16.810

Engineering Design and Rapid Prototyping

Lecture 6

Design Optimization

- Structural Design Optimization -

Instructor(s)

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Massachusetts Institute of Technology

What Is Design Optimization?

Selecting the "best" design within the available means

1. What is our criterion for "best" design?	**Objective function**
2. What are the available means?	**Constraints (design requirements)**
3. How do we describe different designs?	**Design Variables**

Optimization Statement

$$\text{Minimize} \quad f(\mathbf{x})$$

$$\text{Subject to} \quad g(\mathbf{x}) \le 0$$

$$h(\mathbf{x}) = 0$$

Design Variables

For computational design optimization,

⇨ **Objective function and constraints must be expressed as a function of design variables (or design vector X)**

Objective function: $f(\mathbf{x})$ Constraints: $g(\mathbf{x}), h(\mathbf{x})$

Cost = f(design)

Lift = f(design)

Drag = f(design)

Mass = f(design)

What is "f" for each case?

Optimization Statement

$$\text{Minimize} \quad f(\mathbf{x})$$

$$\textit{Subject to} \quad g(\mathbf{x}) \le 0$$

$$h(\mathbf{x}) = 0$$

$f(\mathbf{x})$: **Objective function to be minimized**

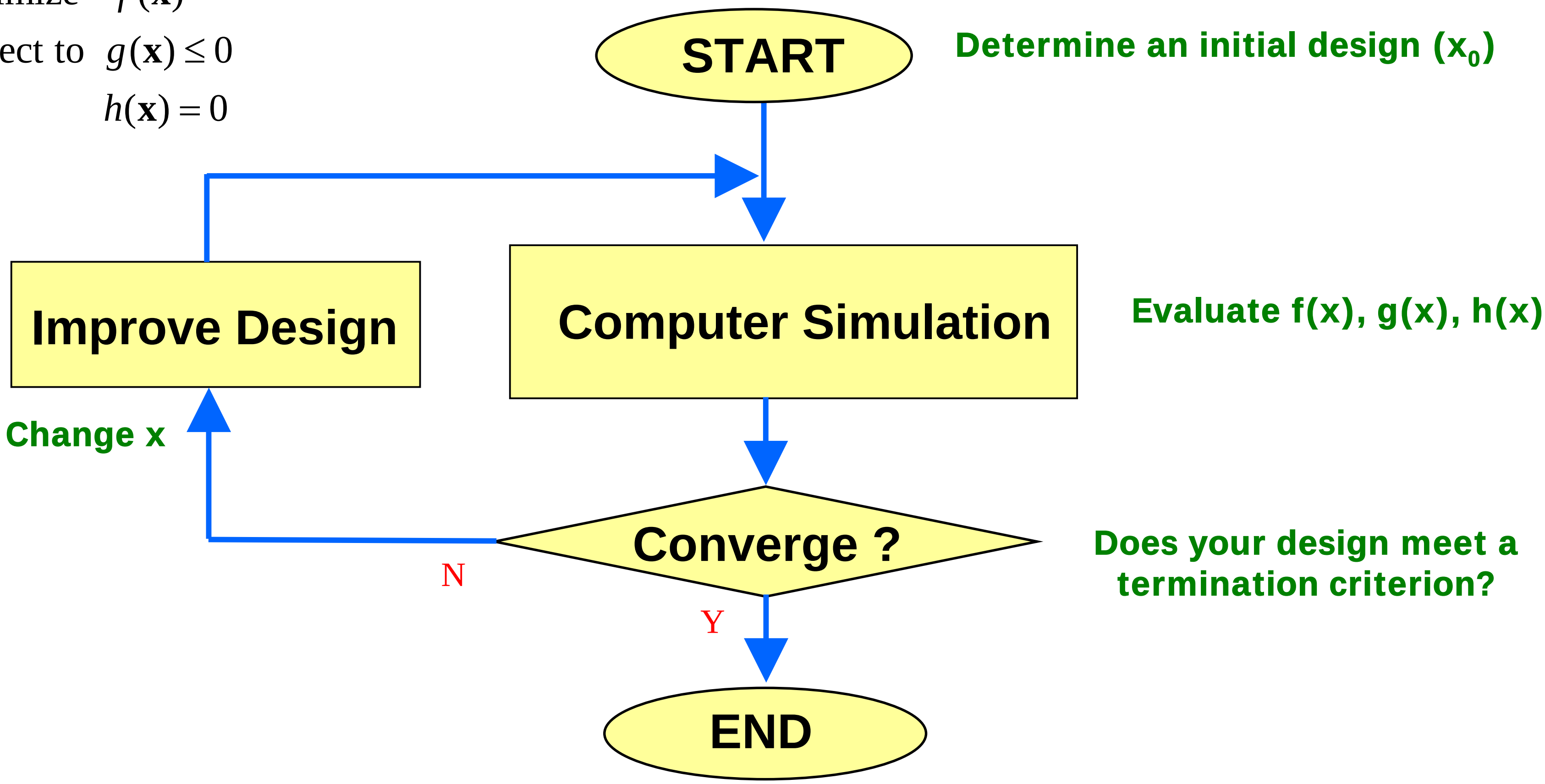
$g(\mathbf{x})$: **Inequality constraints**

$h(\mathbf{x})$: **Equality constraints**

$\mathbf{x}$: **Design variables**

Optimization Procedure

$$\text{Minimize} \quad f(\mathbf{x})$$
$$\text{Subject to} \quad g(\mathbf{x}) \le 0$$
$$h(\mathbf{x}) = 0$$



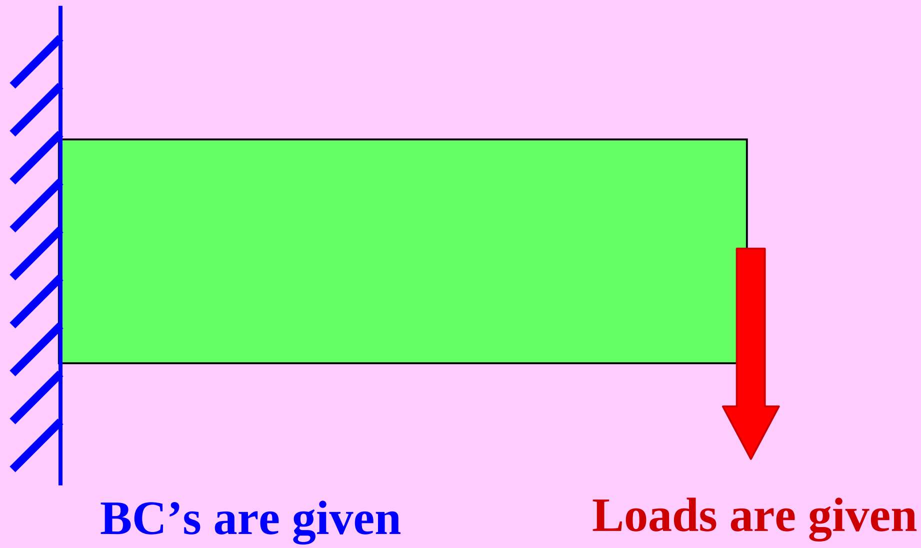
Structural Optimization

Selecting the best "structural" design

- Size Optimization
- Shape Optimization
- Topology Optimization

Structural Optimization

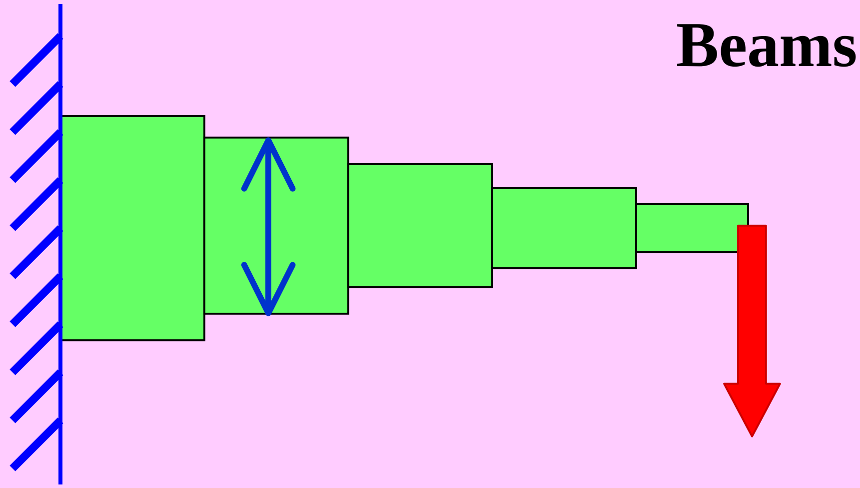
$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g(\mathbf{x}) \le 0 \\ & h(\mathbf{x}) = 0 \end{aligned}$$



