Ventilation

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Natural Ventilation

Can we cool a campus auditorium via ventilation, without running an air conditioner?

Let's get some feeling for the numbers, before taking a look at the physics

Consider, for example, 200 people in an auditorium, each worth (thermally!) about 75 Watts.

$$q = 15 \text{ kW}$$

For now, let's say that all of this heat must be removed by convection. We'll add conduction at the end. From conservation of energy (again!) we have

$$q = \rho C_p \dot{V} (T_{in} - T_{out})$$

where

ρ = density of air, kg/m^3

C_p = specific heat (at constant pressure) $kJ/kg\,K$

$\dot{V}$ = volumetric flow rate, m^3/s

$\rho \approx 1.2$ kg/m^3 for air at indoor temperatures

$C_p \approx 1$ $kJ/kg\,K$

For $q = 15$ kW we can find $\dot{V}$ for different values of $\Delta T = T_{in} - T_{out}$

$\dot{V}$ m^3/s	ACH	v m/s	ΔT K
6.25	5.6	3.1	2
2.5	2,3	1.3	5
1.25	1.1	0.6	10

where the air changes per hour were calculated on the bases of an auditorium volume of $4{,}000\,m^3$ and the air velocity measured at the window opening is based on an area of $2\,m^2$.

Ventilation Tools

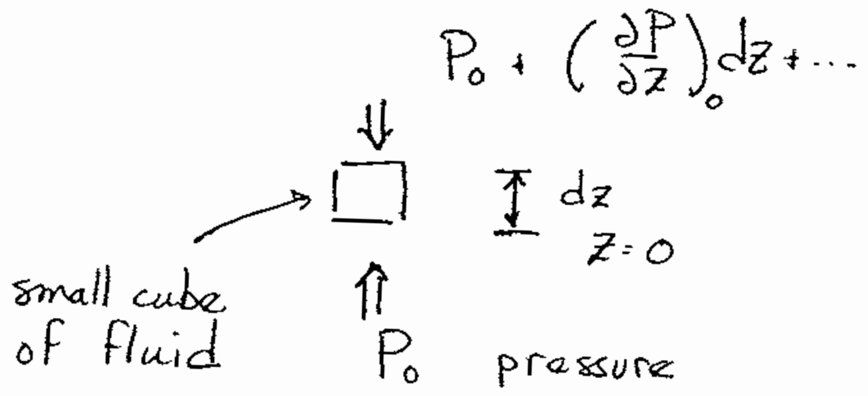
We would like to better understand natural ventilation in a "big picture" way, for its own sake and to support the use of computational fluid dynamics (CFD). CFD simulation programs, such as PHOENICS, produce a single "data point" per run - an airflow pattern that can be analyzed or reduced to an air change rate. To understand trends, many runs must be made. A simplified approach can help. This approach assumes that there are no obstructions within the space in question and yields no information about airflow distribution within the space.

To set up this approach, which we will explore with a spreadsheet, we need some basic components (building blocks or simple tools - call them what you'd like).

Before we start, keep in mind the physics: air moves through an opening (i.e., a window) because there is a pressure difference across the opening. Our job is to understand how differences in temperatures inside and out r wind create that pressure difference, and how the pressure difference is related to the flow.

How to create a pressure difference across a window? Let's start with buoyancy-driven flows, for which we'll need the hydrostatic equation and the ideal gas law.

1. Hydrostatic equation. How does pressure vary with temperature and height? Let's look at force equilibrium. What keeps a small blob of fluid from falling? After all, it has mass. Answer: pressure on top must be slightly lower than on the bottom.



Equilibrium $F_z = 0$

Let's multiply pressures by areas to get forces, and density by volume (to get mass) and the gravitational constant, to get a force.

$$P_0\,dx\,dy - \left[P_0 + \left(\frac{\partial P}{\partial z}\right)_0 dz + \dots\right] dx\,dy$$

$$- \rho g\,dx\,dy\,dz = 0$$

Weight of fluid is balanced by difference in pressure; top and bottom

$$\left(\frac{\partial P}{\partial z}\right)_0 + \rho g = 0$$

$$P_z = P_0 - \int_0^h \rho g\,dz$$

If the density is approximately constant over the heights in question (good for buildings)

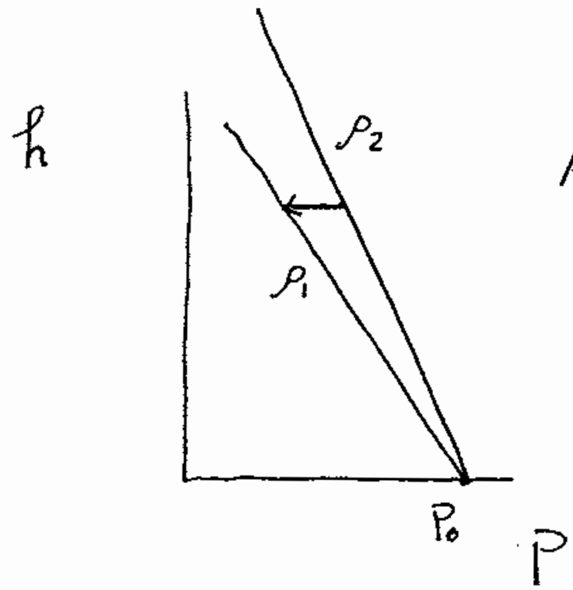
$$P_z = P_0 - \rho g h$$

P : N/m^2 or Pa, where $N = kg\,\frac{m}{s^2}$

g : $9.8\ m/s^2$, the gravitational constant

z, h : m

Let's graph this relationship for two values of density



At a given height, h, air will flow through an opening from the high-pressure side (ρ_2) to the low-pressure side (ρ_1)

This means that ρ_2 is <u>less</u> than ρ_1. But how do we relate air density to temperature?

2. Ideal gas law

$$pV = nRT$$

P Pa

V m^3

$$PV = kg\frac{m}{s^2} \times \frac{1}{m^2} \times m^3 = kg\frac{m^2}{s^2} = J$$

n = number of moles = $\dfrac{\text{mass}}{\text{molecular weight}}$

R = gas constant = 8.314 $\dfrac{kJ}{kg\cdot mole\ {}^\circ K}$

Re-arrange

$$P = \frac{n}{V} RT = \frac{m}{V} \frac{R}{M} T = \rho \frac{R}{M} T$$

where M is the molecular weight

Solve for the density ρ

$$\rho = \frac{P}{T \frac{R}{M}}$$

Density is inversely proportional to temperature

Now let's move on to <u>wind-driven</u> airflows. We need a relation between wind speed and pressure (Bernoulli's Equation), some fudge factors called wind pressure coefficients, and relations that give us wind speed as a function of height above the ground.

3. Bernoulli's Equation

This is an expression of conservation of energy for isothermal, steady flow

$$P + \rho \frac{v^2}{2} + \rho g z = \text{constant}$$

The term $\rho \frac{v^2}{2}$ is called the dynamic pressure.

It is the pressure the wind would exert when it hits a wall of infinite size, has no place to go, and stops completely.

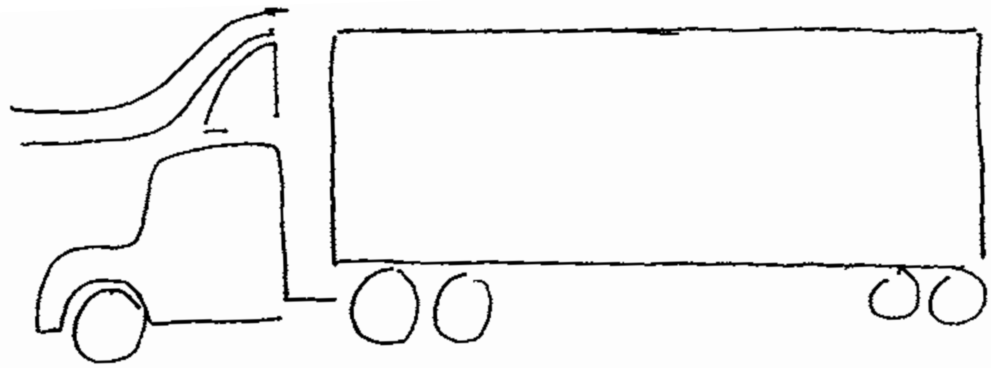
It is (more or less) the same as the pressure you feel on your hand when you hold it outside the window of a moving car. Not quite the same, because your hand is not infinite in size. It's also related to the pressure a car must overcome. For a car, force and not pressure is the issue that translates into required horsepower. The aerodynamic drag is

$$C_d A \frac{\rho v^2}{2}$$

C_d is a drag coefficient. Smaller is better. But a relatively larger C_d may not be a big deal if the cross-sectional area A is small, as for most sports cars.

Note also that the wind pressure or drag scales as v^2. So, in the oil shock days, speed limits dropped to save energy. Comparing 60 and 50 mph, we see that the drag drops to $\left(\frac{50}{60}\right)^2$ or 0.69, of its original value: a 31% decrea for a 17% decrease in speed.

One more word about Cd for vehicles: it explains why the cabs of tractor-trailer rigs have the wind deflectors on top.



4. Wind pressure coefficients (fudge factors) for buildings.

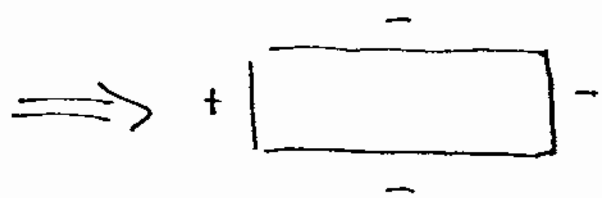
Okay. enough of cars and trucks. What about buildings? Here's what we use:

$$P_\omega = C_\omega \rho \frac{v^2}{2}$$

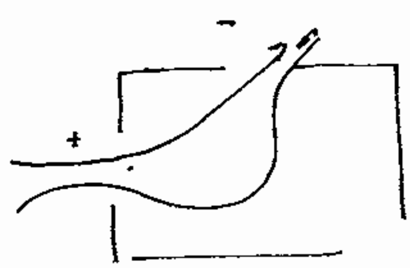
Why does this help? Because buildings are not infinitely large and air moves around them. We therefore expect the peak positive wind pressure to be less than $\rho \frac{v^2}{2}$. In other words, C_ω will be less than 1.0.

The relationship also helps because it applies on the leeward sides of buildings as well, where C_ω will be negative. Wind pressure not only forces air into a building, but it sucks it out as well. Pretty neat!

So, you say, how about some values for wind pressure coefficients? They depend on building shape, wind direction, and the influence of nearby buildings, terrain, and vegetation. ASHRAE claims they must be determined from wind-tunnel studies. Perhaps CFD is now an acceptable alternative. ASHRAE's words of wisdom are attached. A value of 0.7 seems about right, as a single number to remember, for the windward side. Negative values, say −0.5 to −0.7 are appropriate for the leeward sides. See figures 4 and 5. Note that the sides of a building have negative C_w's, even more negative than −1.0.



