Electromagnetic Formation Flight

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TWO-SPACECRAFT NONLINEAR EQUATIONS OF MOTION, INCLUDING GYROSTIFFENING

Nomenclature:

A	Coil Cross-Sectional Area
i	Current Running Through Electromagnetic Coil [A]
$I_{rr,s}$	Spacecraft Mass-Moment of Inertia about Radial Axes [$kg \cdot m^2$]
$I_{rr,w}$	Reaction Wheel Mass-Moment of Inertia about Radial Axes [$kg \cdot m^2$]
$I_{zz,s}$	Spacecraft Mass-Moment of Inertia about Spin Axis [$kg \cdot m^2$]
$I_{zz,w}$	Reaction Wheel Mass-Moment of Inertia about Spin Axis [$kg \cdot m^2$]
F_r, F_ϕ, F_ψ	Forces on Spacecraft
m	Spacecraft Mass
n	Number of Conductor Wraps around Electromagnet
$\vec{r}$	Position Vector of Spacecraft A [m]
r, ϕ, ψ	Position Coordinates of Spacecraft A
RW	Reaction Wheel
T_r, T_ϕ, T_ψ	Torques on Spacecraft about Local r, ϕ, ψ Frame
T_x, T_y, T_z	Torques on Spacecraft about Body-Fixed x, y, z Frame
$\boldsymbol{x}$	State Vector
x, y, z	Local Body-Fixed Coordinates on Spacecraft A
X, Y, Z	Global Coordinates
α_i	i^{th} Euler Angle of Spacecraft A
β_i	i^{th} Euler Angle of Spacecraft B
$\Omega_{z,w}$	Constant Spin Rate of RW
$\vec{\mu}$	Magnetic Moment of Coil [$A\ m^2$]

1. Introduction

The goal of this work is to define the nonlinear equations of motion for a two-spacecraft formation flying array undergoing a steady-state spin maneuver. While these equations will capture the nonlinear dynamics of the system being considered, they will be linearized for purposes of control design and stability analysis. Once a controller has been designed using the linearized *design model* of the dynamics, the original nonlinear equations may serve as an *evaluation model* for simulating the closed-loop behavior of the nonlinear system.

In the following section, we define the geometry of the system being considered. In Section 3, the nonlinear equations of motion are presented, and in Section 4, the equations are linearized.

2. System Description

The two-spacecraft array being considered is depicted in Figure 2.1. The *X*, *Y*, *Z* coordinate frame represents a global, non-rotating frame whose origin lies at the center of mass of the two-spacecraft array. The first spacecraft, denoted as "spacecraft A," lies at coordinates r, ϕ, ψ. Since the global frame's origin coincides with the array's center of mass, and we are considering the two spacecraft to be identical in mass and geometry, the second spacecraft, denoted as "spacecraft B," lies at coordinates r, $\phi + \pi$, ψ (or equivalently r, ϕ, $\psi + \pi$).