1. To make the structure strong → *Min.* $f(\mathbf{x})$
   e.g. Minimize displacement at the tip
2. Total mass ≤ M_C → $g(\mathbf{x}) \le 0$

Size Optimization

$$\text{minimize} \quad f(\mathbf{x})$$
$$\text{subject to} \quad g(\mathbf{x}) \le 0$$
$$h(\mathbf{x}) = 0$$



Design variables (x)

x : thickness of each beam

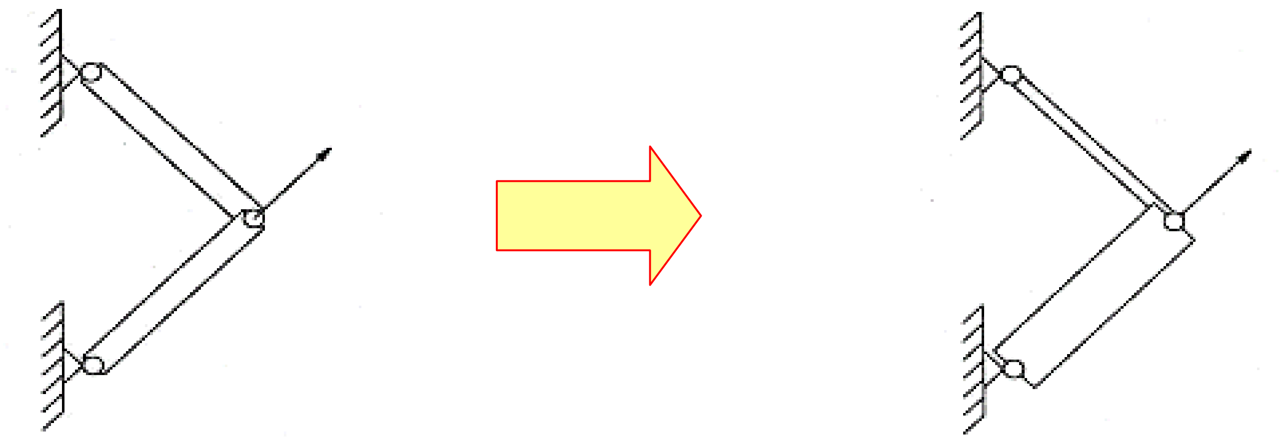
Number of design variables (ndv)

ndv = 5

***f*(x) : compliance**

***g*(x) : mass**

Size Optimization

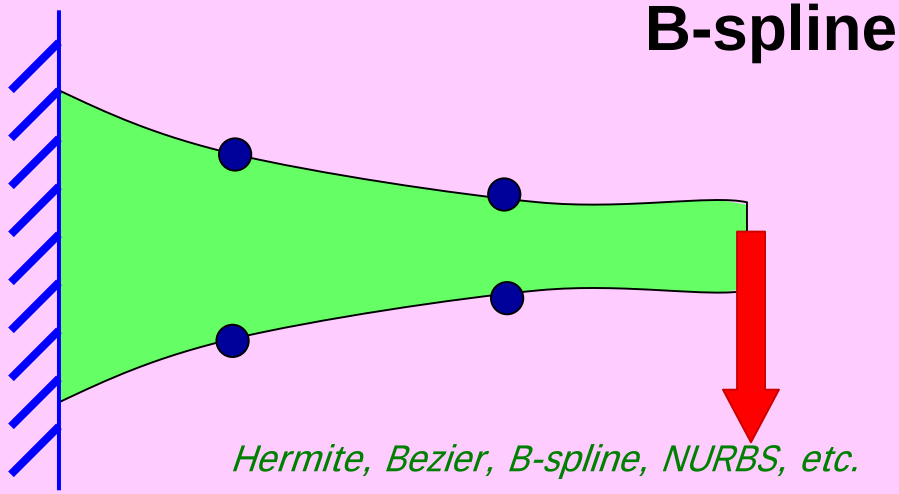


- Shape
  Topology } are given

- **Optimize cross sections**

Shape Optimization

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g(\mathbf{x}) \le 0 \\ & h(\mathbf{x}) = 0 \end{aligned}$$



Design variables (x)

x : control points of the B-spline

(position of each control point)

***f*(x) : compliance**

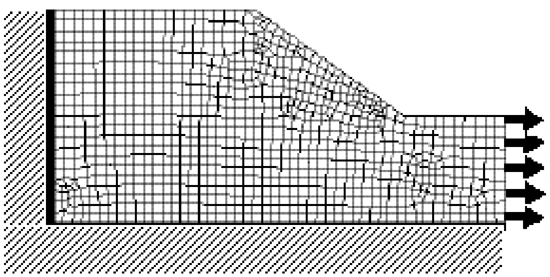
***g*(x) : mass**

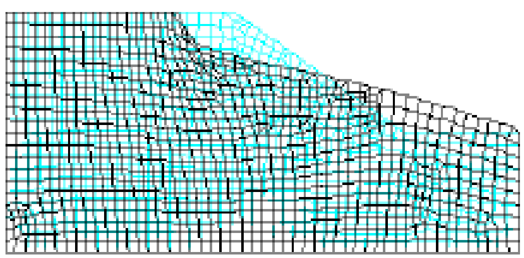
Number of design variables (ndv)

