coefficients

➡ The curve's shape change cannot be intuitively anticipated from changes in these values

Hermite Curves

$$\mathbf{P}(u) = \mathbf{a}_0 + \mathbf{a}_1 u + \mathbf{a}_2 u^2 + \mathbf{a}_3 u^3 \qquad (0 \le u \le 1)$$

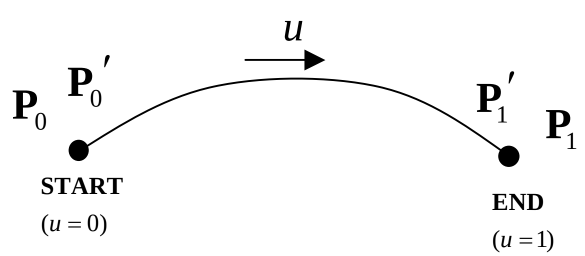
Instead of algebraic coefficients, let's use the position vectors and the tangent vectors at the two end points!

Position vector at starting point: $\mathbf{P}_0 = \mathbf{P}(0) = \mathbf{a}_0$

Position vector at end point: $\mathbf{P}_1 = \mathbf{P}(1) = \mathbf{a}_0 + \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$

Tangent vector at starting point: $\mathbf{P}_0' = \mathbf{P}'(0) = \mathbf{a}_1$

Tangent vector at end point: $\mathbf{P}_1' = \mathbf{P}'(1) = \mathbf{a}_1 + 2\mathbf{a}_2 + 3\mathbf{a}_3$



Blending functions

$$\mathbf{P}(u) = [1 - 3u^2 + 2u^3 \quad 3u^2 - 2u^3 \quad u - 2u^2 + u^3 \quad -u^2 + u^3] \begin{bmatrix} \mathbf{P}_0 \\ \mathbf{P}_1 \\ \mathbf{P}_0' \\ \mathbf{P}_1' \end{bmatrix}$$

: Hermit curve

No algebraic coefficients

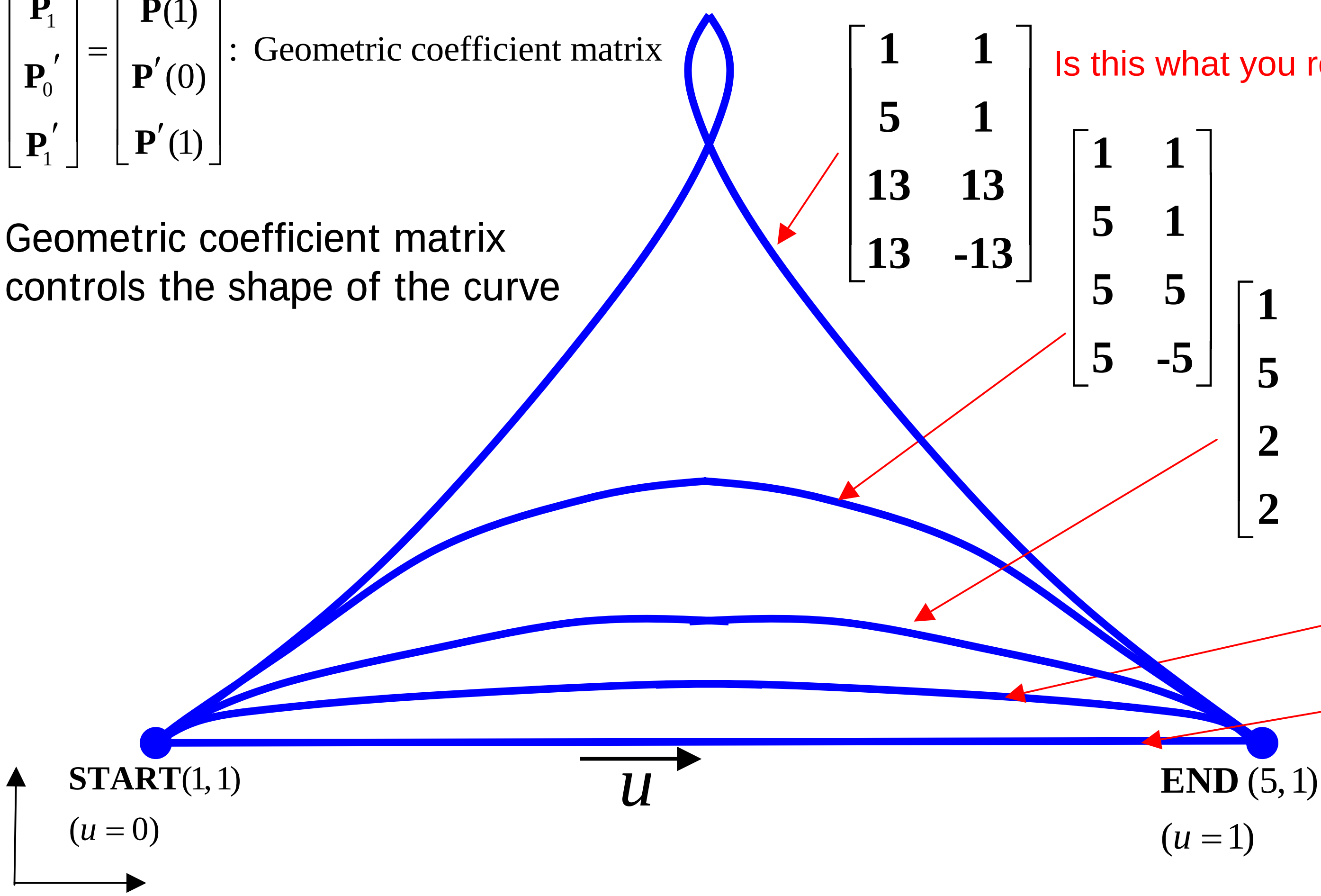
$\mathbf{P}_0, \mathbf{P}_0', \mathbf{P}_1, \mathbf{P}_1'$: Geometric coefficients