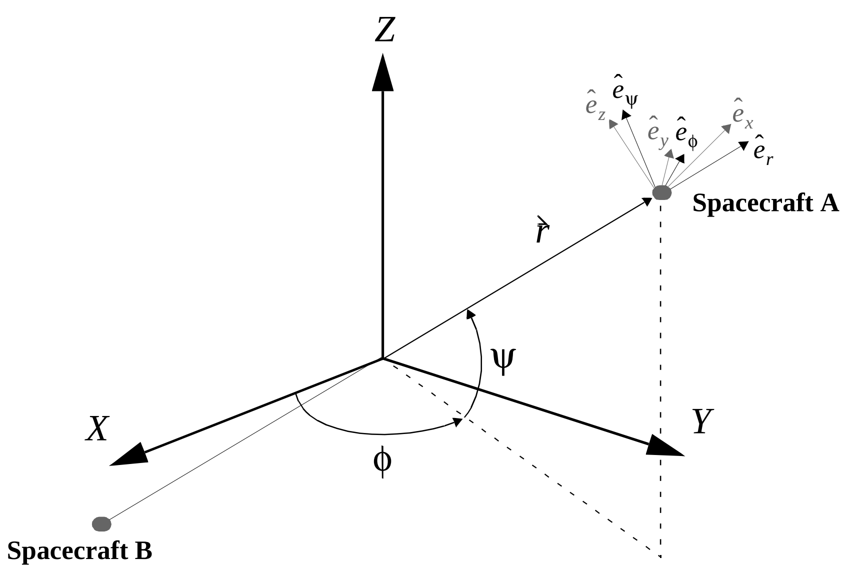

Figure 2.1 Geometry of Two-Spacecraft Array

While the *X*, *Y*, *Z* frame represents a global frame, the r, ϕ, ψ frame represents a local frame whose origin lies at the center of mass of spacecraft A. The r, ϕ, ψ frame is not fixed to the body in that it does not rotate or "tilt" with the spacecraft. Notice the $\hat{e}_r$ vector always aligns with the position vector, $\vec{r}$, of spacecraft A relative to the origin of the global frame. The x, y, z frame, in contrast, is fixed to spacecraft A; it rotates with the body relative to the r, ϕ, ψ frame.

We now define the relative orientations of the two spacecraft using Euler angles. The Euler angles of spacecraft A are α_1, α_2, and α_3, which represent sequential rotations about the body-fixed z, y, and x axes, respectively. Similarly, the orientation of spacecraft B is defined by the Euler angles β_1, β_2, and β_3, which represent three sequential rotations about a body-fixed frame on B that is nominally aligned with the r, ϕ, ψ frame on spacecraft A. The nominal orientation of each spacecraft is such that the x, y, z frame aligns with the r, ϕ, ψ frame. In the following sections, we consider perturbations from this nominal orientation; in other words, we consider the dynamics of the x, y, z frame rotating relative to the r, ϕ, ψ frame.

With the variables defined so far and the constraints on the position of spacecraft B:

$$r_B = r_A = r, \qquad \phi_B = \phi_A + \pi = \phi + \pi, \qquad \psi_B = \psi_A = \Psi \tag{2.1}$$

we have defined 18 state variables that make up the state vector, $\boldsymbol{x}$:

$$\boldsymbol{x} = \begin{bmatrix} r & \phi & \Psi & \alpha_1 & \alpha_2 & \alpha_3 & \beta_1 & \beta_2 & \beta_3 & \dot{r} & \dot{\phi} & \dot{\Psi} & \dot{\alpha}_1 & \dot{\alpha}_2 & \dot{\alpha}_3 & \dot{\beta}_1 & \dot{\beta}_2 & \dot{\beta}_3 \end{bmatrix}^{T}. \tag{2.2}$$

In this analysis, we consider that spacecraft A and B each contain a single electromagnetic dipole oriented along the body-fixed x-axis (and thus aligned with $\hat{e}_r$ when the spacecraft is in its nominal orientation). The magnetic moment of the electromagnet on spacecraft A is defined as:

$$\vec{\mu}_A = \mu_A \hat{e}_x = n_A i_A A_A \hat{e}_x \tag{2.3}$$

where n_A is the number of times the conductor is wrapped around to form the electromagnetic coil, i_A is the current running through the coil, and A_A is the cross-sectional area of the coil system. The magnitude of the magnetic moment, μ_A, is assumed constant in this analysis, although its direction, $\hat{e}_x$, rotates with the spacecraft.

The magnetic moment of the electromagnet on spacecraft B is defined similarly and points along the local body-fixed *x*-axis on spacecraft B. For this analysis, we assume the same geometry for the coils on both spacecraft, so that:

$$n_B = n_A = n, \quad A_B = A_A = A \tag{2.4}$$

However, the currents i_A and i_B are unique and depend on the dynamics and closed-loop control of the system.