ndv = 8

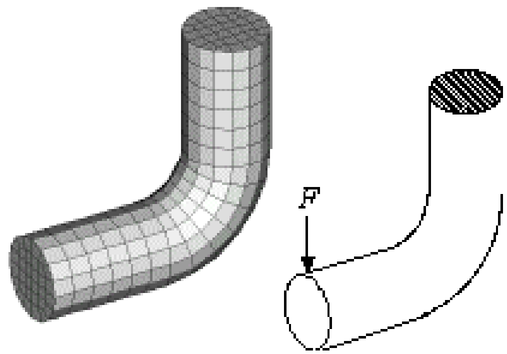
Shape Optimization

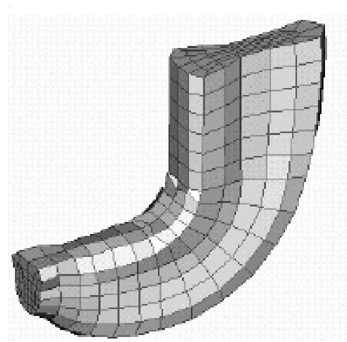
Fillet problem



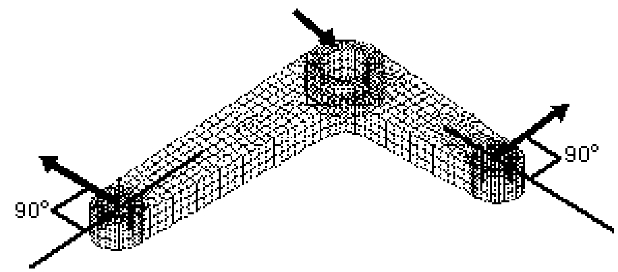


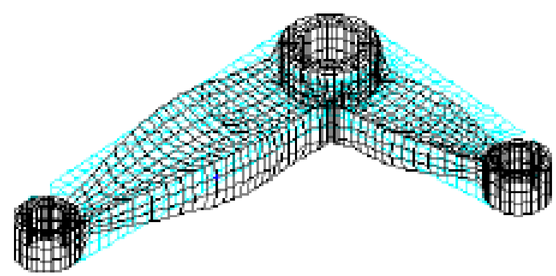
Hook problem





Arm problem





Shape Optimization

Multiobjective & Multidisciplinary Shape Optimization

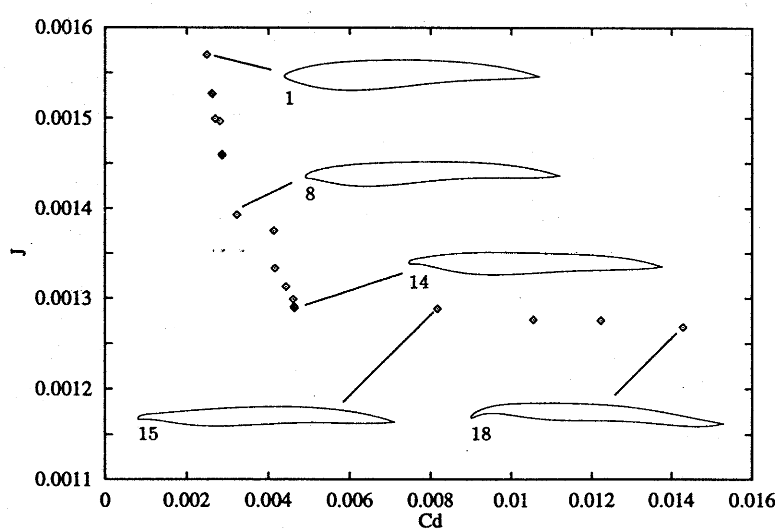
Objective function

1. Drag coefficient,   2. Amplitude of backscattered wave

Analysis

1. Computational Fluid Dynamics Analysis
2. Computational Electromagnetic Wave Field Analysis

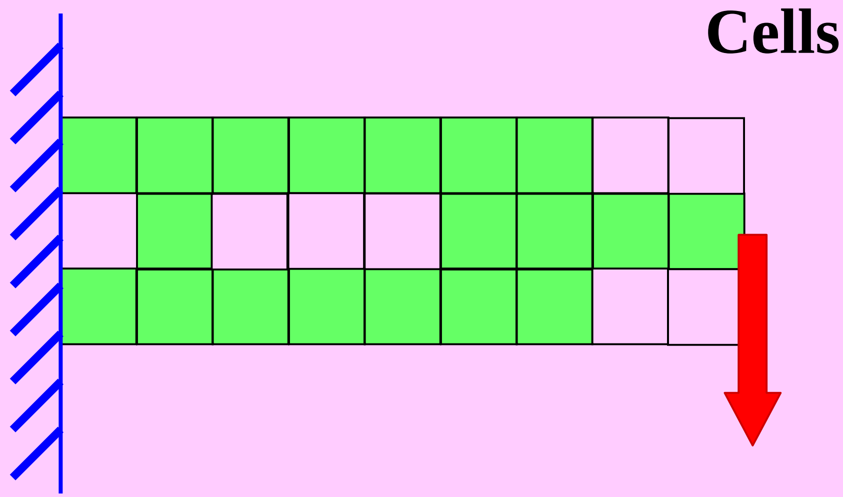
Obtain Pareto Front



Raino A.E. Makinen et al., "Multidisciplinary shape optimization in aerodynamics and electromagnetics using genetic algorithms," International Journal for Numerical Methods in Fluids, Vol. 30, pp. 149-159, 1999

Topology Optimization

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g(\mathbf{x}) \le 0 \\ & h(\mathbf{x}) = 0 \end{aligned}$$



Design variables (x)

x : density of each cell

Number of design variables (ndv)

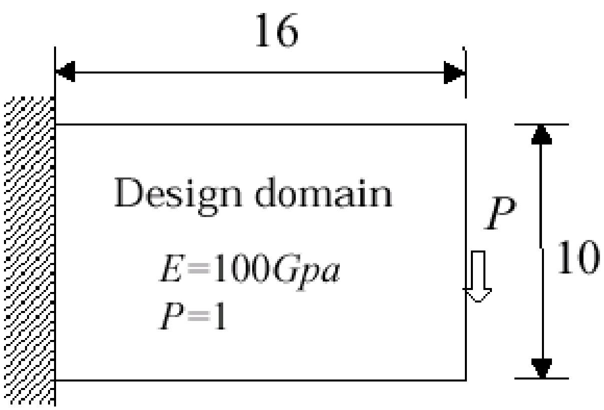
ndv = 27

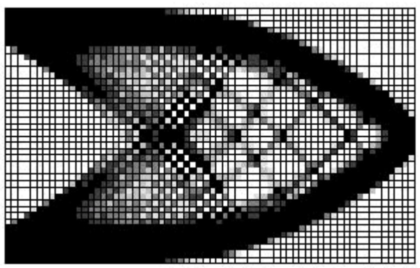
***f*(x) : compliance**

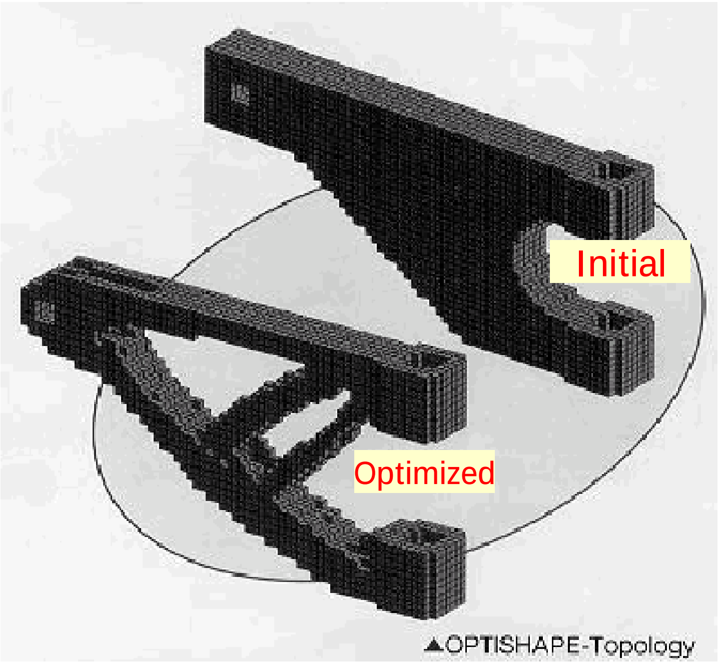
***g*(x) : mass**

Topology Optimization

Short Cantilever problem

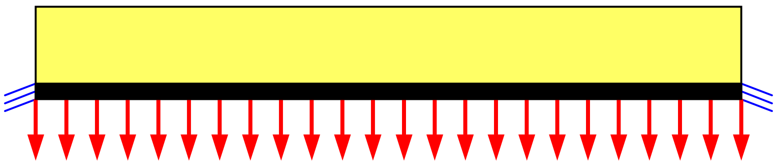






Topology Optimization

Bridge problem

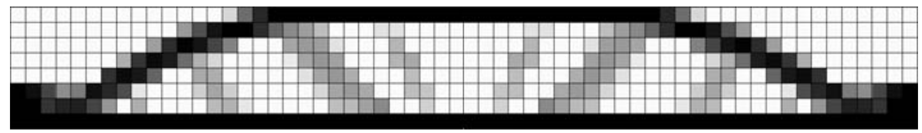


$$Minimize \quad \int_{\Gamma} F^i z^i d\Gamma,$$

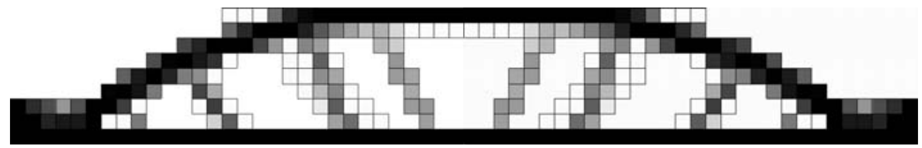
$$Subject\ \ to \int_{\Omega} \rho(x) d\Omega \leq M_o,$$