➡ The curve's shape change can be intuitively anticipated from changes in these values

Effect of tangent vectors on the curve's shape

$$\begin{bmatrix} \mathbf{P}_0 \\ \mathbf{P}_1 \\ \mathbf{P}_0' \\ \mathbf{P}_1' \end{bmatrix} = \begin{bmatrix} \mathbf{P}(0) \\ \mathbf{P}(1) \\ \mathbf{P}'(0) \\ \mathbf{P}'(1) \end{bmatrix}$$: Geometric coefficient matrix

Geometric coefficient matrix controls the shape of the curve



$$\begin{bmatrix} 1 & 1 \\ 5 & 1 \\ 13 & 13 \\ 13 & -13 \end{bmatrix}$$

Is this what you really wanted?

$$\begin{bmatrix} 1 & 1 \\ 5 & 1 \\ 5 & 5 \\ 5 & -5 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \\ 5 & 1 \\ 2 & 2 \\ 2 & -2 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \\ 5 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \\ 5 & 1 \\ 4 & 0 \\ 4 & 0 \end{bmatrix}$$

$$\frac{dy}{dx} = \frac{dy/du}{dx/du} = \frac{0}{4} = 0$$

Bezier Curve

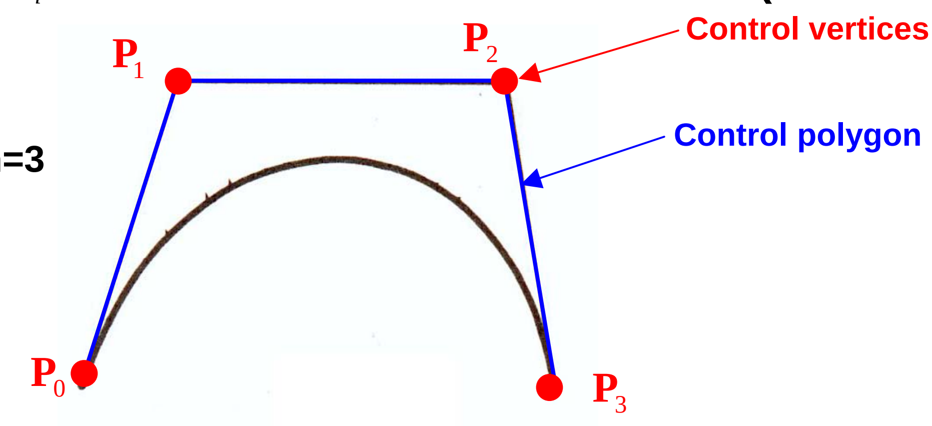
* In case of Hermite curve, it is not easy to predict curve shape according to changes in magnitude of the tangent vectors, $\mathbf{P}_0'$ and $\mathbf{P}_1'$