Finally, we assume that each spacecraft contains a reaction wheel (RW) whose spin axis is aligned with the body-fixed *z*-axis. Each RW is spinning at a constant rate, $\Omega_{z,w}$, necessary to conserve the angular momentum of the spinning array. In other words, the angular momentum stored in the two RWs is equal and opposite to the angular momentum of the two-spacecraft array. Nominally the two spacecraft would assume a circular trajectory in the global *X*, *Y* plane ($\psi = 0$) with a constant angular velocity, $\dot{\phi} = \dot{\phi}_0$. In this case, the conservation of angular momentum is expressed as:

$$I_{zz,w}\Omega_{z,w} = mr_0^2\dot{\phi}_0 \tag{2.5}$$

where $I_{zz,w}$ is the RW mass-moment of inertia about its spin axis, m is the mass of each spacecraft, and r_0 is the nominal array radius.

3. Nonlinear Equations of Motion

3.1 Translational Equations

The translational equations of motion for spacecraft A describe the motion of its center of mass with respect to the global coordinate frame. They may be written as:

$$\ddot{\vec{r}} = \begin{Bmatrix} \ddot{r} - r\dot{\psi}^2 - r\dot{\phi}^2\sin^2\psi \\ 2\dot{r}\dot{\phi}\sin\psi + r\ddot{\phi}\sin\psi + 2r\dot{\phi}\dot{\psi}\cos\psi \\ 2\dot{r}\dot{\psi} + r\ddot{\psi} - r\dot{\phi}^2\sin\psi\cos\psi \end{Bmatrix} = \frac{1}{m}\begin{Bmatrix} F_r \\ F_\phi \\ F_\psi \end{Bmatrix}_A \tag{3.1}$$

where $\begin{Bmatrix} F_r & F_\phi & F_\psi \end{Bmatrix}_A^T$ are the external forces acting on spacecraft A along the local axes $\begin{Bmatrix} \hat{e}_r & \hat{e}_\phi & \hat{e}_\psi \end{Bmatrix}^T$. We now consider only the forces exerted on spacecraft A by the electromagnet on spacecraft B due to the relative positions and orientations of the two spacecraft. With the Euler angles of each spacecraft as defined in Section 2, the forces exerted on spacecraft A due to the electromagnetic interaction with spacecraft B are:

$$\begin{Bmatrix} F_r \\ F_\phi \\ F_\psi \end{Bmatrix}_A = \frac{3\mu_0\mu_A\mu_B}{64\pi r^4}\begin{Bmatrix} s\alpha_1 c\alpha_2 s\beta_1 c\beta_2 - 2c\alpha_1 c\alpha_2 c\beta_1 c\beta_2 + s\alpha_2 s\beta_2 \\ c\alpha_2 c\beta_2(s\alpha_1 c\beta_1 + s\beta_1 c\alpha_1) \\ -c\beta_1 c\beta_2 s\alpha_2 - c\alpha_1 c\alpha_2 s\beta_2 \end{Bmatrix} \tag{3.2}$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ T·m/A is the permeability constant, “s” represents the sine function, and “c” represents the cosine function. **Hence the translational equations of motion for spacecraft A are:**

$$\begin{Bmatrix} \ddot{r} - r\dot{\psi}^2 - r\dot{\phi}^2\sin^2\psi \\ 2\dot{r}\dot{\phi}\sin\psi + r\ddot{\phi}\sin\psi + 2r\dot{\phi}\dot{\psi}\cos\psi \\ 2\dot{r}\dot{\psi} + r\ddot{\psi} - r\dot{\phi}^2\sin\psi\cos\psi \end{Bmatrix} = \frac{3\mu_0\mu_A\mu_B}{64\pi m r^4}\begin{Bmatrix} s\alpha_1 c\alpha_2 s\beta_1 c\beta_2 - 2c\alpha_1 c\alpha_2 c\beta_1 c\beta_2 + s\alpha_2 s\beta_2 \\ c\alpha_2 c\beta_2(s\alpha_1 c\beta_1 + s\beta_1 c\alpha_1) \\ -c\beta_1 c\beta_2 s\alpha_2 - c\alpha_1 c\alpha_2 s\beta_2 \end{Bmatrix} \tag{3.3}$$