$$0 \leq \rho(x) \leq 1$$

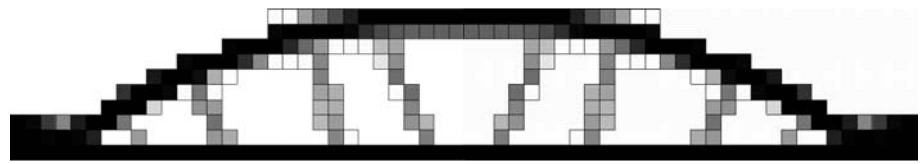
Mass constraints: 35%



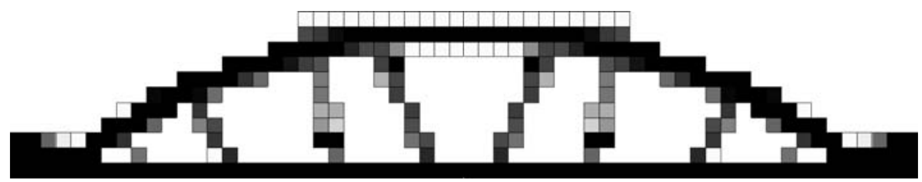
Obj = 4.16×10^5



Obj = 3.29×10^5



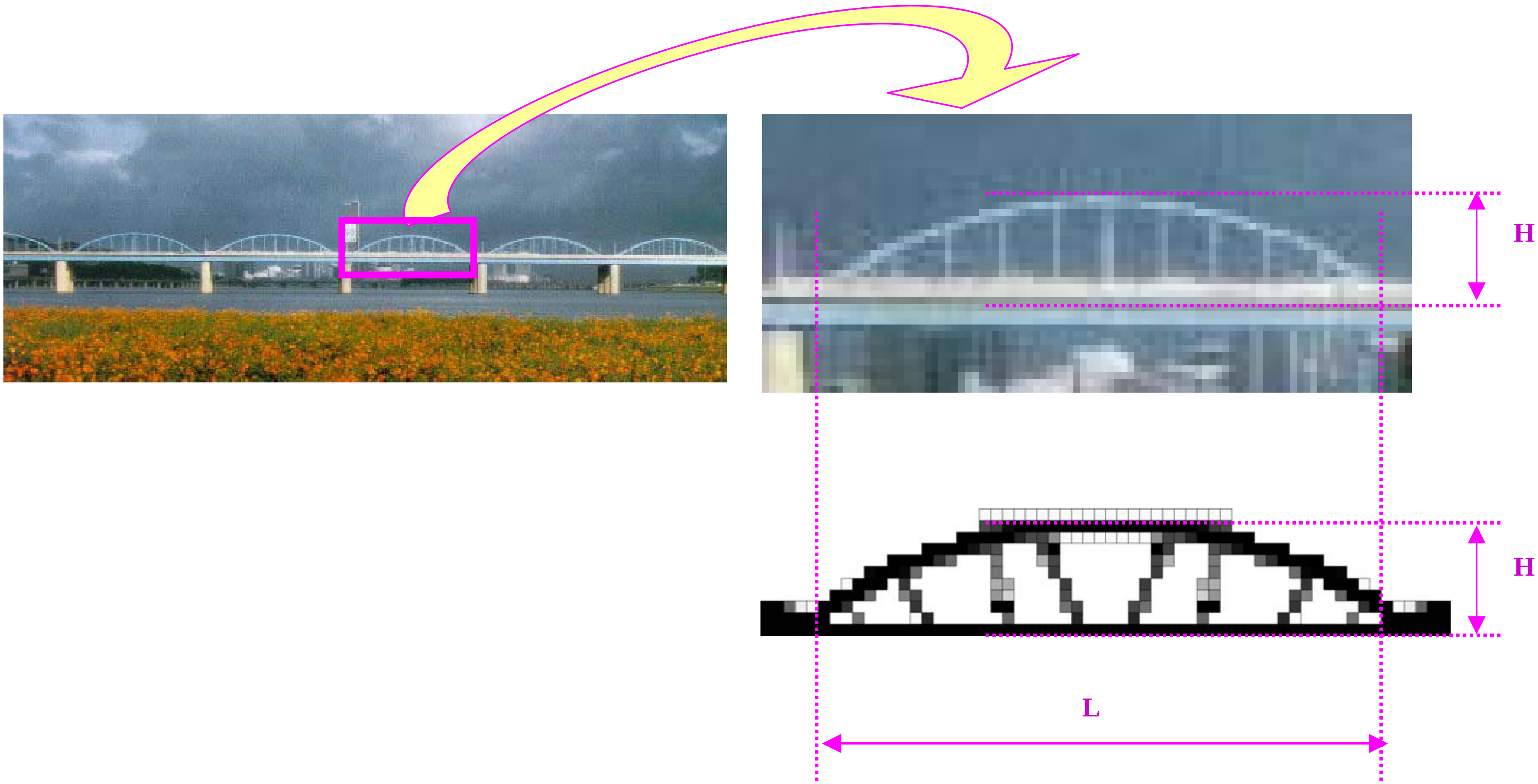
Obj = 2.88×10^5



Obj = 2.73×10^5

Topology Optimization

DongJak Bridge in Seoul, Korea



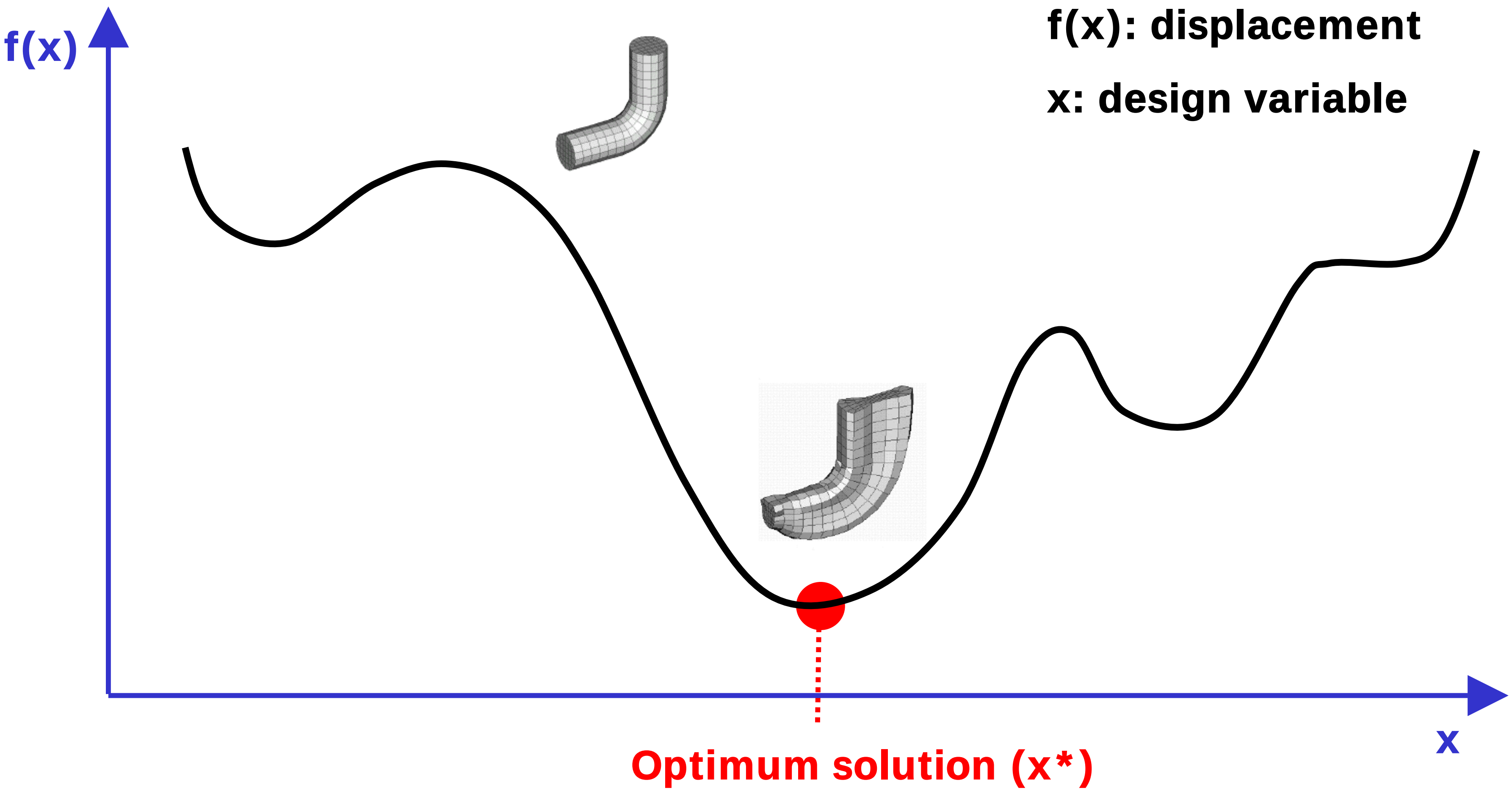
Structural Optimization

What determines the type of structural optimization?

Type of the design variable

(How to describe the design?)

