I Code

I.1 Control Test Code (LS, AB)

I.1.1 Test Case 1 Code

```
%r=0.1;       % Effective Radius [m]
%m=2;         % Estimated Body Mass [kg]
%I=0.5*m*r^2; % Estimated Inertia [kg-m^2]

I = 1;        % Estimated Inertia [kg-m^2]

A=[0 1 ; 0 0 ];
B=[0 ; 1/I];

lamda=1;
rho=lamda/4;

type=input('Input case(in quotes)'); % 0 for Regulator, 1 for Slew to Reference
switch type
case 'a'
   Rxx=[1e-5 0 ; 0 lamda];
   Ruu=rho;
   [K,S,E]=lqr(A,B,Rxx,Ruu)
case 'b'
   n=1e-2;
   Rxx=[lamda 0 ; 0 lamda/1000];
   Ruu=rho;
   [K,S,E]=lqr(A,B,Rxx,Ruu)
   C=[n^3 1];
   D=0;
   G = B;
   H = 0;
   sys=ss(A,[B G],C,[D H]);
   [Kest,L,P]=kalman(sys,n,n,n)
end
```

I.1.2 Test Case 2 Code

```
clear all;
mu_b1 = 100*40*(.75/2)^2*pi; %Guess at value
mu_b2 = 1/10*mu_b1;         %arbitrary decision
mu_0  = 4*pi*1e-7;
ma = 15;                    %Estimate in kg
Ia = 1;                     %Inertia of a and b in z direction
Ib = 1;                     %Change this!!!!
r = 2;                      %separation distance in meters
n = 1e-5;
rho = 1/4;                  %weighting factor

c = input('Which case(in quotes)');

A = zeros(8);
A(1:4, 5:8) = eye(4);

B = zeros(8,4);
B(5,1) = -15*mu_b1*mu_0/(2*pi*r^4*ma);
B(5,2) = 15*mu_b2*mu_0/(4*pi*r^4*ma);
B(6,1) = 15*mu_b2*mu_0/(4*pi*r^4*ma);
B(6,2) = 15*mu_b1*mu_0/(4*pi*r^4*ma);
B(7,1) = -mu_b2*mu_0/(pi*r^3*Ia);
B(7,2) = -2*mu_b1*mu_0/(pi*r^3*Ia);
B(7,3) = 1/Ia;
B(8,1) = -2*mu_b2*mu_0/(pi*r^3*Ib);
B(8,2) = -mu_b1*mu_0/(pi*r^3*Ib);
B(8,4) = 1/Ib;

switch c

case 'a' %two vehicles, 1 fixed, disturbance rejection
   A=A([1:6], [2:7]);
   B=B([2:7], [1:3]);

   Q = n*eye(6);
   Q(4:6, 4:6) = eye(3);

   R = diag([rho, rho, rho/2]);

   [Ka, S, E] = lqr(A, B, Q, R);
   Ka
case 'b'  %two vehicles, 1 fixed, tracking
   A=A([1:6], [2:7]);
   B=B([2:7], [1:3]);
```

```
   Q = n*eye(6);
   Q(1:3, 1:3) = eye(3);

   R = diag([rho, rho, rho/2]);

   [Kb, S, E] = lqr(A, B, Q, R);
   Kb
case 'c'  %two vehicles, both free, disturbance rejection
   Q = n*eye(8);
   Q(5:8, 5:8) = eye(4);

   R = diag([rho, rho, rho/2, rho/2]);
   [Kc, S, E] = lqr(A, B, Q, R);
   Kc
case 'd' %two vehicles, both free, tracking
   Q = n*eye(8);
   Q(1:4, 1:4) = eye(4);

   R = diag([rho, rho, rho/2, rho/2]);
   [Kd, S, E] = lqr(A,B,Q,R);
   Kd
end
```

I.1.3 Test Case Three Code

```
clear
mu_b1 = 100*40*(.75/2)^2*pi; %Guess at value
mu_b2 = 1/10*mu_b1;          %arbitrary decision
mu_0  = 4*pi*1e-7;
ma = 15;                     %Estimate in kg
mb = 15;                     %Estimate in kg
mc = 15;                     %Estimate in kg
Ia = .5*ma*(.75/2)^2;        %Inertia estimates of a and b in z direction
Ib = .5*mb*(.75/2)^2;        %Inertia estimates of a and b in z direction
Ic = .5*mc*(.75/2)^2;        %Inertia estimates of a and b in z direction
r_ab = 2;
r_bc = 2;
n = 1e-5;

c = input('Which case(in quotes)');

A=zeros(14);
A(1:7,8:14)=eye(7);

B=zeros(14,7);
B(8,1:2)=(15*mu_0/(2*pi*ma*r_ab^4))*[-mu_b1 mu_b2/2];
B(9,3:4)=(15*mu_0/(2*pi*mc*r_bc^4))*[-mu_b1 mu_b2/2];
B(10,1:2)=(15*mu_0/(4*pi*ma*r_ab^4))*[mu_b2 mu_b1];
B(11,3:4)=(15*mu_0/(4*pi*mc*r_bc^4))*[mu_b2 mu_b1];