While similar equations of motion may be written for the motion of spacecraft B due to the forces exerted by the electromagnet on spacecraft A, these equations are not necessary for a dynamic simulation; rather, the constraints defined by Equation 2.1, along with a knowledge of the position of spacecraft A, are sufficient to determine the position of spacecraft B. Note also that the two spacecraft exert equal and opposite forces on one another, so that:

$$\begin{Bmatrix} F_r \\ F_\phi \\ F_\psi \end{Bmatrix}_A = -\begin{Bmatrix} F_r \\ F_\phi \\ F_\psi \end{Bmatrix}_B . \tag{3.4}$$

3.2 Rotational Equations

We now consider the rotational ("rocking") equations of motion of spacecraft A. They may be written as:

$$\begin{bmatrix} I_{rr,s} + I_{rr,w} & 0 & 0 \\ 0 & I_{rr,s} + I_{rr,w} & 0 \\ 0 & 0 & I_{zz,s} \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \end{Bmatrix}_A + \begin{bmatrix} 0 & \Omega_{z,w} I_{zz,w} & 0 \\ -\Omega_{z,w} I_{zz,w} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_z \end{Bmatrix}_A = \begin{Bmatrix} T_x \\ T_y \\ T_z \end{Bmatrix}_A \tag{3.5}$$

where $\begin{Bmatrix} \dot{\theta}_x & \dot{\theta}_y & \dot{\theta}_z \end{Bmatrix}_A^T$ are the rotation rates of spacecraft A about its body-fixed *x*, *y*, *z* frame, and $\begin{Bmatrix} T_x & T_y & T_z \end{Bmatrix}_A^T$ are the external torques on spacecraft A about its body-fixed frame. $I_{zz,s}$ and $I_{zz,w}$ represent the mass-moments of inertia of the spacecraft and RW, respectively, about the body-fixed *z*-axis. $I_{rr,s}$ and $I_{rr,w}$ represent the spacecraft and RW inertias, respectively, about the body-fixed radial (*x* and *y*) axes. Recall that $\Omega_{z,w}$ is the constant spin rate of the RW, so the skew-symmetric damping matrix in Equation 3.5 represents gyrostiffening effects of the RW.

Since the orientations of the two spacecraft are represented in terms of their Euler angles, we rewrite Equation 2.4 in terms of the Euler angles. The rotational rates and accelerations are:

$$\begin{Bmatrix} \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_z \end{Bmatrix}_A = \begin{Bmatrix} \dot{\alpha}_3 - \dot{\alpha}_1 s\alpha_2 \\ \dot{\alpha}_2 c\alpha_3 + \dot{\alpha}_1 c\alpha_2 s\alpha_3 \\ \dot{\alpha}_1 c\alpha_2 c\alpha_3 - \dot{\alpha}_2 s\alpha_3 \end{Bmatrix} \tag{3.6}$$

$$\begin{Bmatrix} \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \end{Bmatrix}_A = \begin{Bmatrix} \ddot{\alpha}_3 - \ddot{\alpha}_1 s\alpha_2 - \dot{\alpha}_1\dot{\alpha}_2 c\alpha_2 \\ \ddot{\alpha}_2 c\alpha_3 - \dot{\alpha}_2\dot{\alpha}_3 s\alpha_3 + \ddot{\alpha}_1 c\alpha_2 s\alpha_3 - \dot{\alpha}_1\dot{\alpha}_2 s\alpha_2 s\alpha_3 + \dot{\alpha}_1\dot{\alpha}_3 c\alpha_2 c\alpha_3 \\ \ddot{\alpha}_1 c\alpha_2 c\alpha_3 - \dot{\alpha}_1\dot{\alpha}_2 s\alpha_2 c\alpha_3 - \dot{\alpha}_1\dot{\alpha}_3 c\alpha_2 s\alpha_3 - \ddot{\alpha}_2 s\alpha_3 - \dot{\alpha}_2\dot{\alpha}_3 c\alpha_3 \end{Bmatrix}. \quad (3.7)$$