Optimum Solution
– Graphical Representation

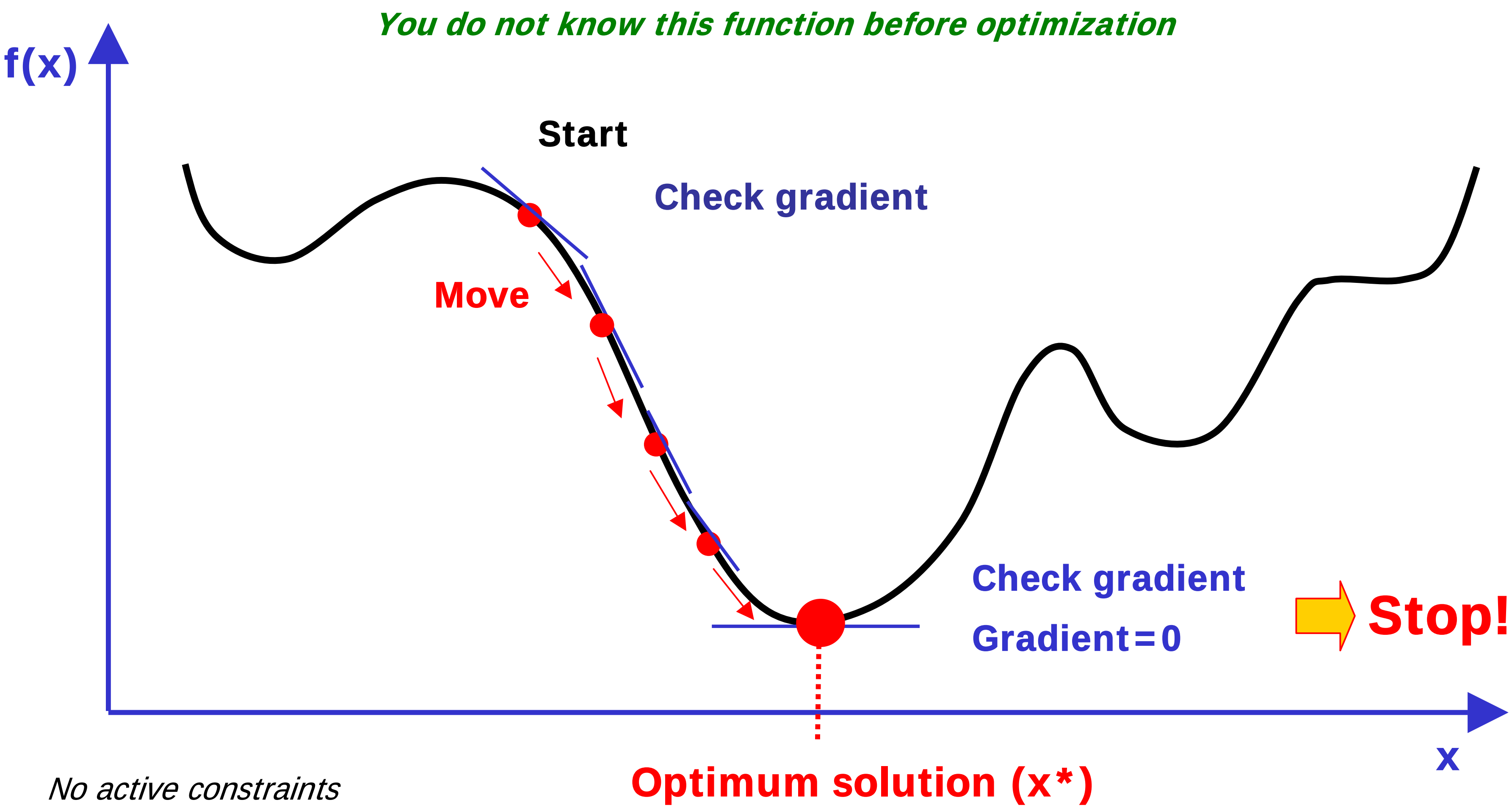


Optimization Methods

Gradient-based methods

Heuristic methods

Gradient-based Methods



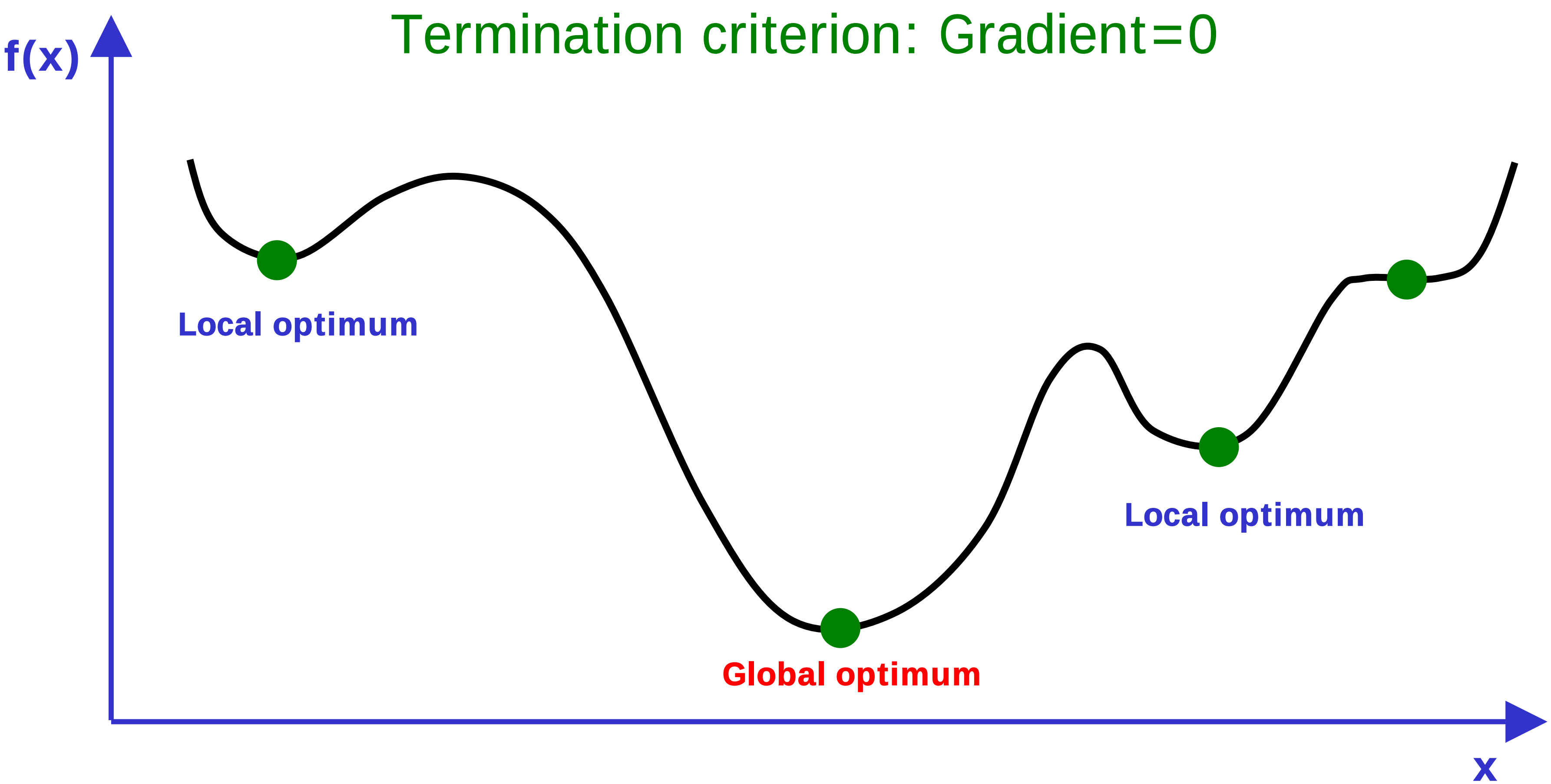
No active constraints

(Termination criterion: Gradient=0)

Gradient-based Methods

Steepest Descent	**UNCONSTRAINED**
Conjugate Gradient	
Quasi-Newton	
Newton	
Simplex – linear	**CONSTRAINED**
SLP – linear	
SQP – nonlinear, expensive, common in engineering applications	
Exterior Penalty – nonlinear, discontinuous design spaces	
Interior Penalty – nonlinear	
Generalized Reduced Gradient – nonlinear	
Method of Feasible Directions – nonlinear	
Mixed Integer Programming	

Global optimum vs. Local optimum



No active constraints

Heuristic Methods

- Heuristics Often Incorporate Randomization