* Bezier Curve can control curve shape more easily using several control points (Bezier 1960)

$$\mathbf{P}(u) = \sum_{i=0}^{n} \binom{n}{i} u^i (1-u)^{n-i} \mathbf{P}_i \quad , \qquad \text{where } \binom{n}{i} = \frac{n!}{i!(n-i)!}$$

$\mathbf{P}_i$: Position vector of the *i* th vertex (control vertices)

n=3



* Number of vertices: n+1
  (No of control points)

* Number of segments: n

* Order of the curve: n

* The order of Bezier curve is determined by the number of control points.

n control points ⇨ Order of Bezier curve: n-1

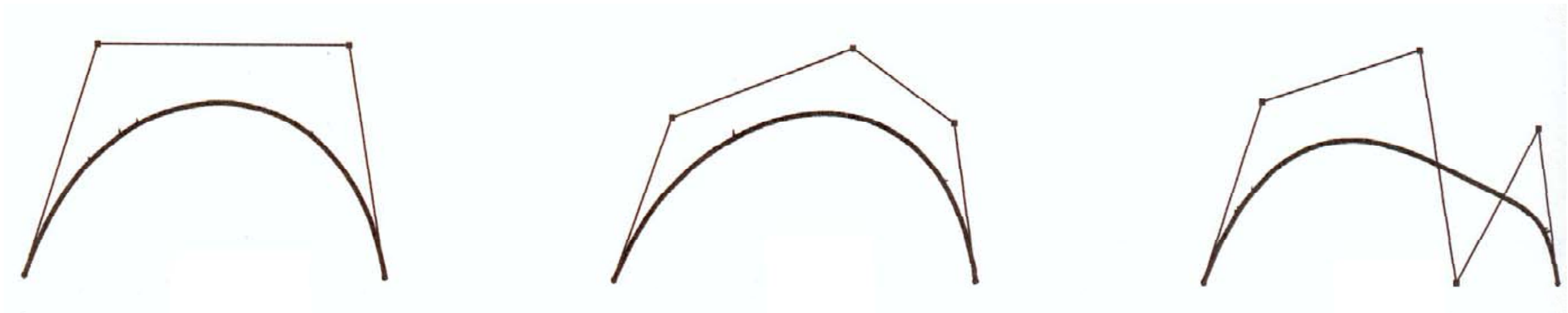
Bezier Curve

Properties

- The curve passes through the first and last vertex of the polygon.

- The tangent vector at the starting point of the curve has the same direction as the first segment of the polygon.

- The *n*th derivative of the curve at the starting or ending point is determined by the first or last (*n*+1) vertices.



Two Drawbacks of Bezier curve

(1) For complicated shape representation, higher degree Bezier curves are needed.

→ Oscillation in curve occurs, and computational burden increases.

(2) Any one control point of the curve affects the shape of the entire curve.

→ Modifying the shape of a curve locally is difficult.

(Global modification property)

Desirable properties :

1. **Ability to represent complicated shape with low order of the curve**
2. **Ability to modify a curve's shape locally**

⇨ B-spline curve!

B-Spline Curve

* Developed by Cox and Boor (1972)

$$\mathbf{P}(u) = \sum_{i=0}^{n} N_{i,k}(u)\mathbf{P}_i$$

where

$\mathbf{P}_i$: Position vector of the ith control point

$$N_{i,k}(u) = \frac{(u - t_i)N_{i,k-1}(u)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - u)N_{i+1,k-1}(u)}{t_{i+k} - t_{i+1}}$$

$$N_{i,1}(u) = \begin{cases} 1 & t_i \le u \le t_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

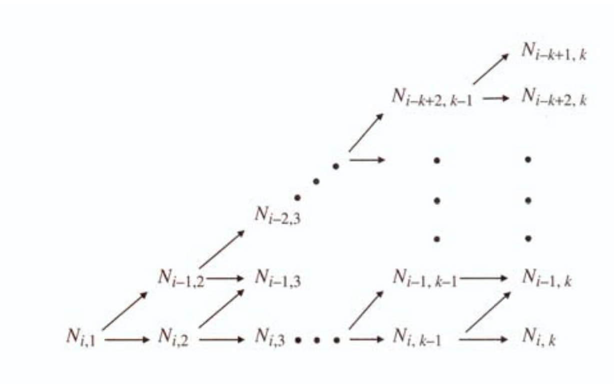
$$t_i = \begin{cases} 0 & 0 \le i < k \\ i - k + 1 & k \le i \le n \\ n - k + 2 & n < i \le n + k \end{cases}$$