The torques are easily expressed in the r, ϕ, ψ frame:

$$\begin{Bmatrix} T_r \\ T_\phi \\ T_\psi \end{Bmatrix}_A = \frac{-\mu_0\mu_A\mu_B}{32\pi r^3}\begin{Bmatrix} s\alpha_2 s\beta_1 c\beta_2 - s\alpha_1 c\alpha_2 s\beta_2 \\ c\alpha_1 c\alpha_2 s\beta_2 + 2 s\alpha_2 c\beta_1 c\beta_2 \\ c\alpha_1 c\alpha_2 s\beta_1 c\beta_2 + 2 s\alpha_1 c\alpha_2 c\beta_1 c\beta_2 \end{Bmatrix} \quad (3.8)$$

and must be transformed to the body-fixed x, y, z frame:

$$\begin{Bmatrix} T_x \\ T_y \\ T_z \end{Bmatrix}_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha_3 & s\alpha_3 \\ 0 & -s\alpha_3 & c\alpha_3 \end{bmatrix}\begin{bmatrix} c\alpha_2 & 0 & -s\alpha_2 \\ 0 & 1 & 0 \\ s\alpha_2 & 0 & c\alpha_2 \end{bmatrix}\begin{bmatrix} c\alpha_1 & s\alpha_1 & 0 \\ -s\alpha_1 & c\alpha_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} T_r \\ T_\phi \\ T_\psi \end{Bmatrix}_A \quad (3.9)$$

Hence while the rotational equation of motion for spacecraft A, Equation 3.5, appears to be linear in form, it is actually nonlinear once Equations 3.6-3.9 are substituted and the rotations are expressed in terms of Euler angles.

The nonlinear rotational equations of motion for spacecraft B are similar to those for A:

$$\begin{bmatrix} I_{rr,s} + I_{rr,w} & 0 & 0 \\ 0 & I_{rr,s} + I_{rr,w} & 0 \\ 0 & 0 & I_{zz,s} \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \end{Bmatrix}_B + \begin{bmatrix} 0 & \Omega_{z,w} I_{zz,w} & 0 \\ -\Omega_{z,w} I_{zz,w} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_z \end{Bmatrix}_B = \begin{Bmatrix} T_x \\ T_y \\ T_z \end{Bmatrix}_B \quad (3.10)$$

The angular rates and accelerations for spacecraft B are found by substituting the Euler angles β_1, β_2, and β_3 in place of α_1, α_2, and α_3 in Equations 3.6 and 3.7. The torques acting on spacecraft B due to the electromagnet on spacecraft A are:

$$\begin{Bmatrix} T_r \\ T_\phi \\ T_\psi \end{Bmatrix}_B = \frac{-\mu_0\mu_A\mu_B}{32\pi r^3}\begin{Bmatrix} s\alpha_1 c\alpha_2 s\beta_2 - s\alpha_2 s\beta_1 c\beta_2 \\ s\alpha_2 c\beta_1 c\beta_2 - 4c\alpha_1 c\alpha_2 s\beta_2 \\ s\alpha_1 c\alpha_2 c\beta_1 c\beta_2 - 4c\alpha_1 c\alpha_2 s\beta_1 c\beta_2 \end{Bmatrix} \tag{3.11}$$

Expressed about the body-fixed frame on spacecraft B, these torques become:

$$\begin{Bmatrix} T_x \\ T_y \\ T_z \end{Bmatrix}_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\beta_3 & s\beta_3 \\ 0 & -s\beta_3 & c\beta_3 \end{bmatrix}\begin{bmatrix} c\beta_2 & 0 & -s\beta_2 \\ 0 & 1 & 0 \\ s\beta_2 & 0 & c\beta_2 \end{bmatrix}\begin{bmatrix} c\beta_1 & s\beta_1 & 0 \\ -s\beta_1 & c\beta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} T_r \\ T_\phi \\ T_\psi \end{Bmatrix}_B . \tag{3.12}$$