B(12,1:2)=(-mu_0/(pi*Ia*r_ab^3))*[mu_b2 2*mu_b1];
B(13,1:4)=(-mu_0/(pi*Ib))*[2*mu_b2/(r_ab^3) mu_b1/(r_ab^3) 2*mu_b2/(r_bc^3)
mu_b1/(r_bc^3)];
B(14,3:4)=(-mu_0/(pi*Ic*r_bc^3))*[mu_b2 2*mu_b1];
B(12,5)= 1/Ia;
B(13,6)= 1/Ib;
B(14,7)= 1/Ic;

switch c

case 'a' %test case a is three vehicles with the center fixed doing distrubance rejection
rho = 1/4;

A=A([1:5 7:12 14], [1:5 7:12 14]);
B=B([1:5 7:12 14], [1:5 7]);

Q=n*eye(6);
Q(7:12,7:12)=eye(6);

R=diag([rho rho rho rho rho/2 rho/2]);
```

```
[Ka, S, E] = lqr(A, B, Q, R);
Ka

case 'b' %test case b is three vehicles with the center vehicle fixed doing tracking
rho = 1/4;

A=A([1:5 7:12 14], [1:5 7:12 14]);
B=B([1:5 7:12 14], [1:5 7]);

Q=eye(6);
Q(7:12,7:12)=n*eye(6);

R=diag([rho rho rho rho rho/2 rho/2]);

[Kb, S, E] = lqr(A, B, Q, R);
Kb

case 'c', %test case c is three free vehicles, disturbance rejection
rho = 1/4;

Q=n*eye(7);
Q(8:14,8:14)=eye(7);

R=diag([ rho rho rho rho rho/2 rho/2 rho/2]);

[Kc, S, E] = lqr(A, B, Q, R);
Kc

case 'd', %test case d is three vehicles, tracking
rho = 1/4;

Q=eye(7);
Q(8:14,8:14)=n*eye(7);

R=diag([ rho rho rho rho rho/2 rho/2 rho/2]);

[Kd, S, E] = lqr(A, B, Q, R);
Kd
end
```

I.1.4 Spin-up Approach 1 Code

```
%  Spinup dymanics
%  Calculated and written by Laila Elias, Leah Soffer, and Andre' Bosch
%  See Andre's Lab book page 14 with any questions

clear all;
close all;

ramp=2;          % 0 for constant k, 1 for ramp k, 2 for ramp higher than s.s. value
switch ramp
case 0
   steps = 200;                          % number of timesteps
   total_time = 25;
case 1
   steps = 100;                          % number of timesteps
   total_time = 85;
case 2
   steps = 100;                          % number of timesteps
   total_time = 10;
end
dt = total_time/steps;                   % timestep in seconds (time for total
maneuver/timesteps)
time = dt:dt:steps*dt;
mass = 20;                              % in kg
I=1;                                   %inertia
thetadotdot = zeros(steps,1);               % enough for time steps of .05 seconds
thetadot = zeros(steps,1);
theta_a = 0;                            % coordinate system fixed on vehicle a
theta_b = zeros(steps,1);
theta_bdot = zeros(steps,1);
r = 1;                                 % radius is one meter
Frad = zeros(steps,1);                   % radial EM force
Ftan = zeros(steps,1);                   % tangential EM force
%F_rad = 3/(2*pi)*mu0*mua*mub/(2*r)^4 = m*omega^2*r
%k = 3*mu0*mua*mub/(4*pi);
mu0 = 4*pi*1e-7;
omega = 2*pi/60;                        %in steady state
%mu = sqrt(32*mass*(omega^2)*(r^5)/(3*mu0)*pi);   %mu for each coil in steady state;
d_coil= 0.83;                           %diameter of large coil in meters
```

```
n = 100;                                %number of wraps in coil;

k = 40*(pi^2)/3600*16;                  % constant terms in front, also takes into
% account the mu's since we are saying they
% are constant in this case.  See lab book!k

switch ramp
case 0
   k=k*ones(1,steps);
case 1
   root_k = sqrt(k);
   ramp_time = 69.7;
   dk = root_k/ramp_time*dt;
   ramp_k = [dk:dk:root_k root_k*ones(1,(steps-ramp_time/dt))];