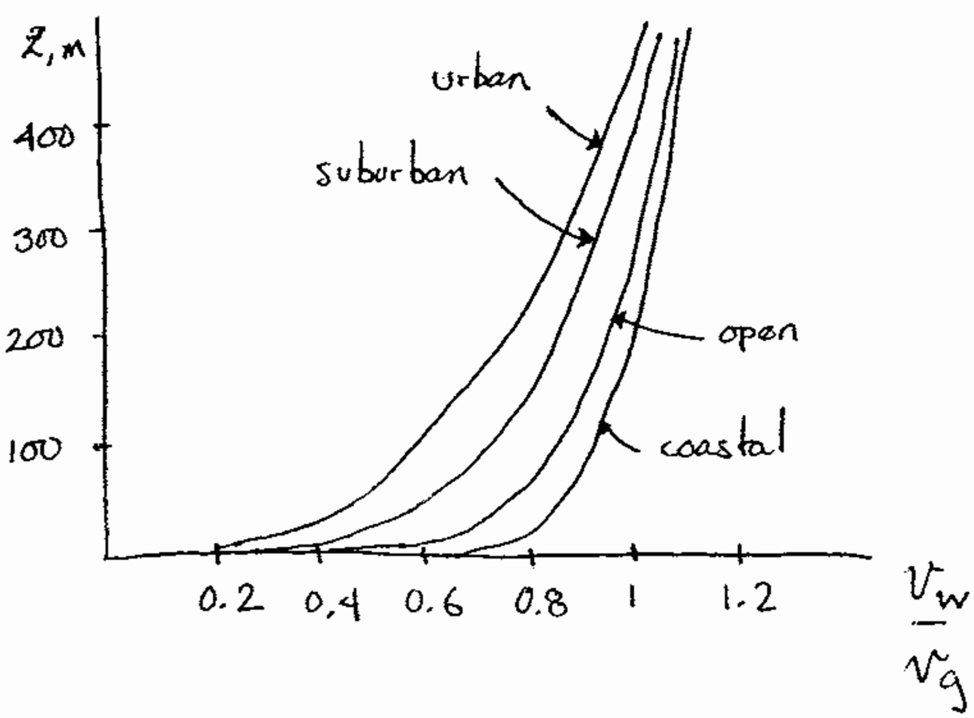
This is why cross ventilation works in corner rooms



5. Wind variation with height above the ground. ASHRAE speaks to this as well, as do other sources. Here's a typical expression for wind speed:

$$v_W = v_G \left(\frac{Z}{\delta} \right)^{\alpha}$$

v_G is the wind velocity at and above the boundary layer or gradient height, δ. The boundary layer thickness and the power-law coefficient depend on the general roughness of the upwind terrain.



Here are values for the boundary layer height and the exponential factor:

'Use ASHRAE	α	δ m
	—	—
urban	0.33	460
suburban	0.22	370
open	0.14	270
coastal	0.10	210

OK! Now we need just one more relationship, something that tells us how much airflow we get for a given pressure difference

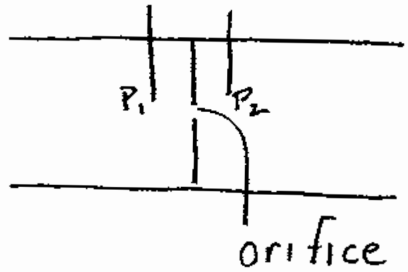
6. Orifice equation

The name comes from pipe flow, where the set-up looks like

See ASHRAE for more information about working w/ wind speeds.

V_{met} is usually measured flat, open terrain, for which $\alpha = 0.14$ and $\delta = 270$ m, at a height of 10 m.

$$\therefore \quad v = v_{met} \underbrace{\left(\frac{270}{10}\right)^{0.14}}_{1.58} \left(\frac{z}{\delta}\right)^{\alpha}$$



Pressure is measured just upstream and downstream of the small hole in the plate. From a bunch of measurements, the following empirical relationship was derived

$$\dot{V} = C_d A \sqrt{\frac{2 \Delta P_{loss}}{\rho}}$$

where $\dot{V}$ is the volumetric flow m^3/s

C_d is the discharge coefficient, dimensionless

A is the area of the orifice, m^2

ΔP_{loss} is the pressure drop across the orifice, Pa or N/m^2

ρ is the density of the fluid entering the orifice, $\frac{kg}{m^3}$

Wind flowing through a window is usually turbulent. For turbulent flow, C_d has a constant value of 0.6.

Air leaking into a building through small cracks (check out an electrical outlet in an outside wall when it's cold) is laminar (slow, smooth). For laminar flow, C_d varies with ΔP_{loss}

$Cd \propto \sqrt{\Delta P_{loss}}$.

So,

$$V \propto \sqrt{\Delta P_{loss}} \quad \text{for turbulent flow}$$

$$\propto \Delta P_{loss} \quad \text{for laminar flow}$$

In practice - i.e., in real buildings - there is a mix of turbulent and laminar flow and

$$V \propto \Delta P_{loss}^{n}$$

where $0.5 \leq n \leq 1$

It's reasonable to drop the subscript "loss" and simply refer to ΔP, which we take to be the pressure difference across an opening in the direction of the flow.

The orifice equation merits a little more attention, on three fronts:

a. Orifice means small hole. Well, a dictionary says "mouth" or "aperture," but in pipe flow it is a small hole. The relationship means something for buildings when windows are small relative to wall size. Before you label this an unreasonable abstraction, please note that our two-stage use of PHOENICS involves a similar approximation. We model outdoor airflows to get pressures at the boundary of a building, then use those pressures as boundary conditions when modeling indoor airflows. More or less fine, but the first stage is done with the building modeled as a blockage - a solid lump with no openings. The presence of openings, as in a real building, affects the pressure at the façade. More openings mean more influence.

b. What happens if we have combined wind-and buoyancy-driven flows? Let's say, in a building or a pumpkin, you measure 2 ACH for buoyancy flows and 6 ACH for wind flows (the former with no wind and the latter with no temperature difference inside and out). What happens if you have both? Would you expect 8 ACH? No!!
Here's why:

$$V_{total} \propto \sqrt{\Delta P_{total}}$$

$$\propto \sqrt{\Delta P_{buoyancy} + \Delta P_{wind}}$$

$$V_{total}^2 \propto \Delta P_{buoyancy} + \Delta P_{wind}$$

But $V_{buoyancy}^2 \propto \Delta P_{buoyancy}$ and a similar relation holds for wind. Therefore

$$V_{total}^2 = V_{buoyancy}^2 + V_{wind}^2$$

or $$V_{total} = \sqrt{V_{buoyancy}^2 + V_{wind}^2}$$

In other words, the flows add in quadrature. Let's return to our hypothetical case.

$$V_{total} = \sqrt{2^2 + 6^2}$$
$$= \sqrt{40}$$
$$= 6.3 \text{ ACH} \quad \text{Not } 8 \text{ !!}$$

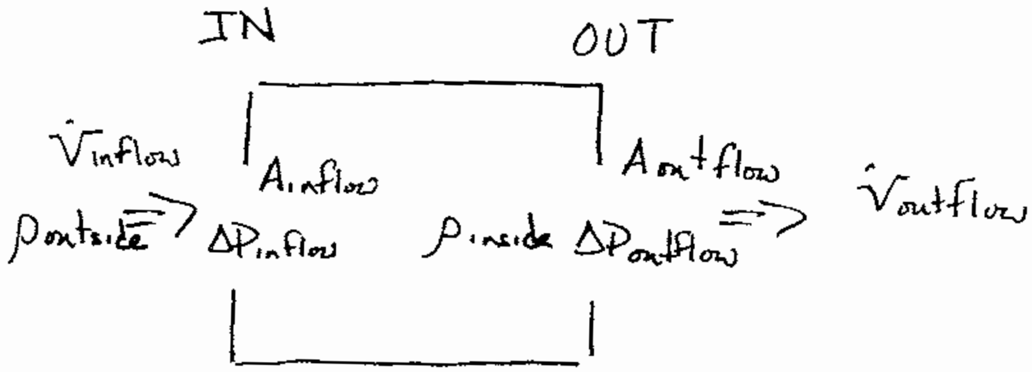
c. So, how do we use the orifice equation? We insist on conservation of mass: mass flow in = mass flow out.

$$\rho_{out} V_{in} = \rho_{in} V_{out}$$

where V_{in} is the flow of outdoor air <u>into</u> our building and V_{out} is the flow of indoor air <u>out</u> of our building.

We then substitute the orifice equation:

$$\rho_{out}\, C_d\, A_{in} \sqrt{\frac{2\,\Delta P_{in}}{\rho_{out}}} = \rho_{in}\, C_d\, A_{out} \sqrt{\frac{2\,\Delta P_{out}}{\rho_{in}}}$$



Got the picture? So, we have one equation, which means we can solve for one unknown. Which unknown?

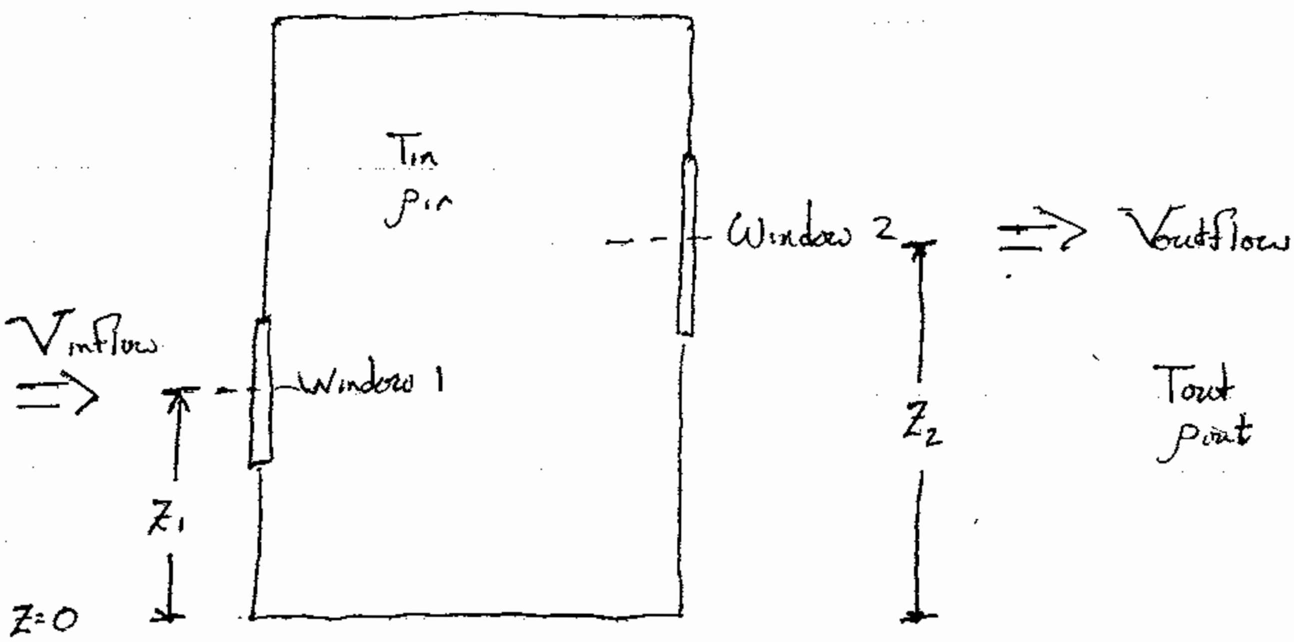
$$\Delta P_{in} = P_{out}\big|_{inlet} - P_{in}$$

$$\Delta P_{out} = P_{in} - P_{out}\big|_{outlet}$$

The outside pressures are easy. We know them. What we solve for is P_{in}. With that in hand, we can calculate ΔP_{in} or ΔP_{out} and use the orifice equation to get the flow. Presto!

"Presto" may be too much but nature works the same way and it really is magic. The inside pressure automatically adjusts itself such that mass is conserved.