- **3 Most Common Heuristic Techniques**
  - Genetic Algorithms
  - Simulated Annealing
  - Tabu Search

Optimization Software

- iSIGHT
- DOT
- Matlab (fmincon)

16.810 Topology Optimization Software

- **ANSYS**

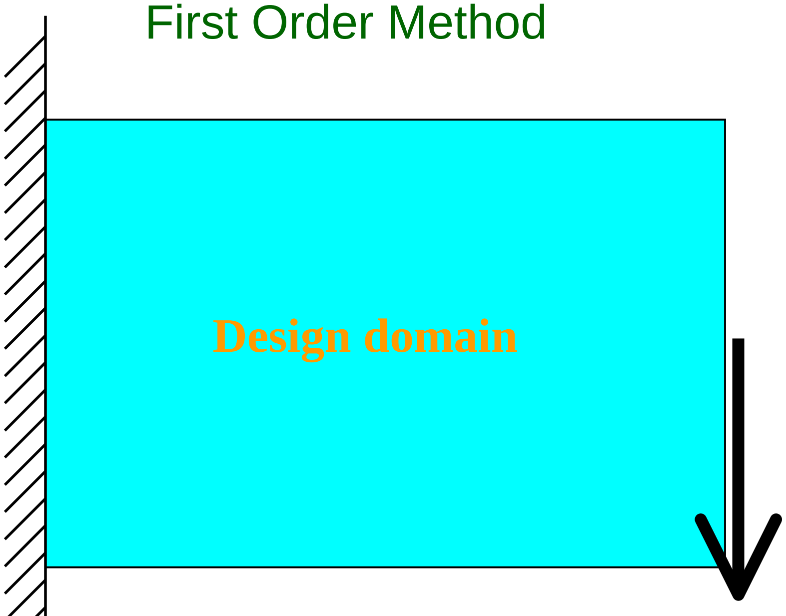
  Static Topology Optimization

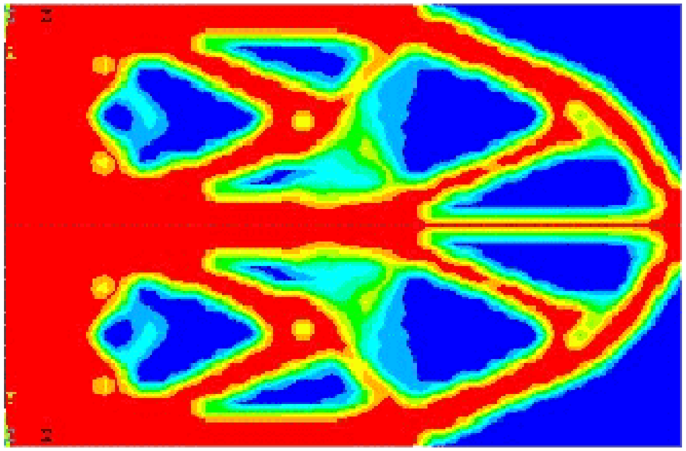
  Dynamic Topology Optimization

  Electromagnetic Topology Optimization

  Subproblem Approximation Method

  First Order Method



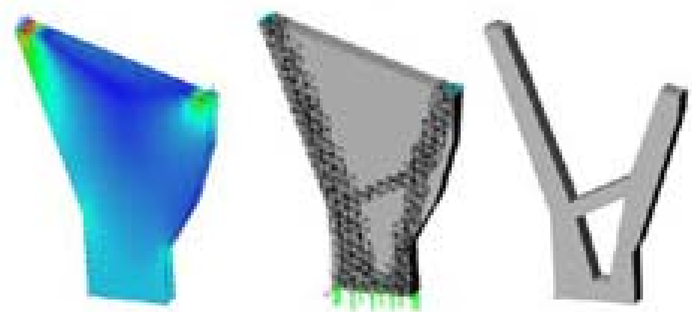


Topology Optimization Software

- **MSC. Visual Nastran FEA**

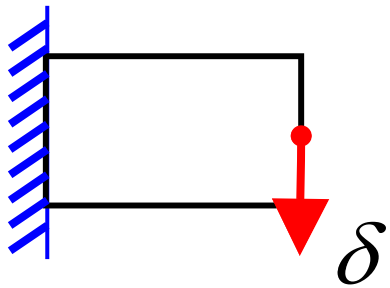
Elements of lowest stress are removed gradually.

Optimization results



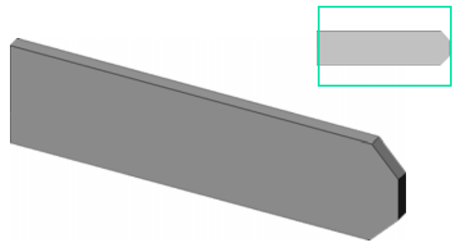
Optimization results illustration

Design Freedom



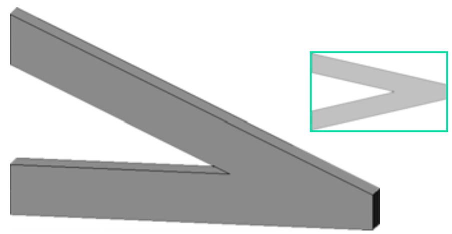
Volume is the same.