   k = ramp_k.*ramp_k;
   %theta = zeros(steps,1);
case 2
   root_k = sqrt(k);
   max_root_k=1.5*root_k;
   ramp_time_1 = 1;
   ramp_time_2 = 8; % max_ramp_k time should be 8 or less to make sense
   ramp_time_3 = 1;
   ramp_time_4 = total_time-(ramp_time_1+ramp_time_2+ramp_time_3);
   dk_1 = max_root_k/(ramp_time_1)*dt;
   dk_3 = -(max_root_k-root_k)/ramp_time_3*dt;
   ramp_k = [dk_1:dk_1:max_root_k max_root_k*ones(1,ramp_time_2/dt)
max_root_k+dk_3:dk_3:root_k root_k*ones(1,ramp_time_4/dt)];
   k = ramp_k.*ramp_k;
   %theta = zeros(steps,1);
end

% initial conditions      NOTE Coriolis effect not in thetadotdot
theta_b(1) = pi/2;
thetadotdot(1) =  (k(1)/mass/((2*r)^4)/r)*(sin(theta_b(1))*cos(theta_a)...
   +sin(theta_a)*cos(theta_b(1)));

for count = 2:(steps);
   thetadot(count) = thetadot(count-1) + thetadotdot(count-1)*dt;
   %theta(count) = theta(count-1) + thetadotdot(count-1)/2*dt^2;  %needs to add integral
of thetadot term
   Frad(count) = -((thetadot(count))^2)*r*mass;

   theta_b(count) = acos(-Frad(count)*(2*r)^4/k(count)/2);

   if imag(theta_b(count)) ~=0
      theta_b(count) = 0;
```

```
    end
    theta_bdot(count) = (theta_b(count) - theta_b(count-1))/dt;
    Ftan(count) = k(count)/((2*r)^4)*((sin(theta_b(count))*cos(theta_a))...
        +(sin(theta_a)*cos(theta_b(count))));    %r^5????
    thetadotdot(count) = Ftan(count)/mass/r;

end
real_k = k/3/(2*r)^3;               %k defined in packet as mu0*muA*muB/(4*pi*d^3)
delta = 0; chi = 0;                  %no rotation in plane
%EM Torques on vehicle noted
%T_EMxA = real_k'.*(sin(theta_a)*sin(theta_b)*sin(chi-delta));
%T_EMyA =
real_k'.*(cos(theta_a)*sin(theta_b)*sin(delta)+2*sin(theta_a)*cos(theta_b)*sin(chi));
T_EMzA = -
real_k'.*(cos(theta_a)*sin(theta_b)*cos(delta)+2*sin(theta_a)*cos(theta_b)*cos(chi));
%T_EMxB = real_k'.*(sin(theta_a)*sin(theta_b)*sin(delta-chi));
%T_EMyB =
real_k'.*(2*cos(theta_a)*sin(theta_b(count))*sin(delta)+sin(theta_a)*cos(theta_b(count))
*sin(chi));
T_EMzB = -
real_k'.*(2*cos(theta_a)*sin(theta_b)*cos(delta)+sin(theta_a)*cos(theta_b)*cos(chi));

%Torque on reaction wheels
theta_b_dotdot=[diff(diff(theta_b))]/dt^2;
theta_b_dotdot = [theta_b_dotdot; theta_b_dotdot(end); theta_b_dotdot(end)];
T_RW_A = -(mass*r^2+I)*thetadotdot + T_EMzA;             %Torque History Needed
on RW A to Maintain Theta A at Zero
T_RW_B = -(mass*r^2+I)*thetadotdot +I*theta_b_dotdot + T_EMzB;    % Torque
History Needed on RW B to Maintain the Theta B History Calculated Above


%figure(5); set(gca,'fontsize',14);
subplot(2,2,2);
plot(time, theta_b);
xlabel('Time [s]')
ylabel('\theta_b [rad]')
title('Angle of Spacecraft B from Radial Line')

%figure(6); set(gca,'fontsize',14);
subplot(2,2,3);
plot(time, thetadotdot);
xlabel('Time [s]')
ylabel('\Theta_{dd} [rad/s^2]')
title('Angular Acceleration of Array')

%figure(7); set(gca,'fontsize',14);
```

```
subplot(2,2,4);
plot(time, thetadot);
xlabel('Time [s]')
ylabel('\Theta_{d} [rad/s]')
title('Angular Rate of Array')

mu = sqrt(k*4*pi/(3*mu0));
i= mu/(n*pi*(d_coil/2)^2);          %amps needed in coil for steady
state??????????????????????????????