Airflow Schematic



ΔP_{inflow} is pressure drop, out-to-in, across window 1

$\Delta P_{outflow}$ is pressure drop, in-to-out, across window 2

Wind pressure at window 1 is $\quad C_{w1} \dfrac{\rho_{out}}{2} V_{w1}^2$, relative to P_o, the atmospheric pressure

Wind pressure at window 2 is $\quad C_{w2} \dfrac{\rho_{out}}{2} V_{w2}^2$

where $\quad v_{w1} = v_4 \left(\dfrac{Z_1}{\delta} \right)^{\alpha}$

$$v_{w2} = v_4 \left(\frac{Z_2}{\delta} \right)^{\alpha}$$

Buoyancy-driven flows

Consider conservation of mass, whereby the mass flow entering the single zone through the lower window equals that leaving through the upper window:

$$\rho_{out} C_d A_1 \sqrt{\frac{2\Delta P_{\inf low}}{\rho_{out}}} = \rho_{in} C_d A_2 \sqrt{\frac{2\Delta P_{outflow}}{\rho_{in}}}$$

$$A_1^2 \rho_{out} \Delta P_{\inf low} = A_2^2 \rho_{in} \Delta P_{outflow}$$

$$A_1^2 \frac{\Delta P_{\inf low}}{T_{out}} = A_2^2 \frac{\Delta P_{outflow}}{T_{in}}$$

$$\frac{A_1^2}{T_{out}}\left[\left(P_{out} - \rho_{out} g z_1\right) - \left(P_{in} - \rho_{in} g z_1\right)\right] = \frac{A_2^2}{T_{in}}\left[\left(P_{in} - \rho_{in} g z_2\right) - \left(P_{out} - \rho_{out} g z_2\right)\right]$$

$$\left(P_{out} - P_{in}\right) = \left(\rho_{out} - \rho_{in}\right) g z_2 \frac{\left[\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) \frac{z_1}{z_2} + 1\right]}{\left[\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) + 1\right]}$$

With the pressure difference calculated, we can now determine the volumetric flow in through the lower window:

$$V_{\inf low} = C_d A_1 \left[\frac{2}{\rho_{out}}\left[\left(P_{out} - P_{in}\right) - \left(\rho_{out} - \rho_{in}\right) g z_1\right]\right]^{0.5}$$

$$= C_d A_1 \left[2 \frac{T_{in} - T_{out}}{T_{in}} g z_2 \frac{\left[\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) \frac{z_1}{z_2} + 1\right]}{\left[\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) + 1\right]} - \frac{z_1}{z_2}\right]^{0.5}$$

$$= C_d A_1 \left[2 \frac{T_{in} - T_{out}}{T_{in}} g \left(z_2 - z_1\right) \left[\frac{1}{\left[\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) + 1\right]}\right]\right]^{0.5}$$

If z_1 is set to zero and, for simplicity z_2 is defined as z,

$$V_{\inf low} = C_d A_1 \left[2\frac{\Delta T}{T_{in}} gz \frac{1}{\left[\left(\frac{A_1}{A_2} \right)^2 \left(\frac{T_{in}}{T_{out}} \right) + 1 \right]} \right]^{0.5}$$

Note that if z equals zero, there is no buoyancy-driven flow, as expected. If $T_{out} = T_{in}$, there is also no flow, again as expected.

The above equation is exact. Introducing our first approximation, let

$$\frac{T_{in}}{T_{out}} \approx 1$$

Note that the temperature difference must be retained. Then

$$V_{\inf low} \approx C_d A_1 \left[2\frac{\Delta T}{T_{in}} gz \frac{1}{\left[\left(\frac{A_1}{A_2} \right)^2 + 1 \right]} \right]^{0.5}$$

$$= C_d \left[\frac{A_1^2 A_2^2}{A_1^2 + A_2^2} \right]^{0.5} \left[2gz \frac{\Delta T}{T_{in}} \right]^{0.5} = C_d A_{eff} \left[2gz \frac{\Delta T}{T_{in}} \right]^{0.5}$$

The effective area is sometimes defined with a factor of two, as follows:

$$A_{eff}{}' = \left[\frac{2 A_1^2 A_2^2}{A_1^2 + A_2^2} \right]^{0.5}$$

in which case $A_{eff}' = A_1 = A_2 = A$ if the two windows are equal in area and

$$V_{\inf low} = C_d A_{eff}{}' \left[gz \frac{\Delta T}{T_{in}} \right]^{0.5}$$

This equation takes a slightly different form when the height is defined in terms of a neutral plane, where the indoor and outdoor pressures are equal:

$V_{inflow} = C_d\, A_1\, [\, 2g\, \Delta h_{NPL}\, (T_i - T_o)/T_i\,]^{0.5}$

where C_d = discharge coefficient
Δh_{NPL} = height from midpoint of lower opening to NPL (m)

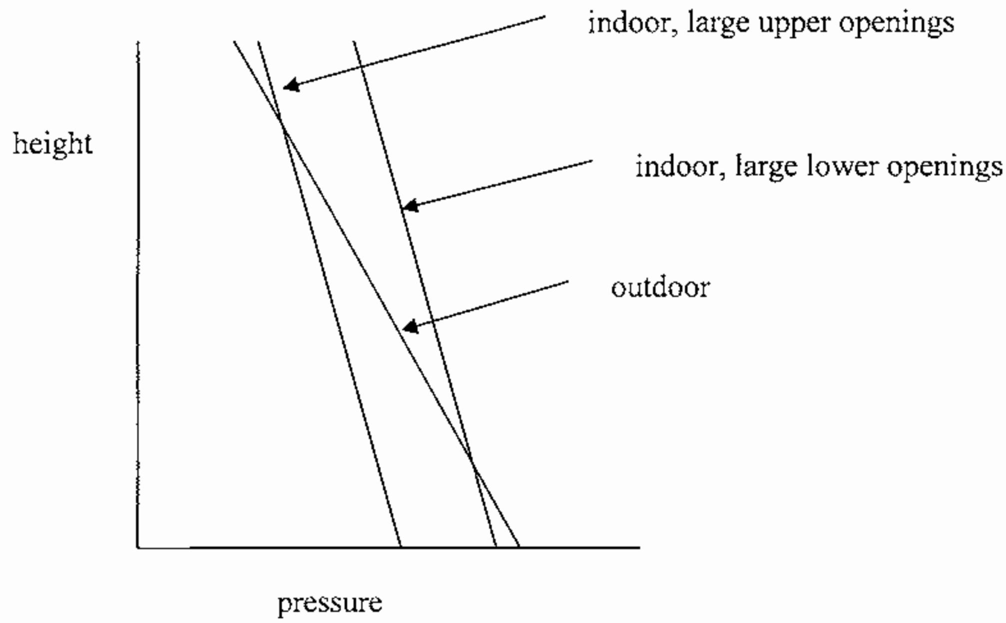
$C_d = 0.40 + 0.0045\,|T_i - T_o|$ if there is bidirectional flow in a single opening and 0.65 otherwise.

However, this form is somewhat difficult to use because the location of the neutral plane must be determined. Comparing expressions above we find that

$$V_{\inf low} \approx C_d A_1 \left[2\frac{\Delta T}{T_{in}} gz \frac{1}{\left[\left(\frac{A_1}{A_2} \right)^2 + 1 \right]} \right]^{0.5} = C_d A_1 \left[2\frac{\Delta T}{T_{in}} g\Delta h_{NPL} \right]^{0.5}$$

$$\Delta h_{NPL} = z \frac{1}{\left[\left(\frac{A_1}{A_2} \right)^2 + 1 \right]}$$

If there are two windows and they are of equal area, neutral pressure is half-way between the two windows, as intuitively expected. If the lower window is much larger, the neutral-pressure point shifts toward the lower window. If the upper window is larger, the neutral-pressure point shifts upward. The neutral-pressure height is more difficult to determine with multiple openings.



Example

Determine the volumetric inflow for the following case:

T_{in} = 30 °C
T_{out} = 20 °C
Z = 3 m
A_{eff}' = 2 m^2

Solution:

$$V_{inflow} = C_d A_{eff}' \left[gz \frac{\Delta T}{T_{in}} \right]^{0.5} = 0.6 \cdot 2 \cdot \left[9.8 \cdot 3 \cdot \frac{10}{303} \right]^{0.5} \approx 1.2 \frac{m^3}{s}$$

Note that absolute temperature is required, because the ideal gas law was used to replace densities with temperatures.

Wind-driven flows

We start with the same expression of conservation of mass as for buoyancy-driven flows. For wind-driven flows, we could choose to let Tin equal Tout, which implies that ρin equals ρout. Flows are driven by wind and not temperature differences. However, if there are heat sources inside the building, Tin will be higher than Tout and we will therefore retain distinct indoor and outdoor temperatures and densities.

$$\rho_{out} C_d A_1 \sqrt{\frac{2\Delta P_{\inf low}}{\rho_{out}}} = \rho_{in} C_d A_2 \sqrt{\frac{2\Delta P_{outflow}}{\rho_{in}}}$$

$$A_1^2 \rho_{out} \Delta P_{\inf low} = A_2^2 \rho_{in} \Delta P_{outflow}$$

$$A_1^2 \frac{\Delta P_{\inf low}}{T_{out}} = A_2^2 \frac{\Delta P_{outflow}}{T_{in}}$$

$$\frac{A_1^2}{T_{out}}\left[\left(P_{out} + \frac{1}{2} C_{p1} \rho_{out} v_{w1}^2\right) - P_{in}\right] = \frac{A_2^2}{T_{in}}\left[P_{in} - \left(P_{out} + \frac{1}{2} C_{p2} \rho_{out} v_{w2}^2\right)\right]$$

$$\left(P_{out} - P_{in}\right) = -\frac{1}{2} \rho_{out} C_{p1} v_{w1}^2 \frac{\left[\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) + \frac{C_{p1}}{C_{p2}} \left(\frac{v_{w2}}{v_{w1}}\right)^2\right]}{\left[\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) + 1\right]}$$

If the inflow and outflow windows are at different heights, the wind speed associated with these openings may vary, according to

$$v = v_G \left(\frac{z}{\delta}\right)^{\alpha}$$

where v_G = gradient or free-stream velocity
δ = gradient or boundary layer height
α = height exponent

With the pressure difference calculated, we can now determine the volumetric flow in through the windward window:

$$V_{\inf low} = C_d A_1 \left[\frac{2}{\rho_{out}}\left[\left(P_{out} - P_{in}\right) + \frac{1}{2} C_{p1} \rho_{out} v_{w1}^2\right]\right]^{0.5}$$

$$= C_d A_1 \left[C_{p1} v_{w1}^2 \left[\frac{1 - \frac{C_{p2}}{C_{p1}}\left(\frac{v_{w2}}{v_{w1}}\right)^2}{\left[\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) + 1\right]}\right]\right]^{0.5}$$

$$= C_v A_1 v_{w1}$$

where C_v represents the effectiveness of the opening area

$$C_v = C_d\left[C_{p1}\left[\frac{1-\frac{C_{p2}}{C_{p1}}\left(\frac{v_{w2}}{v_{w1}}\right)^2}{\left(\frac{A_1}{A_2}\right)^2\left(\frac{T_{in}}{T_{out}}\right)+1}\right]\right]^{0.5}$$

This expression is exact, within the framework of the model. To simplify, we will equate the indoor and outdoor temperatures and the heights of the windward and leeward openings, such that the wind velocities are equal. In this case

$$\begin{aligned} V_{\inf low} &= C_d A_1\left[C_{p1}v_{w1}^2\left[\frac{1-\frac{C_{p2}}{C_{p1}}}{\left(\frac{A_1}{A_2}\right)^2+1}\right]\right]^{0.5} \\ &= C_v A_{eff}{}' v \end{aligned}$$

where

$$A_{eff}{}' = \left[\frac{2A_1^2A_2^2}{A_1^2+A_2^2}\right]^{0.5}$$

$$C_v = C_d\left[\frac{C_{p1}-C_{p2}}{2}\right]^{0.5}$$

Finally, if the leeward wind-pressure coefficient is equal in magnitude to the windward coefficient and opposite in sign,

$$C_v = C_d C_p^{0.5}$$

C_v equals 0.5 to 0.6 for perpendicular winds and 0.25 - 0.35 for diagonal winds.