4. Linearization of Equations of Motion

We now linearize the dynamic equations of motion for spacecraft A and B about some nominal state by assuming that all motions are small relative to the nominal trajectories. We define the nominal trajectories along a circle in the global *X*, *Y* plane with a constant angular velocity. Hence the nominal state vector is:

$$\begin{aligned} \boldsymbol{x}_0 &= \left[\, r_0\ \phi_0\ \Psi_0\ \alpha_{1,0}\ \alpha_{2,0}\ \alpha_{3,0}\ \beta_{1,0}\ \beta_{2,0}\ \beta_{3,0}\ \dot{r}_0\ \dot{\phi}_0\ \dot{\Psi}_0\ \dot{\alpha}_{1,0}\ \dot{\alpha}_{2,0}\ \dot{\alpha}_{3,0}\ \dot{\beta}_{1,0}\ \dot{\beta}_{2,0}\ \dot{\beta}_{3,0} \,\right]^{\mathrm{T}} \\ &= \left[\, r_0\ \phi_0(t)\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \dot{\phi}_0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \,\right]^{\mathrm{T}} \end{aligned} \tag{4.1}$$

Substituting the perturbed state, $\boldsymbol{x} = \boldsymbol{x}_0 + \Delta\boldsymbol{x}$, into the nonlinear equations of motion presented in Section 3 results in the following linearized equations of motion for the two-spacecraft system. For the translational degrees of freedom of spacecraft A, the linearized equations of motion are:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & r_0 & 0 \\ 0 & 0 & r_0 \end{bmatrix} \begin{Bmatrix} \Delta\ddot{r} \\ \Delta\ddot{\phi} \\ \Delta\ddot{\psi} \end{Bmatrix} + \begin{bmatrix} 0 & -2r_0\dot{\phi}_0 & 0 \\ 2\dot{\phi}_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta\dot{r} \\ \Delta\dot{\phi} \\ \Delta\dot{\psi} \end{Bmatrix} + \begin{bmatrix} -\dot{\phi}_0^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & r_0\dot{\phi}_0^2 \end{bmatrix} \begin{Bmatrix} \Delta r \\ \Delta\phi \\ \Delta\psi \end{Bmatrix} + \begin{Bmatrix} -r_0\dot{\phi}_0^2 \\ 0 \\ 0 \end{Bmatrix}$$

$$= \frac{-3\mu_0\mu_A\mu_B}{64\pi m r_0^4} \begin{Bmatrix} 2 - \dfrac{8\Delta r}{r_0} \\ \Delta\alpha_2 + \Delta\beta_2 \\ -\Delta\alpha_1 - \Delta\beta_1 \end{Bmatrix} \tag{4.2}$$

Recall that the position of spacecraft B is determined from the position of A and the constraints in Equation 2.1.

For the rotational degrees of freedom of spacecraft A, the linearized equations of motion are:

$$\begin{bmatrix} I_{rr,s} + I_{rr,w} & 0 & 0 \\ 0 & I_{rr,s} + I_{rr,w} & 0 \\ 0 & 0 & I_{zz,s} \end{bmatrix} \begin{Bmatrix} \Delta\ddot{\alpha}_3 \\ \Delta\ddot{\alpha}_2 \\ \Delta\ddot{\alpha}_1 \end{Bmatrix} + \begin{bmatrix} 0 & \Omega_{z,w} I_{zz,w} & 0 \\ -\Omega_{z,w} I_{zz,w} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta\dot{\alpha}_3 \\ \Delta\dot{\alpha}_2 \\ \Delta\dot{\alpha}_1 \end{Bmatrix}$$

$$= \frac{-\mu_0\mu_A\mu_B}{32\pi r_0^3} \begin{Bmatrix} 0 \\ \Delta\alpha_2 + \Delta\beta_2 \\ \Delta\alpha_1 + \Delta\beta_1 \end{Bmatrix} . \tag{4.3}$$

and for the rotational degrees of freedom of spacecraft B, the linearized equations of motion are:

$$\begin{bmatrix} I_{rr,s}+I_{rr,w} & 0 & 0 \\ 0 & I_{rr,s}+I_{rr,w} & 0 \\ 0 & 0 & I_{zz,s} \end{bmatrix} \begin{Bmatrix} \Delta\ddot{\beta}_3 \\ \Delta\ddot{\beta}_2 \\ \Delta\ddot{\beta}_1 \end{Bmatrix} + \begin{bmatrix} 0 & \Omega_{z,w} I_{zz,w} & 0 \\ -\Omega_{z,w} I_{zz,w} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta\dot{\beta}_3 \\ \Delta\dot{\beta}_2 \\ \Delta\dot{\beta}_1 \end{Bmatrix} = \frac{-\mu_0 \mu_A \mu_B}{32\pi r_0^3} \begin{Bmatrix} 0 \\ \Delta\alpha_2 - 4\Delta\beta_2 \\ \Delta\alpha_1 - 4\Delta\beta_1 \end{Bmatrix} \quad . \tag{4.4}$$