%figure(11); set(gca,'fontsize',14);
subplot(2,2,1);
plot(time,i);
grid
xlabel('Time [s]')
ylabel('i [A]')
title('Current in EM')

figure;
%figure(8); set(gca,'fontsize',14);
subplot(2,1,1);
plot(time,T_EMzA, time, T_EMzB);
xlabel('Time [s]')
ylabel('Torque [Nm]')
title('Torque Needed From EM')
legend('T_{EMzA}' , 'T_{EMzB}' ,0)

%figure(9); set(gca,'fontsize',14);
subplot(2,1,2);
plot(time,T_RW_A, time, T_RW_B);
xlabel('Time [s]')
ylabel('Torque [Nm]')
title('Torque Needed From Reaction Wheel A and B')
legend('T_{RWA}', 'T_{RWB}', 0)

if 0
   figure(10); set(gca,'fontsize',14);
   plot(time,theta);
   xlabel('Time [s]')
   ylabel('')
   title('')
end
```

I.1.5 Spin-up Approach 2 Code (has bugs)

```
%  Spinup dymanics
%  Calculated and written by Laila Elias, Leah Soffer, and Andre' Bosch
%  See Andre's Lab book page 14 with any questions

clear all;
close all;


steps = 200;                       % number of timesteps
total_time = 25;

dt = total_time/steps;                  % timestep in seconds (time for total
maneuver/timesteps)
time = dt:dt:steps*dt;
mass = 20;                         % in kg
I=1;                          %inertia
thetadotdot = zeros(steps,1);            % enough for time steps of .05 seconds
thetadot = zeros(steps,1);
theta_a = 0;                       % coordinate system fixed on vehicle a
theta_b = zeros(steps,1);
theta_bdot = zeros(steps,1);
theta_b_dotdot = zeros(steps,1);
r = 1;                          % radius is one meter
Frad = zeros(steps,1);                 % radial EM force
Ftan = zeros(steps,1);                 % tangential EM force
mu0 = 4*pi*1e-7;
mu2 = zeros(steps,1);                  %mu squared (muA*muB)
omega = 2*pi/60;                      %in steady state
%mu = sqrt(32*mass*(omega^2)*(r^5)/(3*mu0)*pi);   %mu for each coil in steady state;
d_coil= 0.83;                        %diameter of large coil in meters
n = 100;                          %number of wraps in coil;

k = 3/(4*pi)*mu0/(2*r)^4;


% initial conditions      NOTE Coriolis effect not in thetadotdot
theta_b(1) = pi/2;
mu2(1) = 44.7*44.7;                    %steady state mu
```

```
thetadotdot(1) =  (k*mu2(1)/mass/r)*(sin(theta_b(1))*cos(theta_a)...
    +sin(theta_a)*cos(theta_b(1)));

for count = 2:(steps);
    theta_b(count) = pi/4*cos(pi/total_time*time(count))+pi/4;

    thetadot(count) = thetadot(count-1) + thetadotdot(count-1)*dt;
    Frad(count) = -((thetadot(count))^2)*r*mass;

    mu2(count) = -
Frad(count)/k/(2*cos(theta_a)*cos(theta_b(count))+sin(theta_b(count))*sin(theta_a));
    Ftan(count) = k*mu2(count)*((sin(theta_b(count))*cos(theta_a))...