Combined flows

Again, let's start with conservation of mass, this time including both buoyancy and wind terms in the indoor and outdoor pressures.

Note

$$\rho_{out} C_d A_1 \sqrt{\frac{2\Delta P_{\inf low}}{\rho_{out}}} = \rho_{in} C_d A_2 \sqrt{\frac{2\Delta P_{outflow}}{\rho_{in}}}$$

$$A_1^2 \rho_{out} \Delta P_{\inf low} = A_2^2 \rho_{in} \Delta P_{outflow}$$

$$A_1^2 \frac{\Delta P_{\inf low}}{T_{out}} = A_2^2 \frac{\Delta P_{outflow}}{T_{in}}$$

$$\frac{A_1^2}{T_{out}} \left[\left((P_{out} - \rho_{out} g z_1) + \frac{1}{2} C_{p1} v_1^2 \right) - (P_{in} - \rho_{in} g z_1) \right] =$$

$$\frac{A_2^2}{T_{in}} \left[(P_{in} - \rho_{in} g z_2) - \left((P_{out} - \rho_{out} g z_2) + \frac{1}{2} C_{p2} v_2^2 \right) \right]$$

$$(P_{out} - P_{in}) = (\rho_{out} - \rho_{in}) g z_2 \frac{\left[\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) \frac{z_1}{z_2} + 1 \right] - \frac{1}{2} C_{p1} \rho_{out} v_1^2 \left[\left(\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) + \frac{C_{p2}}{C_{p1}} \left(\frac{v_2}{v_1}\right)^2 \right) \right]}{\left[\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) + 1 \right]}$$

Note that we have two terms on the right side of the equation: a buoyancy-driven term followed by a wind-driven term. Both are exactly as we calculated earlier. In other words, pressure differences, NOT flows, add.

The inflow can be computed as

$$V_{\inf low} = C_d A_1 \left[2 \frac{\Delta T}{T_{in}} g (z_2 - z_1) \left[\frac{1}{\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) + 1} \right] - C_{p1} v_1^2 \left[\frac{\frac{C_{p2}}{C_{p1}} \left(\frac{v_2}{v_1}\right)^2 - 1}{\left(\frac{A_1}{A_2}\right)^2 \left(\frac{T_{in}}{T_{out}}\right) + 1} \right] \right]^{0.5}$$

The first term under the radical is what we had before for buoyancy-driven flow and the second is as before for wind-driven flow.

If we again set the temperature ratio to one

$$V_{\inf low} = C_d A_{eff}\left[\frac{2\Delta T}{T_{in}} gh + C_{p1} v_1^2 \left[1 - \frac{C_{p2}}{C_{p1}}\left(\frac{v_2}{v_1}\right)^2\right]\right]^{0.5}$$

$$A_{eff} = \left[\frac{1}{\frac{1}{A_1^2} + \frac{1}{A_2^2}}\right]^{0.5}$$

Using an alternative formulation for the wind-driven term we have

$$V_{\inf low} = C_d A_{eff}\left[\frac{2\Delta T}{T_{in}} gh + C_{p1} v_1^2 - C_{p2} v_2^2\right]^{0.5}$$

The windows must be at different heights for the buoyancy force to work. If, however, the height differential is such that the wind velocities are essentially the same, and if the leeward wind-pressure coefficient is equal in magnitude and opposite and sign to the windward coefficient,

$$V_{\inf low} = C_d A_{eff}\left[\frac{2\Delta T}{T_{in}} gh + 2C_p v^2\right]^{0.5} = C_d A_{eff}'\left[\frac{\Delta T}{T_{in}} gh + C_p v^2\right]^{0.5}$$

Complex building geometries

The expressions developed above are adequate for single-zone buildings with no internal obstructions and simple arrangements of windows. For more complex configurations, such computer programs as CONTAM are very useful. CONTAM, freeware from NIST, accounts for forced and natural ventilation, with multiple zones and multiple openings.

Infiltration

Infiltration is from the combined effect of buoyancy and wind. It can be estimated as an air-change rate on the basis of experience or calculated from leakage areas and the orifice equation, using the crack method:

$$Q = A\ C\ \Delta p^n$$

where A = effective leakage area of cracks (m^2)
C = flow coefficient
Δp = pressure difference between outside and inside

Infiltration in houses from Sherman-Grimsrud model

The Sherman-Grismrud model, presented in Chapter 26 of the 2001 ASHRAE Handbook of Fundamentals, is a version of the crack method that is often used for residential buildings:

$$Q = L\,(C_s\,\Delta T + C_w\,V^2)^{0.5}$$

where Q = infiltration rate (L/s)
L = effective leakage area (cm^2)
C_s = stack coefficient ($L^2\,s^{-2}\,cm^{-4}\,K^{-1}$)
ΔT = average indoor-outdoor temperature difference (K)
C_w = wind coefficient ($L^2\,s^{-4}\,cm^{-4}\,K^{-1}\,m^{-2}$)
V = average wind speed measured at local weather station (m/s)

The stack coefficient depends solely on house height, in stories, and the wind coefficient is a function of height and the extent to which the building is sheltered.

Example

Estimate the infiltration at design conditions for a two-story house in Boston. The house has an effective leakage area of 500 cm^2, a volume of 340 m^3, and is surrounded by a thick hedge (shielding class 3).

Solution:

Under winter design conditions: T_o = -13 °C, V = 8 m/s, T_i = 22 °C.

From Tables 7 and 9 from Chapter 26 of the 2001 ASHRAE Handbook of Fundamentals, C_s = 0.000290 and C_w = 0.000231.

$$Q = L\,(C_s\,\Delta T + C_w\,V^2)^{0.5}$$
$$= 500\,(0.000290 \times (22 - (-13)) + 0.00231 \times 8^2)^{0.5}$$
$$= 79 \text{ L/s} = 284 \text{ m}^3/\text{h}$$

Air change rate = 284 / 340 = 0.84 ACH.

Air exchange method for houses:

extremely low	-	0.1 ACH (air change rate per hour)
low	-	0.5 ACH
normal	-	1 ACH
high	-	2 ACH
extremely high	-	3 ACH

Combined Energy and Ventilation Analysis

Consider what is usually of more interest than airflow: indoor temperature. What happened when we open the windows, with a given indoor heat load? Nature adjusts:

- Indoor pressure to conserve mass
- Indoor temperature to conserve energy.

Let's look at the two conservation laws:

$$Q = \rho_{out} C_p V_{\inf low} (T_{in} - T_{out}) + \sum_i U_i A_i (T_{in} - T_{out})$$

$$V_{\inf low} = C_d A_{eff} \left[\frac{\Delta T}{T_{in}} gh + \Delta P_w \right]^{0.5}$$

Substituting for V_{inflow} in the first equation yields a cubic equation in ΔT.

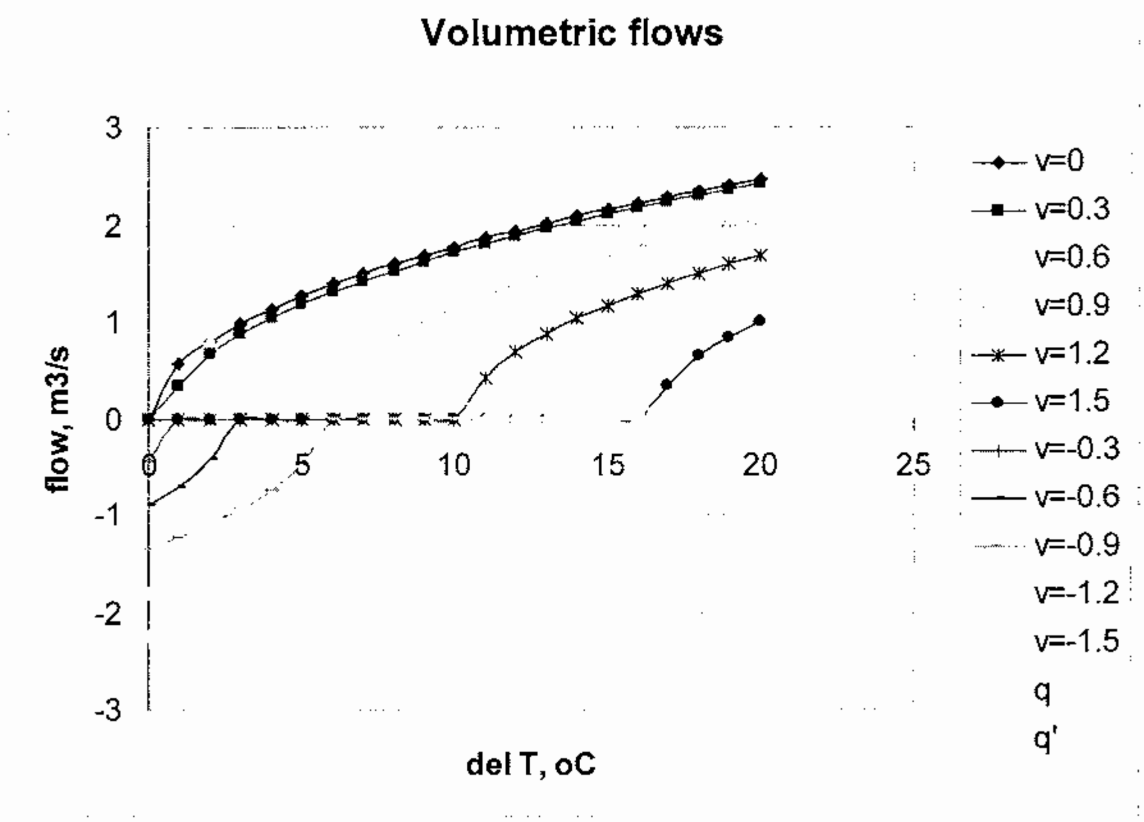
$$Q = \left[\rho_{out} C_p C_d A_{eff} \left[\frac{\Delta T}{T_{in}} gh + \Delta P_w \right]^{0.5} + \sum_i U_i A_i \right] (T_{in} - T_{out})$$

Solve for the temperature difference, then substitute and solve for the flow. Alternatively, solve the following equation for the flow and then substitute to obtain the temperature difference.