1 bar



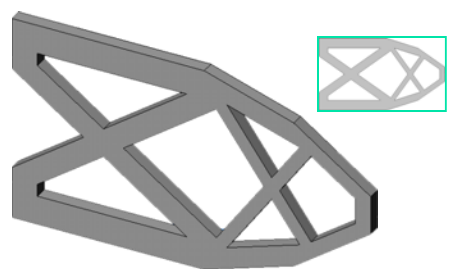
$$\delta = 2.50\ mm$$

2 bars



$$\delta = 0.80\ mm$$

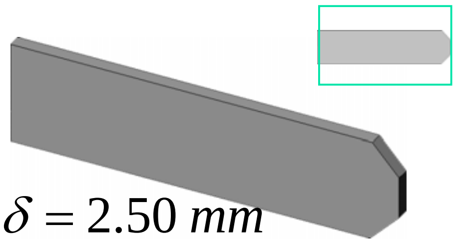
17 bars



$$\delta = 0.63\ mm$$

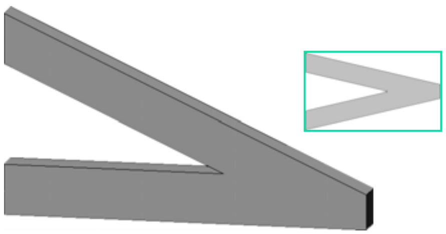
Design Freedom

1 bar



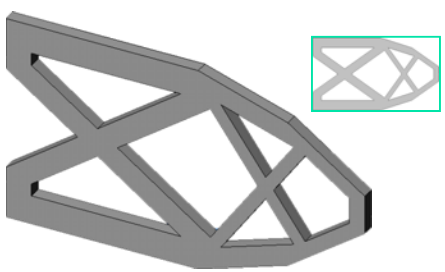
$\delta = 2.50\ mm$

2 bars



$\delta = 0.80\ mm$

17 bars



$\delta = 0.63\ mm$

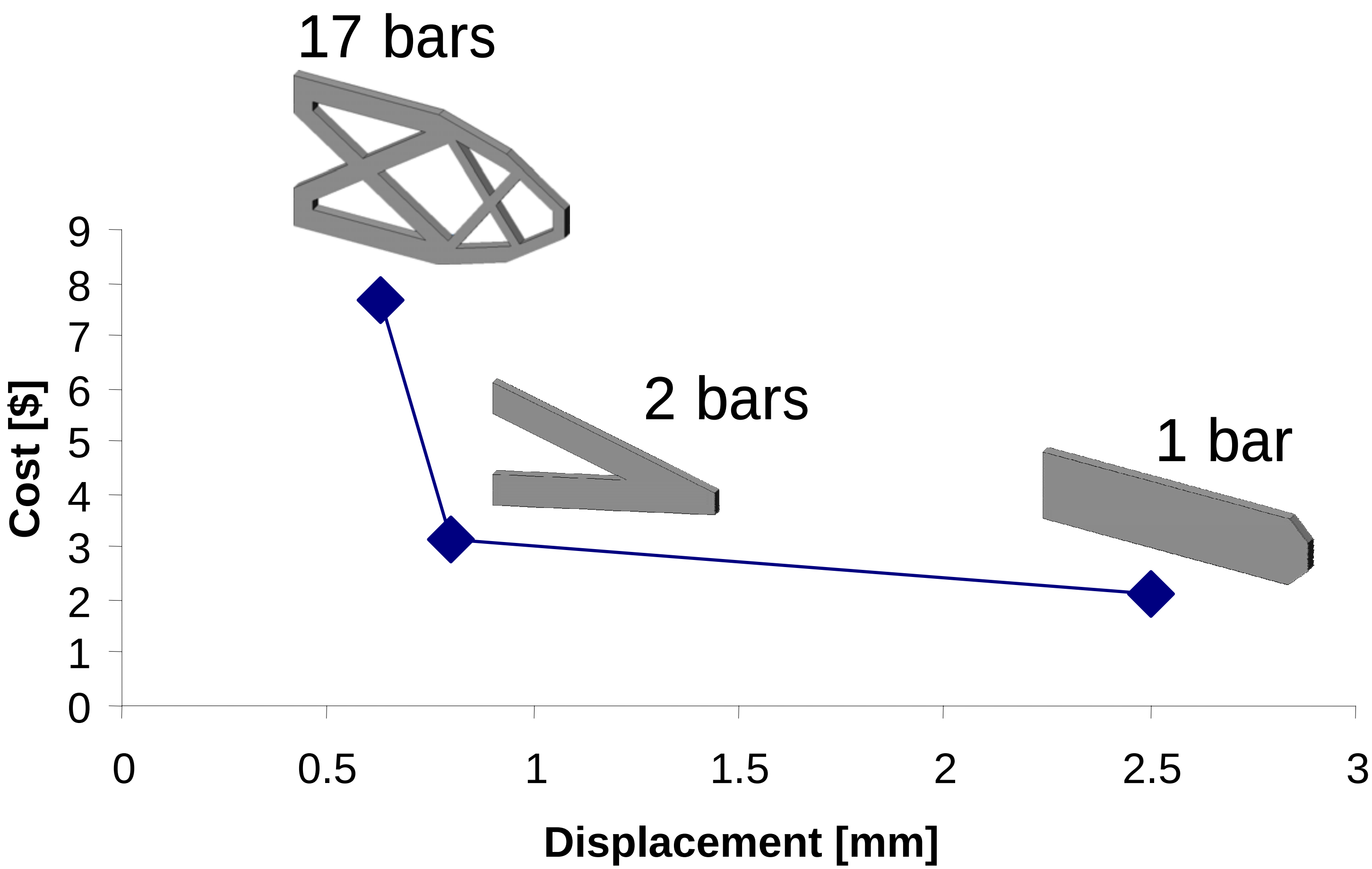
More design freedom

(Better performance)

More complex

(More difficult to optimize)

Cost versus Performance



References

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O. de Weck and K. Willcox, Multidisciplinary System Design Optimization, MIT lecture note, 2003

M. O. Bendsoe and N. Kikuchi, “Generating optimal topologies in structural design using a homogenization method,” comp. Meth. Appl. Mech. Engng, Vol. 71, pp. 197-224, 1988

Raino A.E. Makinen et al., “Multidisciplinary shape optimization in aerodynamics and electromagnetics using genetic algorithms,” International Journal for Numerical Methods in Fluids, Vol. 30, pp. 149-159, 1999

Il Yong Kim and Byung Man Kwak, “Design space optimization using a numerical design continuation method,” International Journal for Numerical Methods in Engineering, Vol. 53, Issue 8, pp. 1979-2002, March 20, 2002.

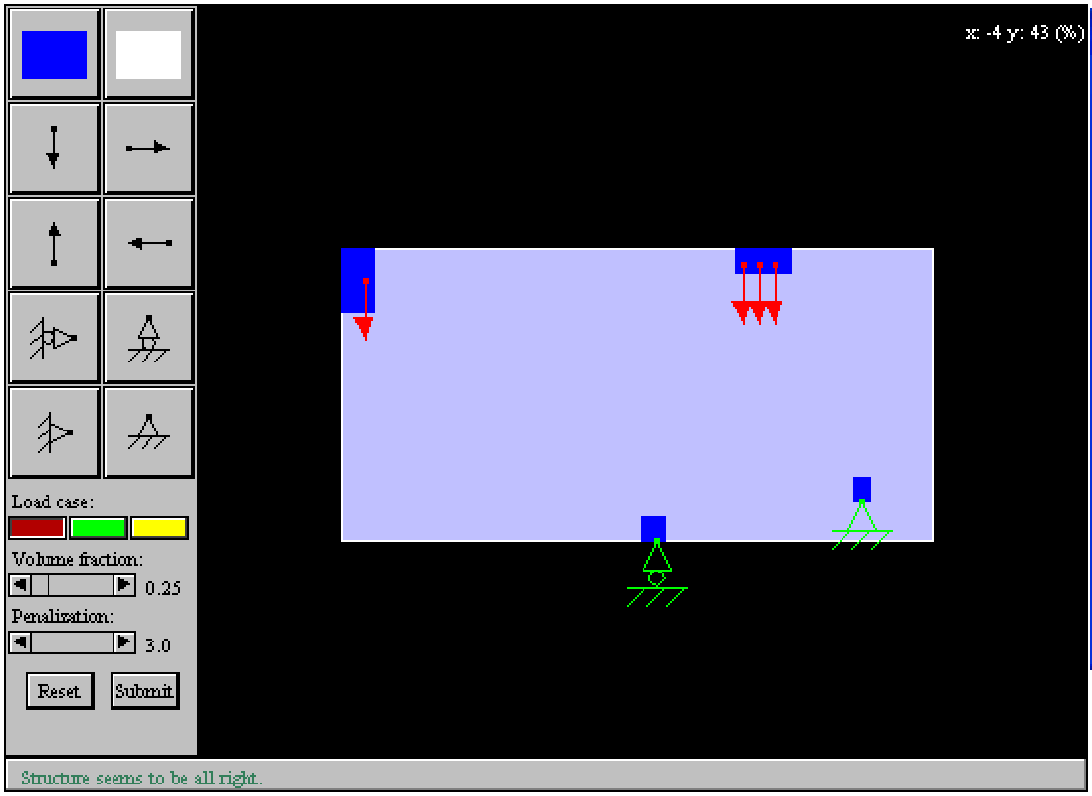
16.810 Web-based topology optimization program

Developed and maintained by Dmitri Tcherniak, Ole Sigmund, Thomas A. Poulsen and Thomas Buhl.