        +(sin(theta_a)*cos(theta_b(count))));
    thetadotdot(count) = Ftan(count)/mass/r;

end
real_k = k/3*(2*r)*mu2;              %k defined in packet as mu0*muA*muB/(4*pi*d^3)
delta = 0; chi = 0;                  %no rotation in plane
%EM Torques on vehicle noted
%T_EMxA = real_k.*(sin(theta_a)*sin(theta_b)*sin(chi-delta));
%T_EMyA =
real_k.*(cos(theta_a)*sin(theta_b)*sin(delta)+2*sin(theta_a)*cos(theta_b)*sin(chi));
T_EMzA = -
real_k.*(cos(theta_a)*sin(theta_b)*cos(delta)+2*sin(theta_a)*cos(theta_b)*cos(chi));
%T_EMxB = real_k.*(sin(theta_a)*sin(theta_b)*sin(delta-chi));
%T_EMyB =
real_k.*(2*cos(theta_a)*sin(theta_b(count))*sin(delta)+sin(theta_a)*cos(theta_b(count))
*sin(chi));
T_EMzB = -
real_k.*(2*cos(theta_a)*sin(theta_b)*cos(delta)+sin(theta_a)*cos(theta_b)*cos(chi));

%Torque on reaction wheels
theta_bdot=diff(theta_b)/dt;
theta_bdot= [theta_bdot(1); theta_bdot];
theta_b_dotdot=[diff(theta_bdot)]/dt;
theta_b_dotdot = [theta_b_dotdot(3); theta_b_dotdot(3); theta_b_dotdot(3);
theta_b_dotdot(3:end)];
T_RW_A = -(mass*r^2+I)*thetadotdot + T_EMzA;            %Torque History Needed
on RW A to Maintain Theta A at Zero
T_RW_B = -(mass*r^2+I)*thetadotdot +I*theta_b_dotdot + T_EMzB;    % Torque
History Needed on RW B to Maintain the Theta B History Calculated Above


%figure(5); set(gca,'fontsize',14);
subplot(2,2,2);
plot(time, theta_b);
```

```
xlabel('Time [s]')
ylabel('\theta_b [rad]')
title('Angle of Spacecraft B from Radial Line')

%figure(6); set(gca,'fontsize',14);
subplot(2,2,3);
plot(time, thetadotdot);
xlabel('Time [s]')
ylabel('\Theta_{dd} [rad/s^2]')
title('Angular Acceleration of Array')

%figure(7); set(gca,'fontsize',14);
subplot(2,2,4);
plot(time, thetadot);
xlabel('Time [s]')
ylabel('\Theta_{d} [rad/s]')
title('Angular Rate of Array')

mu = sqrt(k*4*pi/(3*mu0));
i= mu/(n*pi*(d_coil/2)^2);          %amps needed in coil for steady
state??????????????????????????????
%figure(11); set(gca,'fontsize',14);
subplot(2,2,1);
plot(time,i);
grid
xlabel('Time [s]')
ylabel('i [A]')
title('Current in EM')

figure;
%figure(8); set(gca,'fontsize',14);
subplot(2,1,1);
plot(time,T_EMzA, time, T_EMzB);
xlabel('Time [s]')
ylabel('Torque [Nm]')
title('Torque Needed From EM')
legend('T_{EMzA}' , 'T_{EMzB}' ,0)

%figure(9); set(gca,'fontsize',14);
subplot(2,1,2);
plot(time,T_RW_A, time, T_RW_B);
xlabel('Time [s]')
ylabel('Torque [Nm]')
title('Torque Needed From Reaction Wheel A and B')
legend('T_{RWA}', 'T_{RWB}', 0)
```

```
if 0
   figure(10); set(gca,'fontsize',14);
   plot(time,theta);
   xlabel('Time [s]')
   ylabel('')
   title('')
end
theta_b_dot = diff(theta_b)/dt^2;
figure;
subplot(3,1,1);
plot(time, theta_b);
xlabel('Time [s]')
ylabel('\theta_b [rad]')
subplot(3,1,2);
plot(time(1:end-1), theta_b_dot);
xlabel('Time [s]')
ylabel('\theta_{d}_b [rad/s]')
subplot(3,1,3);
plot(time, theta_b_dotdot);
xlabel('Time [s]')
ylabel('\theta_{dd}_b [rad/s^2]')
```

References

Schweighart, Samuel A. Two Satellite Spin-up. Massachusetts Institute of Technology, 2002.