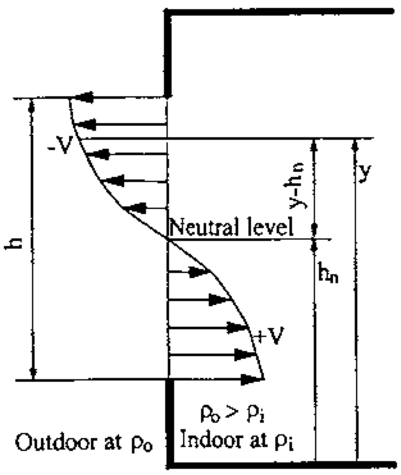
$$V_{\inf low} = C_d A_{eff} \left[\frac{Q}{\rho_{out} C_p V_{\inf low} + \sum_i U_i A_i} \frac{gh}{T_{in}} + \Delta P_w \right]^{0.5}$$

Either way, recognize that cubic equations have three solutions, more than one of which may be physically realizable. One such case concerns a wind that opposes the flow direction established by buoyancy-driven flows. One solution has a relatively high indoor temperature and a net flow out of the top window. Another has a lower temperature, smaller buoyancy force, and flow out the lower window. The time history of the flows determines which solution is present under given conditions. An interesting discussion is found in Li et al., *Building and Environment* 36(2001):851-58.

One way of looking at multiple solutions is shown below. This figure graphs the conservation-of-mass relations for various wind speeds and the conservation of energy relations. Intersection points indicate that both conservation laws are satisfied.


Volumetric flows
3
2
1
0
-1
-2
-3
0
5
10
15
20
25
flow, m3/s
del T, oC
v=0
v=0.3
v=0.6
v=0.9
v=1.2
v=1.5
v=-0.3
v=-0.6
v=-0.9
v=-1.2
v=-1.5
q
q'

Flow in two directions through a single window or door



For consistency with presentation that follows, replace "y" with "z"

$$v = - C_d \frac{z - h_n}{|z - h_n|} \sqrt{2g \frac{|\Delta\rho|}{\rho_r} |z - h_n|}$$

where

v is the horizontal air velocity, m/s, at a vertical distance z, positive for the incoming stream direction and negative for the outgoing

$\Delta\rho$ is the difference in air density between outdoors and indoors

ρ_r is the reference density, taken to be the outdoor air density for the incoming air stream and indoor air density for the outgoing stream

h_n is the neutral level, at which there is no pressure difference and therefore no flow across the opening

This equation is a special form of the familiar orifice equation

$$v = C_d \sqrt{\frac{2\Delta P}{\rho}}$$

Here $\rho = \rho_{out}$ for inflow and ρ_{in} for outflow. We can use the shorthand ρ_r.

$$\Delta P = P_{out} - P_{in} \text{ for inflow}$$
$$= P_{in} - P_{out} \text{ for outflow}$$

$$P_{out} = P_0 - \rho_{out}\, g\, z$$
$$P_{in} = P_I - \rho_{in}\, g\, z$$

where P_0 and P_I are measured at $z = 0$. We can reset this reference plane <u>anywhere</u> we'd like. Why not pick the neutral plane, $z = h_n$? By definition, $\Delta P = 0$ here. If this is our reference, we must compute heights from this level. So...

$$P_{out} = P_0 - \rho_{out}\, g\,(z - h_n)$$
$$P_{in} = P_I - \rho_{in}\, g\,(z - h_n)$$

But, $P_{out} = P_{in}$ at $z = h_n$, which means that

$$P_0 = P_I \quad \text{and}$$

$$\Delta P = -(\rho_{out} - \rho_{in})\, g\, (z - h_n) \quad \text{inflow, } z < h_n$$

$$= (\rho_{out} - \rho_{in})\, g\, (z - h_n) \quad \text{outflow, } z > h_n$$

$$= |\Delta\rho|\, g\, |z - h_n|$$

Why? Because ΔP is positive in both cases - it must be!

Note, too, in passing, that this case assumes $T_{in} > T_{out}$.

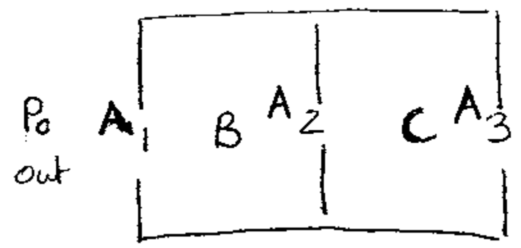
Now we have $\quad v = C_d \sqrt{2g \dfrac{|\Delta\rho|\,|z - h_n|}{\rho_r}}$

What's missing? Only a sign! If we want v to be positive for inflow and negative for outflow (one reasonable choice), we can simply introduce the factor

$$-\frac{(z - h_n)}{|z - h_n|}$$

2/17/00

Airflow calculations w/ internal obstructions



① $V_1 = A_1 C_d \sqrt{\frac{2\Delta P_1}{\rho_{out}}}$

② $V_2 = A_2 C_d \sqrt{\frac{2\Delta P_2}{\rho_{in}}}$

③ $V_3 = A_3 C_d \sqrt{\frac{2\Delta P_3}{\rho_{in}}}$

$V_1 = V_2 = V_3$

$V_1^2 = V_2^2 = V_3^2$

$$\frac{A_1^2 \Delta P_1}{\rho_0} = \frac{A_2^2 \Delta P_2}{\rho_B} = \frac{A_3^2 \Delta P_2}{\rho_C}$$

Wind-driven flows, $\rho_{out} \approx \rho_{in}$ or $\rho_0 = \rho_B = \rho_C$

$A_1^2 \Delta P_1 = A_2^2 \Delta P_2 = A_3^2 \Delta P_3$

$\Delta P_1 = P_0 + C_{w_1} \frac{\rho v_w^2}{2} - P_B$

$\Delta P_2 = P_B - P_C$

$\Delta P_3 = P_C - \left(P_0 + C_{w_2} \frac{\rho v_w^2}{2} \right)$

2/17/00

$$A_1^2 P_0 + A_1^2 C_{\omega 1} \rho \frac{V_{\omega 1}^2}{2} - A_1^2 P_B - A_2^2 P_B + A_2^2 P_C = 0$$

$$A_2^2 P_B - A_2^2 P_C - A_3^2 P_C + A_3^2 P_0 + A_3^2 C_{\omega 2} \rho \frac{V_{\omega 2}^2}{2} = 0$$

Collect terms in the pressures

$$-\left(A_1^2 + A_2^2\right) P_B + A_2^2 P_C = -A_1^2 P_0 - A_1^2 C_{\omega 1} \rho \frac{V_{\omega 1}^2}{2}$$

normalize areas

$$-\left(1 + \frac{A_1^2}{A_2^2}\right) P_B + P_C = -\frac{A_1^2}{A_2^2} P_0 - \frac{A_1^2}{A_2^2} C_{\omega 1} \rho \frac{V_{\omega 1}^2}{2}$$

$$P_B - \left(1 + \frac{A_3^2}{A_2^2}\right) P_C = -\frac{A_3^2}{A_2^2} P_0 - \frac{A_3^2}{A_2^2} C_{\omega 2} \rho \frac{V_{\omega 2}^2}{2}$$

Appealing symmetry between the two equations!

Take another pass, but this time work with the differences between P_B and P_0, and P_c and P_0

$$A_1^2 \left[C_{\omega 1} \rho \frac{V_{\omega 1}^2}{2} - \left(P_B - P_0\right) \right] =$$

$$A_2^2 \left[\left(P_B - P_0\right) - \left(P_C - P_0\right) \right] =$$

$$A_3^2 \left[\left(P_C - P_0\right) - C_{\omega 2} \rho \frac{V_{\omega 2}^2}{2} \right]$$

2/17/00

Let $P_B - P_0 = \hat{P}_B$

$P_C - P_0 = \hat{P}_C$

$$-\left(1 + \frac{A_1^2}{A_2^2}\right)\hat{P}_B + \hat{P}_C = \frac{A_1^2}{A_2^2} C_{\omega_1} \rho \frac{v_{\omega_1}^2}{2}$$

$$\hat{P}_B - \left(1 + \frac{A_3^2}{A_2^2}\right)\hat{P}_C = \frac{A_3^2}{A_2^2} C_{\omega_2} \rho \frac{v_{\omega_2}^2}{2}$$

2 equations in 2 unknowns

Simplify by letting $v_{\omega_1} = v_{\omega_2} = v_\omega$ (windows are at ~ same height)

If $A_3 = 0$ then $\hat{P}_B = \hat{P}_C$ as expected, from second equation

Then, from first equation, $\hat{P}_B$ is simply the stagnation pressure

Simplify further by letting $A_1 = A_2 = A_3$ Then

$$-2\hat{P}_B + \hat{P}_C = C_{\omega_1} \rho \frac{v_\omega^2}{2}$$

$$\hat{P}_B - 2\hat{P}_C = C_{\omega_2} \rho \frac{v_\omega^2}{2}$$

$$-3\hat{P}_B = \frac{\rho v_\omega^2}{2}\left(2C_{\omega_1} + C_{\omega_2}\right)$$

$$\hat{P}_B = -\frac{\rho v_\omega^2}{6}\left(C_{\omega_2} + 2C_{\omega_1}\right)$$

$$-\left(1+\frac{A_1^2}{A_2^2}\right)\hat{P}_B + \hat{P}_C = \frac{A_1^2}{A_2^2}\, C_{\omega_1}\, \rho\, \frac{v_{\omega_1}^2}{2}$$

$$P_B - \left(1+\frac{A_3^2}{A_2^2}\right)\hat{P}_C = \frac{A_3^2}{A_2^2}\, C_{\omega_2}\, \rho\, \frac{v_{\omega_2}^2}{2}$$

$$\hat{P}_C\left[1-\left(1+\frac{A_1^2}{A_2^2}\right)\left(1+\frac{A_3^2}{A_2^2}\right)\right] = \frac{A_1^2}{A_2^2}\, C_{\omega_1}\, \rho\, \frac{v_{\omega_1}^2}{2} + \left(1+\frac{A_1^2}{A_2^2}\right)\left(\frac{A_3^2}{A_2^2}\right) C_{\omega_2}\, \rho\, \frac{v_{\omega_2}^2}{2}$$

$$-\left(1+\frac{A_1^2}{A_2^2}\right)\left(1+\frac{A_3^2}{A_2^2}\right)\hat{P}_B = \left(1+\frac{A_3^2}{A_2^2}\right)\frac{A_1^2}{A_2^2}\, C_{\omega_1}\, \rho\, \frac{v_{\omega_1}^2}{2} + \frac{A_3^2}{A_2^2}\, C_{\omega_2}\, \rho\, \frac{v_{\omega_2}^2}{2}$$

2/17/00

$$V_1 = A_1 C_d \left[\frac{2}{\rho} \left(C_{\omega_1} \rho \frac{v_\omega^2}{2} - \hat{P}_B \right) \right]^{1/2}$$

$$= A_1 C_d \left[\frac{2}{\rho} \left(C_{\omega_1} \rho \frac{v_\omega^2}{2} - \rho \frac{v_\omega^2}{6} \left(C_{\omega_2} + 2 C_{\omega_1} \right) \right) \right]^{1/2}$$

$$= A_1 C_d v_\omega \left[C_{\omega_1} - \frac{1}{3} \left(C_{\omega_2} + 2 C_{\omega_1} \right) \right]^{1/2}$$

$$= A_1 C_d v_\omega \left[\frac{1}{3} C_{\omega_1} - \frac{1}{3} C_{\omega_2} \right]^{1/2}$$

$$= A_1 C_d v_\omega \sqrt{\frac{1}{3}} \sqrt{C_{\omega_1} - C_{\omega_2}}$$

If $|C_{\omega_1}| \approx |C_{\omega_2}| = |C_\omega|$

$$V_1 = A_1 C_d v_\omega \sqrt{\frac{2}{3}} \sqrt{C_\omega}$$

$$\frac{V_1}{A_1} = C_d v_\omega \sqrt{\frac{2 C_\omega}{3}}$$

This is $\sqrt{\frac{2}{3}}$ lower than before or about 20%

(Recall, in the absence of an internal obstruction but under similar assumptions $\frac{V_1}{A_1} = C_d \sqrt{C_\omega}\, v_\omega$)