Features:

1. 2-D
2. Rectangular design domain
3. 1000 design variables (1000 square elements)
4. Objective function: compliance ($F \times \delta$)
5. Constraint: volume

Web-based topology optimization program



Objective function

-Compliance (F×δ)

Constraint

-Volume

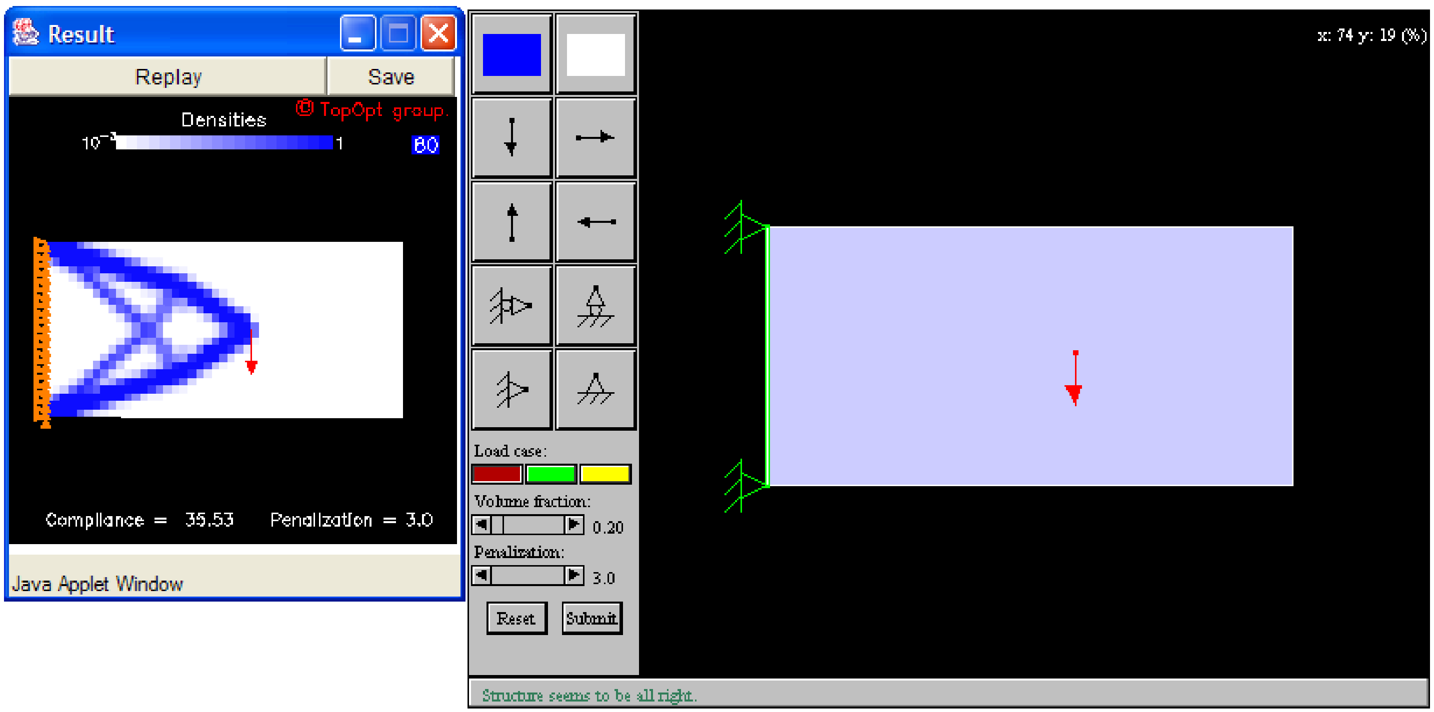
Design variables

- Density of each design cell

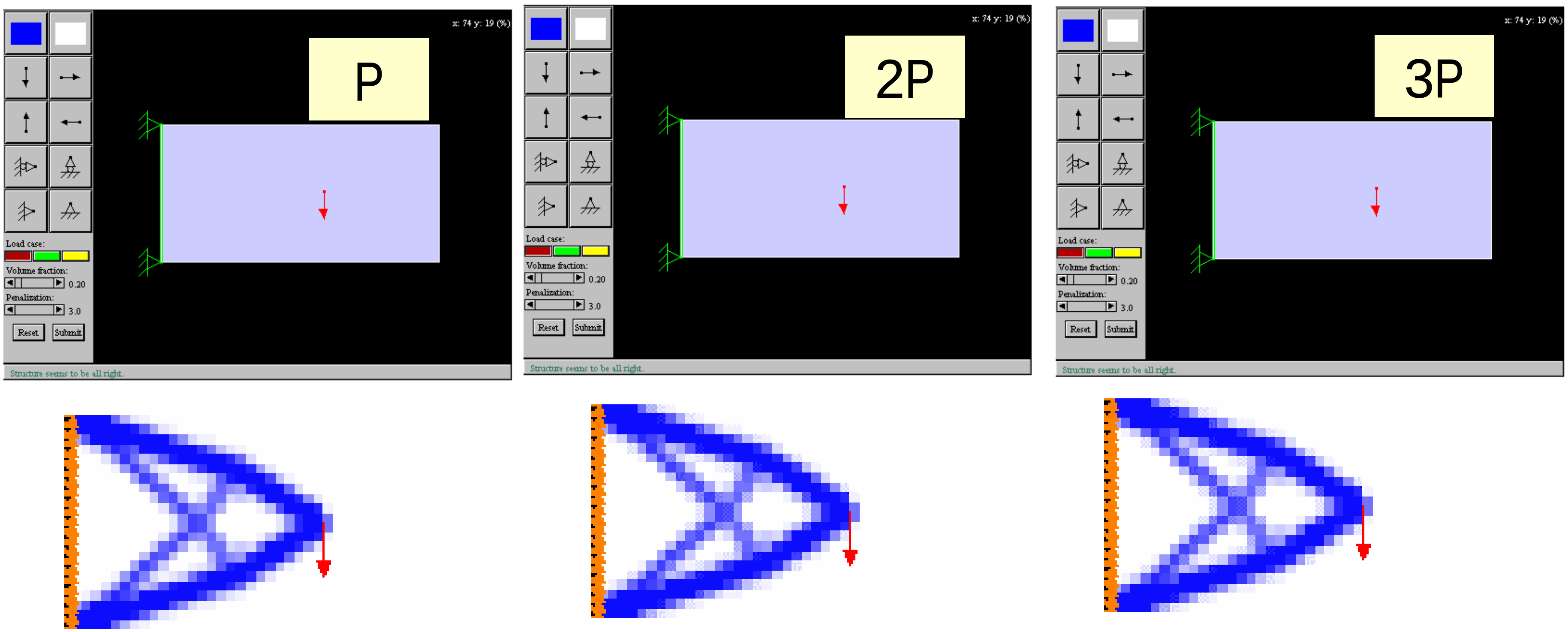
Web-based topology optimization program

No numerical results are obtained.

Optimum layout is obtained.



Web-based topology optimization program



Absolute magnitude of load does not affect optimum solution