The nine linearized equations of motion can now be compiled into the following 9×9 second-order matrix equation:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & r_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & r_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I_{zz,s} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & I_{rr,s}+I_{rr,w} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I_{rr,s}+I_{rr,w} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & I_{zz,s} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{rr,s}+I_{rr,w} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{rr,s}+I_{rr,w}
\end{bmatrix}
\begin{Bmatrix}
\Delta\ddot{r} \\ \Delta\ddot{\phi} \\ \Delta\ddot{\psi} \\ \Delta\ddot{\alpha}_1 \\ \Delta\ddot{\alpha}_2 \\ \Delta\ddot{\alpha}_3 \\ \Delta\ddot{\beta}_1 \\ \Delta\ddot{\beta}_2 \\ \Delta\ddot{\beta}_3
\end{Bmatrix}
$$

$$
+\begin{bmatrix}
0 & -2r_0\dot{\phi}_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2\dot{\phi}_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\Omega_{z,w}I_{zz,w} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Omega_{z,w}I_{zz,w} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Omega_{z,w}I_{zz,w} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \Omega_{z,w}I_{zz,w} & 0
\end{bmatrix}
\begin{Bmatrix}
\Delta\dot{r} \\ \Delta\dot{\phi} \\ \Delta\dot{\psi} \\ \Delta\dot{\alpha}_1 \\ \Delta\dot{\alpha}_2 \\ \Delta\dot{\alpha}_3 \\ \Delta\dot{\beta}_1 \\ \Delta\dot{\beta}_2 \\ \Delta\dot{\beta}_3
\end{Bmatrix}
\tag{4.5}
$$

$$
+\begin{bmatrix}
\frac{8c_1}{r_0}-\dot{\phi}_0^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -c_1 & 0 & 0 & -c_1 & 0 \\
0 & 0 & r_0\dot{\phi}_0^2 & c_1 & 0 & 0 & c_1 & 0 & 0 \\
0 & 0 & 0 & -c_0 & 0 & 0 & -c_0 & 0 & 0 \\
0 & 0 & 0 & 0 & -c_0 & 0 & 0 & -c_0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -c_0 & 0 & 0 & 4c_0 & 0 & 0 \\
0 & 0 & 0 & 0 & -c_0 & 0 & 0 & 4c_0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{Bmatrix}
\Delta r \\ \Delta\phi \\ \Delta\psi \\ \Delta\alpha_1 \\ \Delta\alpha_2 \\ \Delta\alpha_3 \\ \Delta\beta_1 \\ \Delta\beta_2 \\ \Delta\beta_3
\end{Bmatrix}
=
\begin{Bmatrix}
r_0\dot{\phi}_0^2+2c_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{Bmatrix}
$$

where $c_0 \equiv \dfrac{-\mu_0\mu_A\mu_B}{32\pi r_0^3}$ and $c_1 \equiv \dfrac{3c_0}{2mr_0}$.

Equation 4.5, along with the constraint defined in Equation 2.1, is sufficient to completely characterize the linearized dynamics of the system.

5. Summary and Conclusions

In this memo, we have defined the geometry for a sample three-dimensional two-spacecraft electromagnetic formation flying array. We have developed the *nonlinear dynamic equations of motion* (the evaluation model) and linearized these equations to yield the *linearized dynamic equations of motion* (the design model). We can now proceed to perform a stability analysis and control design using the linearized design model. The closed-loop control may then be simulated using the nonlinear evaluation model by substituting the closed-loop magnetic moments into the evaluation model equations.

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