 2/17/00

Try $A_1 = 2A_2 = A_3$ Then

$$-5\hat{P}_B + \hat{P}_C = 4\, C_{\omega 1} \rho \frac{V_\omega^2}{2}$$

$$\hat{P}_B - 5\hat{P}_C = 4\, C_{\omega 2} \rho \frac{V_\omega^2}{2}$$

$$-24\hat{P}_B = \frac{\rho V_\omega^2}{2}\left(20\, C_{\omega 1} + 4\, C_{\omega 2}\right)$$

$$-\hat{P}_B = \frac{\rho V_\omega^2}{24}\left(10\, C_{\omega 1} + 2\, C_{\omega 2}\right)$$

$$= \frac{\rho V_\omega^2}{12}\left(5\, C_{\omega 1} + C_{\omega 2}\right)$$

$$\frac{V_1}{A_1} = C_d \left[\frac{2}{\rho}\left(C_{\omega 1} \rho \frac{V_\omega^2}{2} + \hat{P}_B\right)\right]^{1/2}$$

$$= C_d \left[\frac{2}{\rho}\left(C_{\omega 1} \rho \frac{V_\omega^2}{2} - \left(\frac{\rho V_\omega^2}{12}\left(5\, C_{\omega 1} + C_{\omega 2}\right)\right)\right)\right]^{1/2}$$

Let $|C_{\omega 1}| = |C_{\omega 2}| = C_\omega$

$$\frac{V_1}{A_1} = C_d\, V_\omega \left[C_{\omega 1} - \frac{1}{6}\left(5\, C_{\omega 1} + C_{\omega 2}\right)\right]^{1/2}$$

$$= C_d\, V_\omega \left[C_{\omega 1} - \frac{5}{6} C_{\omega 1} + \frac{1}{6} C_{\omega 2}\right]^{1/2}$$

$$= C_d\, V_\omega \left[\frac{1}{6} C_\omega + \frac{1}{6} C_\omega\right]^{1/2}$$

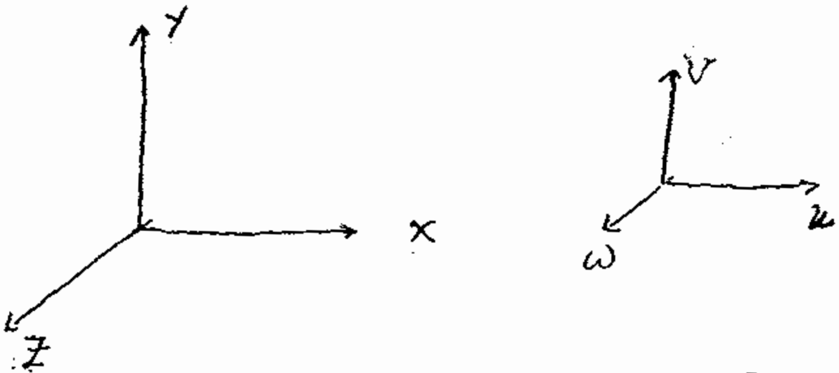
$$= C_d\, V_\omega \sqrt{\frac{1}{3} C_\omega} \checkmark$$

This is $\sqrt{\frac{1}{3}}$ lower than before or about a 40% reduction.

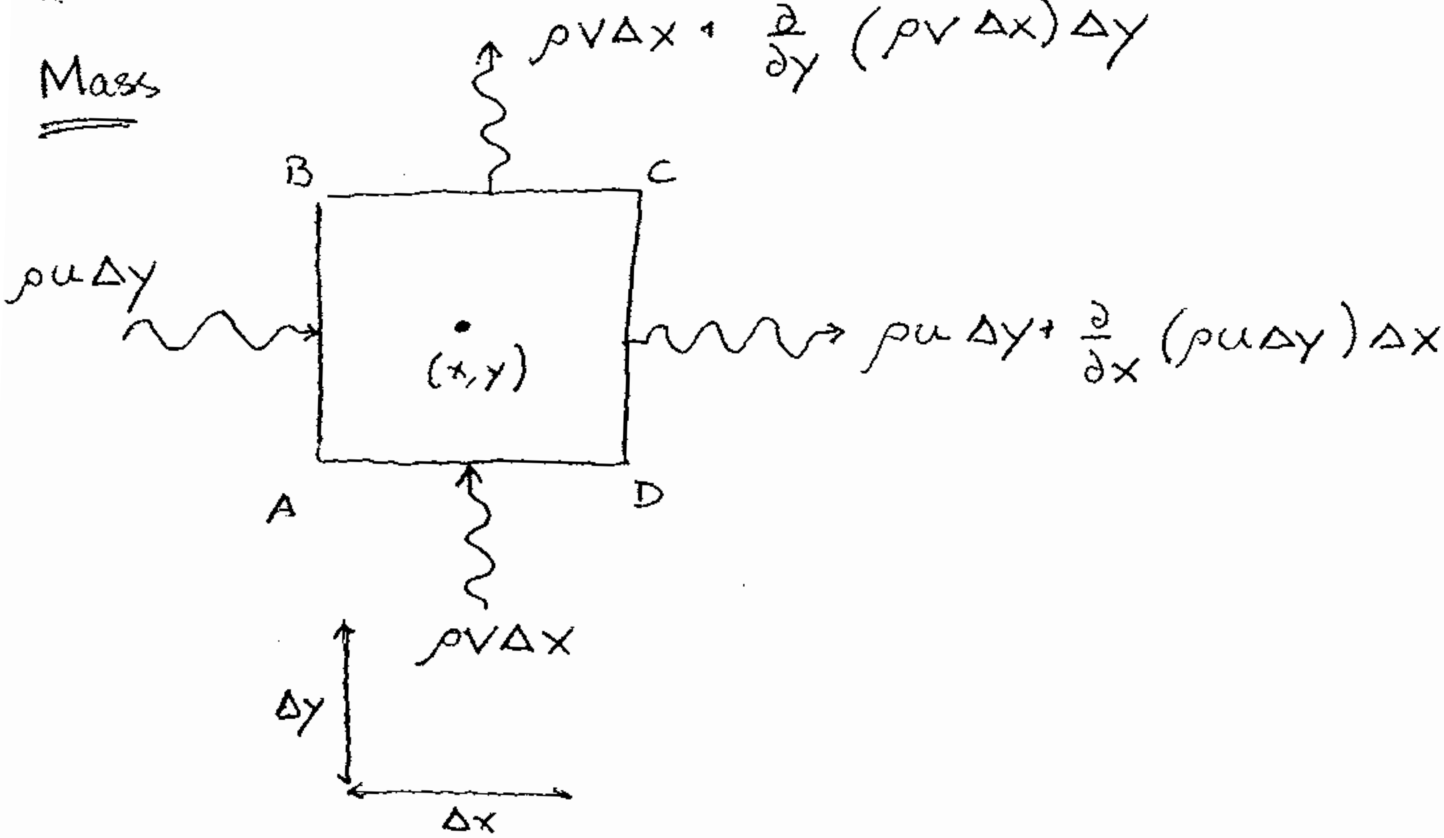
2/22/00

Conservation of momentum.

see Etheridge and Sandberg, pp 287-291



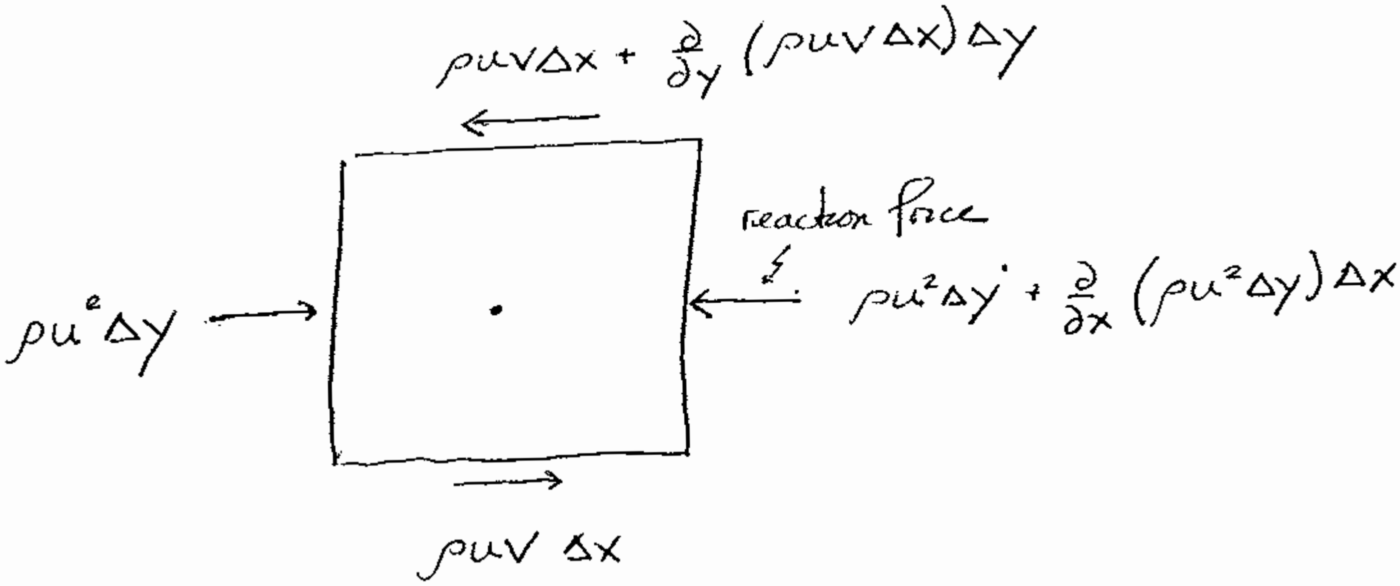
Mass



$$\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v = 0$$

If no source or sink in the control volume

Linear Momentum



Net flux of linear momentum across the surface Δy is

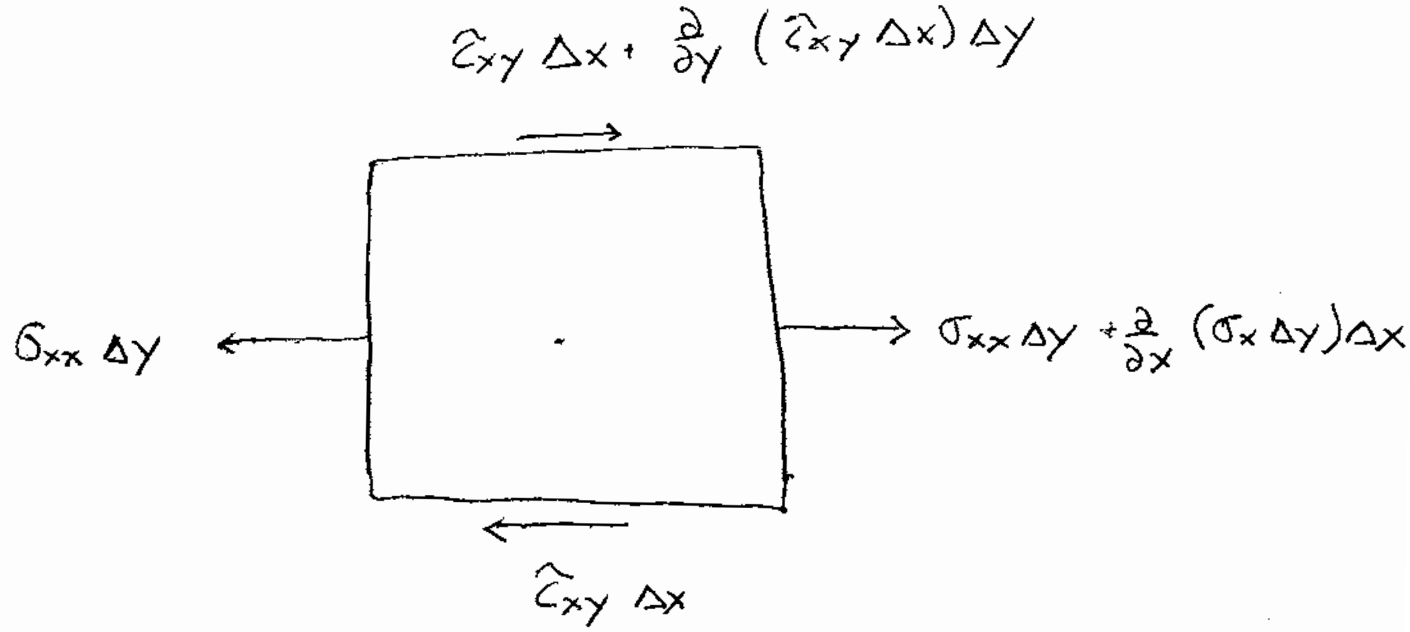
$$\frac{\partial}{\partial x}(\rho u^2)\,\Delta x\,\Delta y$$