(Nonperiodic knots)

k: order of the B-spline curve

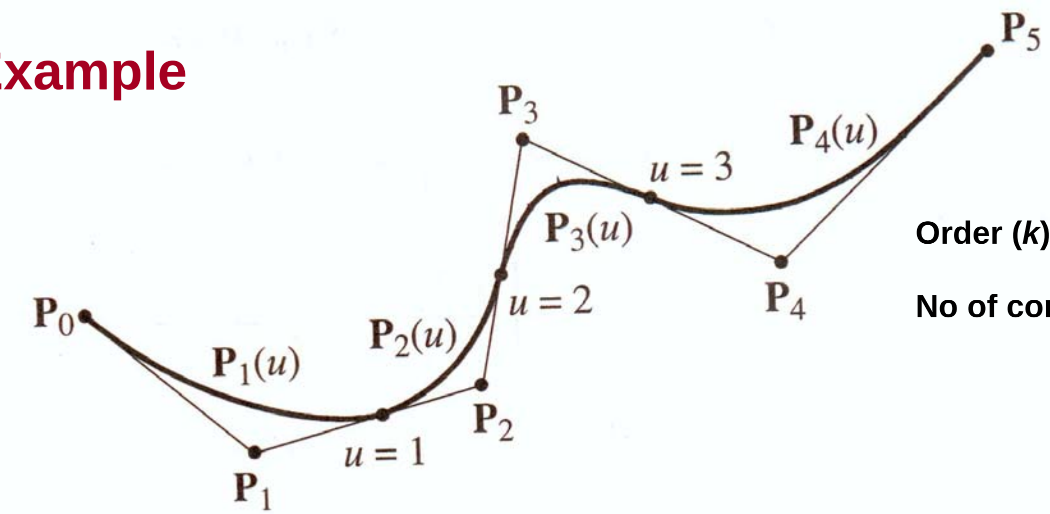
n+1: number of control points

The order of curve is independent of the number of control points!



B-Spline Curve

Example



Order (*k*) = 3 (first derivatives are continuous)

No of control points (*n+1*) = 6

Advantages

(1) The order of the curve is independent of the number of control points (contrary to Bezier curves)

- User can select the curve's order and number of control points separately.
- It can represent very complicated shape with low order

(2) Modifying the shape of a curve locally is easy. (contrary to Bezier curve)

- Each curve segment is affected by *k (order)* control points. (local modification property)

NURBS (Nonuniform Rational B-Spline) Curve

$$\mathbf{P}(u) = \frac{\sum_{i=0}^{n} h_i \mathbf{P}_i N_{i,k}(u)}{\sum_{i=0}^{n} h_i N_{i,k}(u)}$$

$$\left(\text{B-spline}: \ \mathbf{P}(u) = \sum_{i=0}^{n} \mathbf{P}_i N_{i,k}(u) \right)$$

$\mathbf{P}_i$: Position vector of the ith control point

h_i : Homogeneous coordinate

*** If all the homogeneous coordinates (h_i) are 1, the denominator becomes 1**

$$\text{If } h_i = 0 \ \forall i, \text{ then } \sum_{i=0}^{n} h_i N_{i,k}(u) = 1.$$

*** B-spline curve is a special case of NURBS.**

*** Bezier curve is a special case of B-spline curve.**

Advantages of NURBS Curve over B-Spline Curve

(1) More versatile modification capacity

- Homogeneous coordinate h_i, which B-spline does not have, can change.
- Increasing h_i of a control point → Drawing the curve toward the control point.

(2) NURBS can exactly represent the conic curves - circles, ellipses, parabolas, and hyperbolas (B-spline can only approximate these curves)

(3) Curves, such as conic curves, Bezier curves, and B-spline curves can be converted to their corresponding NURBS representations.

Summary

(1) Parametric Equation vs. Nonparametric Equation

(2) Various curves

- Hermite Curve
- Bezier Curve
- B-Spline Curve
- NURBS (Nonuniform Rational B-Spline) Curve

(3) Surfaces

- Bilinear surface
- Bicubic surface
- Bezier surface
- B-Spline surface
- NURBS surface

16.810 SolidWorks Introduction

- SolidWorks
  - Most popular CAD system in education
  - Will be used for this project
  - 40 Minute Introduction by TA
    - http://www.solidworks.com (Student Section)