Net flux of linear momentum across Δx is

$$\frac{\partial}{\partial y}(\rho u v)\,\Delta x\,\Delta y$$

Flow of momentum equals forces acting on the control volume (normal, shear, acceleration)

2/22/00



Surface Forces acting on the volume (x components only)

Net force in the x direction

$$\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right)\Delta x \Delta y$$

Recall $F = ma$:

Apply it

$$\frac{\partial}{\partial t}(\rho u)\Delta x \Delta y = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right)\Delta x \Delta y - \frac{\partial}{\partial x}(\rho u^2)\Delta x \Delta y$$
$$- \frac{\partial}{\partial y}(\rho u v)\Delta x \Delta y$$

$$\frac{\partial}{\partial t}\rho u = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right) - \frac{\partial}{\partial x}\left(\rho u^2\right) - \frac{\partial}{\partial y}\left(\rho u v\right)$$

$$\rho \frac{du}{dt} + u\left[\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}$$

where $\frac{d}{dt} \hat{=} \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$, the total derivative

For two-dimensional flow,

$$\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \quad \text{by conservation of mass}$$

$$\boxed{\rho \frac{du}{dt} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}}$$

The stresses can be related to the pressure, p, and the derivatives of the local flow field

$$\sigma_{xx} = -p + \mu\left(2\frac{\partial u}{\partial x} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right)$$

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

These rely on the Stokes hypothesis that stress (excluding the isotropic component due to p) is proportional to the local rate of change of strain.

2/29/00

Consider the flow to be incompressible (ρ is constant) with constant viscosity μ. Then

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{by conservation of mass}$$

$$\sigma_{xx} = -P + 2\mu \frac{\partial u}{\partial x}$$

$$\tau_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

Momentum in the x and y directions

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

These are the Navier-Stokes equations for an incompressible fluid.

2/29/00

Consider conservation of momentum in the x direction for steady flow $\left(\frac{\partial}{\partial t} = 0\right)$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

where $\nu = \frac{\mu}{\rho}$

If the flow is one dimensional and inviscid,

$$u \frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$\frac{\partial}{\partial x} u^2/2 = -\frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$\frac{\rho u^2}{2} + P = \text{constant}$$ one form of Bernoulli's Eq.

Now, take a key step and make the conservation equation dimensionless

Define $X^* = \frac{X}{L_{ref}}$ $u^* = \frac{u}{V_{ref}}$ $P^* = \frac{P}{\rho V_{ref}^2}$

$Y^* = \frac{Y}{L_{ref}}$ $v^* = \frac{v}{V_{ref}}$

Then $$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial P^*}{\partial x^*} + \frac{\nu}{V_{ref} L_{ref}} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right)$$

All terms are dimensionless, which means than $\frac{\nu}{V_{ref} L_{ref}}$ is also dimensionless

Punch line !! Our model results should match those of the full-size house if

$$\left.\frac{V_{ref} L_{ref}}{\nu}\right|_{model} = Re|_{model} = \left.\frac{V_{ref} L_{ref}}{\nu}\right|_{house} = Re|_{house}$$

Note we're using air for both, so $\nu_{model} = \nu_{house}$

To match the Reynolds numbers, we insist on

$$V_{ref\text{-}model} \, L_{ref\text{-}model} = V_{ref\text{-}house} \, L_{ref\text{-}house}$$

$$V_{ref\text{-}model} = V_{ref\text{-}house} \frac{L_{ref\text{-}house}}{L_{ref\text{-}model}}$$

$$= V_{ref\text{-}house} \cdot \eta$$

where η is our model scale (i.e., 10:1, 20:1)

Note we use the linear dimension of the building as a very reasonable length scale, or L_{ref}.

Let's use the free-stream air speed as our velocity scale

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Example $V_{ref\text{-}house}$ = 2 m/s (appropriate for Beijing)

$n = 20$

Then, to preserve Reynolds similitude,

$V_{ref\text{-}model}$ = 40 m/s, or about 80 mph!

This problem plagues much wind-tunnel work.

If $V_{ref\text{-}model}$ is smaller, we can still predict ACH_{house}, but our prediction may be inaccurate. Why? Two reasons:

1. Recall the orifice equation

$$\dot{V} = A\, C_d \sqrt{\frac{2\Delta p}{\rho}}$$

C_d is ~ constant for turbulent flow but $\propto \sqrt{\Delta p}$ for laminar flow. What distinguishes these two flow regimes? Re! For flow through a window, use the window width as the scaling length

$$Re \approx \frac{1\ m/s \times .04\ m}{1.5 \times 10^{-5}\ m^2/s} \approx 2.7 \times 10^3$$

At the laminar/turbulent flow boundary

2. The structure of the air jet at a window (or door) also depends on Re.

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Air Jets

See Awbi, Ventilation of Buildings

I. Motivation

- Sharpen our understanding of airflow through windows, with a view toward better understanding airflow distribution in rooms
- Over what distance, in from a window, does a jet retain its jet-like character?
- How much does it spread out?
- What about entrainment? If there is cross-ventilation, is it entrainment or diffusion that exchanges air and removes heat?

II Definition and Structure

- Definition: no solid boundaries
  static pressure within jet equals that of surroundings
- There is a shear layer around its boundary, as a result of the discontinuity in velocity
- The thickness of the free shear layer increases with axial distance until the central region of the jet, called the potential core, is consumed.

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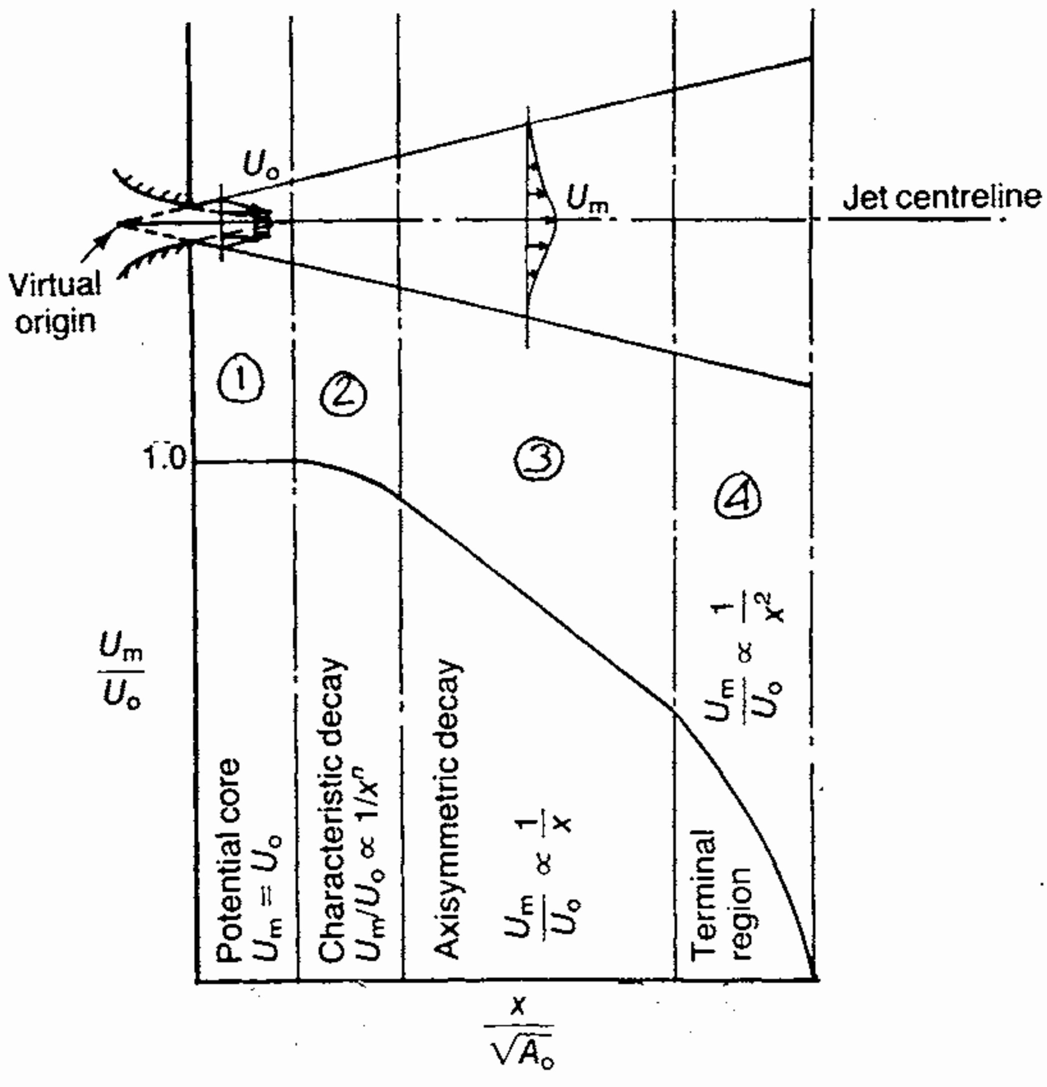
- Downstream of the core, the flow becomes more turbulent and the centerline velocity decreases

III Detailed look at structure



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1. Potential core

- Mixing of jet fluid with surrounding fluid is not complete
- Length depends on type of opening and turbulence of air supply but usually extends 5-10 equivalent opening diameters
- Centerline velocity U_m equals U_o, the supply velocity.

2. Characteristic decay region

- After consumption of the potential core by the free shear layer, the centerline velocity gradually decreases

$$\frac{U_m}{U_o} \propto \frac{1}{x^n} \qquad 0.33 \leq n \leq 1$$

- The length over which this functional form holds and the value of n depend on the shape of the supply opening. This region is associated with openings with large aspect ratios (ℓ / h) and is negligible for square or circular openings.

3. Axisymmetric decay region

- Highly turbulent flow generated by viscous shear at the edge of the shear layer

- For 3-D jets, this is the "fully developed flow region" where the spread angle is a constant, the value of which depends on the geometry of the opening.
- This is the predominant region for jets from low-aspect ratio openings but is insignificant for high-aspect ratio openings
- The region extends to ≈ 100 equivalent diameters
- $\dfrac{U_m}{U_o} \propto \dfrac{1}{x}$

4. Terminal region

- Region of rapid diffusion. The jet becomes indistinguishable from the surrounding air
- $\dfrac{U_m}{U_o} \propto \dfrac{1}{x^2}$

5 Summary of main regions

Plane jet (2-D)	Axisymmetric jet (3-D)
potential core	potential core
characteristic decay	axisymmetric decay

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IV Principles for analysis

A. Momentum across the jet is constant

$$M_x = M_0 = \int_0^A \rho u^2 \, dA$$

$$M_0 = \rho U_0^2 A_0$$

B. Profile of u across the jet has the same shape at different axial distances from the supply opening

C. The static pressure across the jet is constant and equal to the surrounding pressure

Applying these assumptions and a suitable turbulence model, it is possible to generate solutions for:

- velocity profiles (radial)
- decay of the centerline velocity
- Flow entrainment

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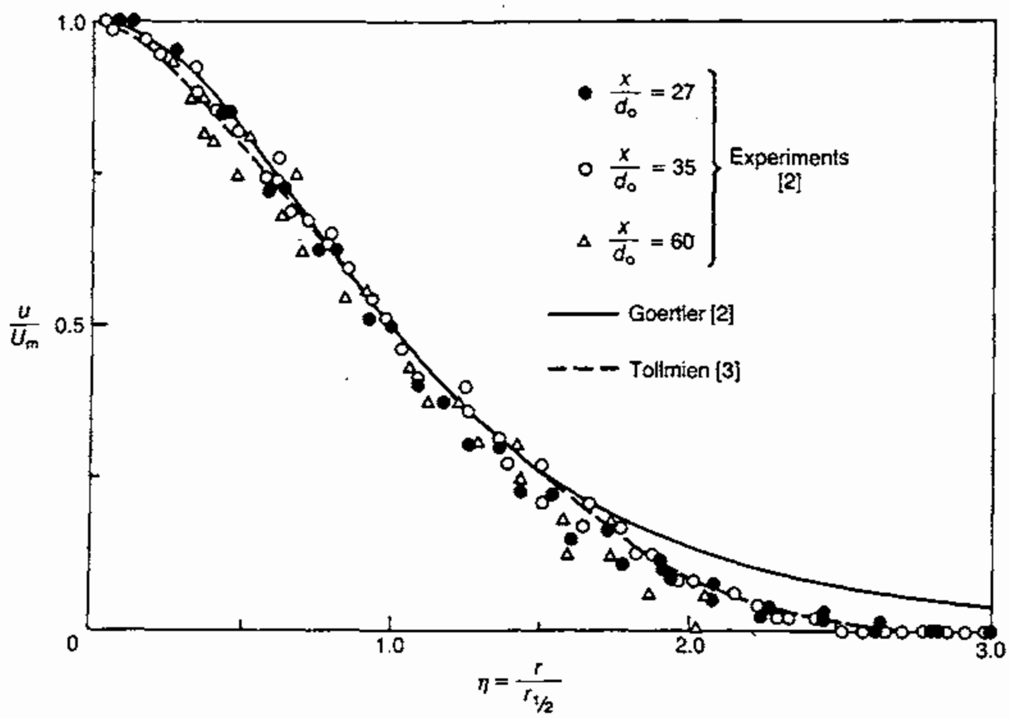
V Circular jets

A. Velocity profile



$r_{1/2}$: radius at which $\frac{U}{U_m} = 0.5$

$r_{1/2} = 0.1x$ (for circular jets)

B. Centerline velocity decay

$$\frac{U_m}{U_o} = \frac{K_v}{(x/d_o)}$$

K_v : throw constant

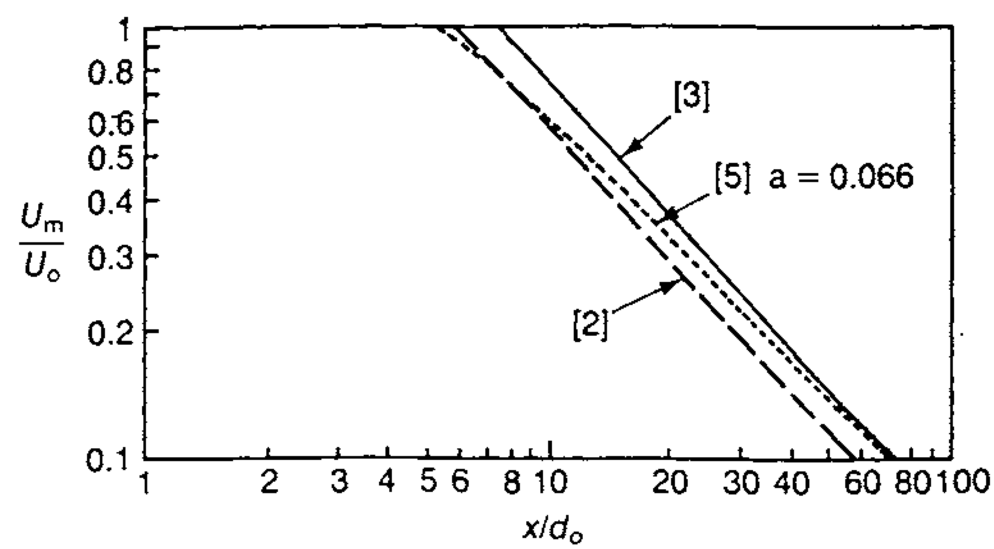
d_o : effective diameter of opening

K_v : 7.32 Tollmien
6.3 Rajaratnam
5.75 Goertler
} theory

Empirical relationship

$$\frac{U_m}{U_o} = \frac{0.48}{(ax/d_o) + 0.145}$$

a : f (opening type)



C. Spread angle

The spread angle is the angle of the outer envelope of the jet where the velocity is zero. It is larger for jets having higher turbulence and swirl at the opening.

α = 25-27° convergent nozzle
29 cylindrical tube
:
85 swirl diffuser

D. Flow entrainment

Empirical relation for the volumetric flow entrained by a circular jet

$$\frac{Q}{Q_o} = 0.32 \left(\frac{x}{d_o} \right)$$

where

Q = volumetric Flow at x

Q_o = supply flow rate

Entrainment velocity (transverse flow at jet boundary)

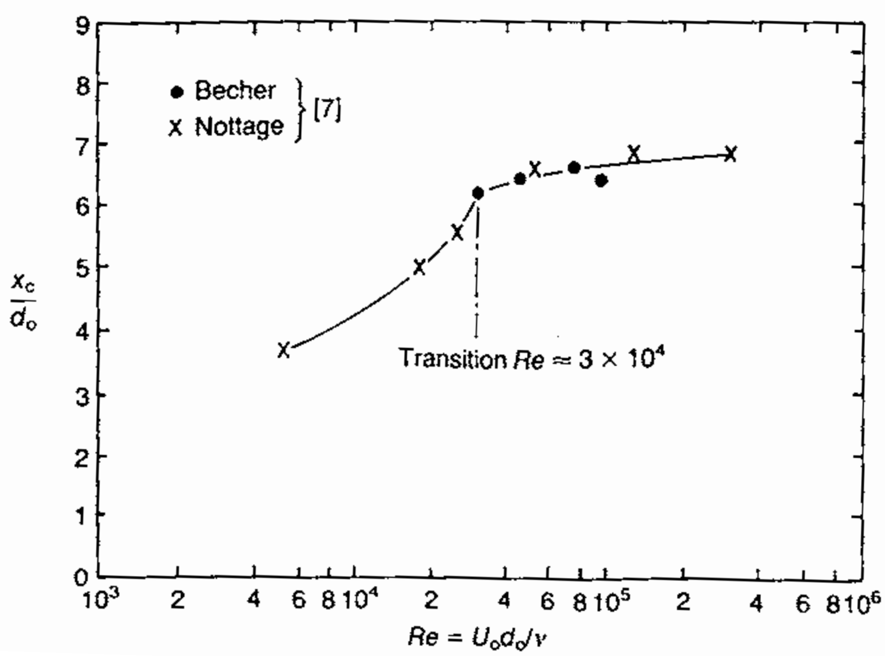
$$V_e = C_e U_m$$

where

C_e = entrainment coefficient
= 0.026 for a circular jet

E. Length of core region

Influenced by Re and turbulence intensity at outlet



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VI Example (!!) Circular jets for flow through windows in a Chinese apartment

A Room geometry.

$$0 \leq x \leq 6\ m$$

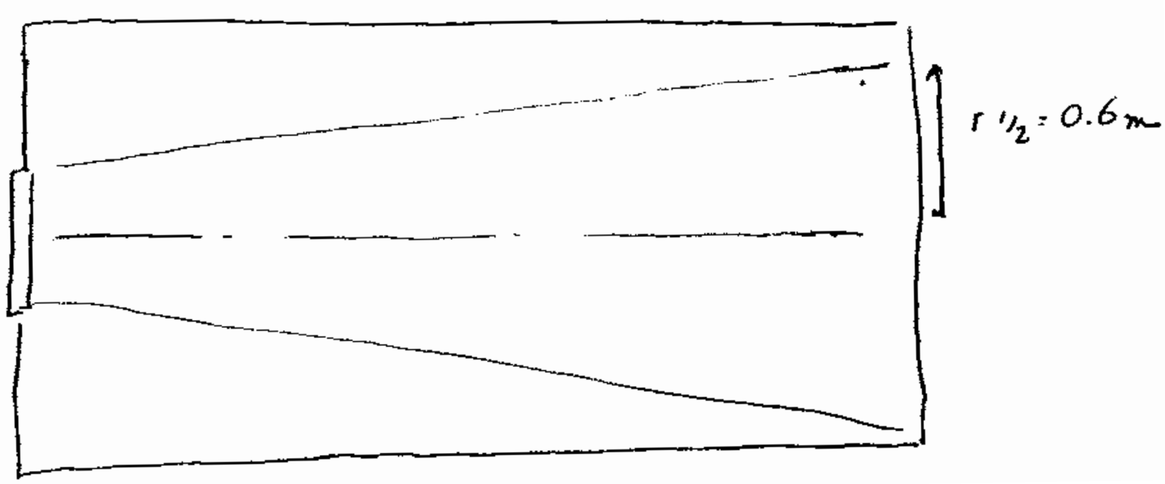
B Window

$$d_0 = 0.6\ m$$

C. Spread

$$r_{1/2} = 0.1\, x$$

At far end of room, $r_{1/2} = 0.6$ or $2 \times r_0$



D. Centerline velocity decay

$$\frac{U_m}{U_0} = K_v / (x/d_0)$$

Take $K_v \approx 6$

Then $\frac{U_m}{U_0} = 1$ for $\frac{x}{d_0} = 6$

This defines end of the potential core region, after which U_m decreases. (Check Re later)

$U_m = U_0 \qquad 0 \le x \le 6\, d_0$

$U_m = 0.6\, U_0$ at $x = 10\, d_0$, the rear wall

E. Flow entrainment

At the rear wall, $\frac{Q}{Q_0} = 0.32 \left(\frac{x}{d_0}\right) = 3.2$

F. Re Is the flow really turbulent?

$U_0 = 1$ m/s

$d_0 = 0.6$ m

$\nu = 1.5 \times 10^{-5}\ m^2/s$

$Re \approx 4 \times 10^4$

Turbulent !! Gives near-maximum extent of core.