**basic elements**

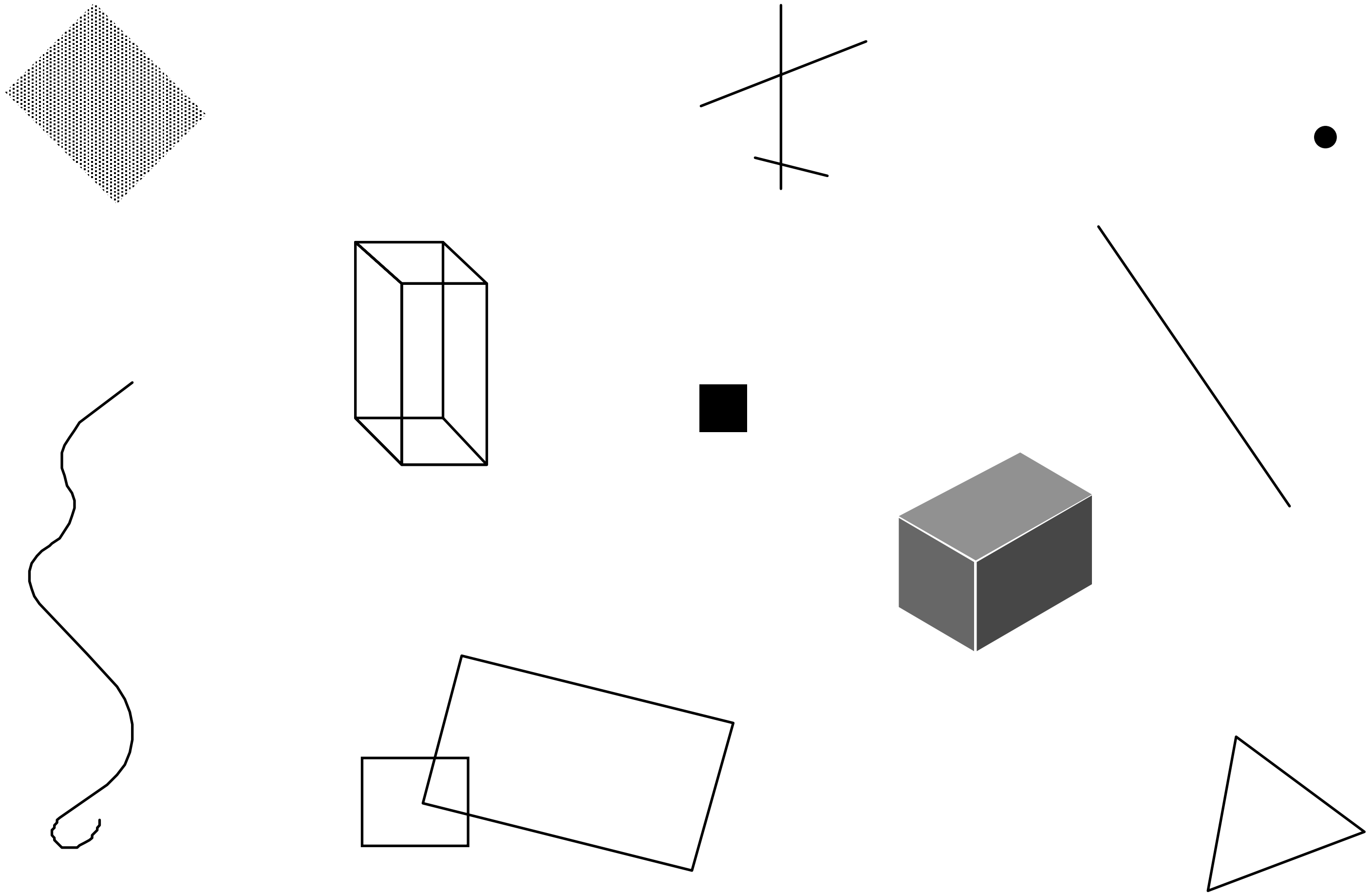
space

| | points | lines | planes | volumes |
|---|---|---|---|---|
| 0D | √ | | | |
| 1D | √ | √ | | |
| 2D | √ | √ | √ | |
| 3D | √ | √ | √ | √ |

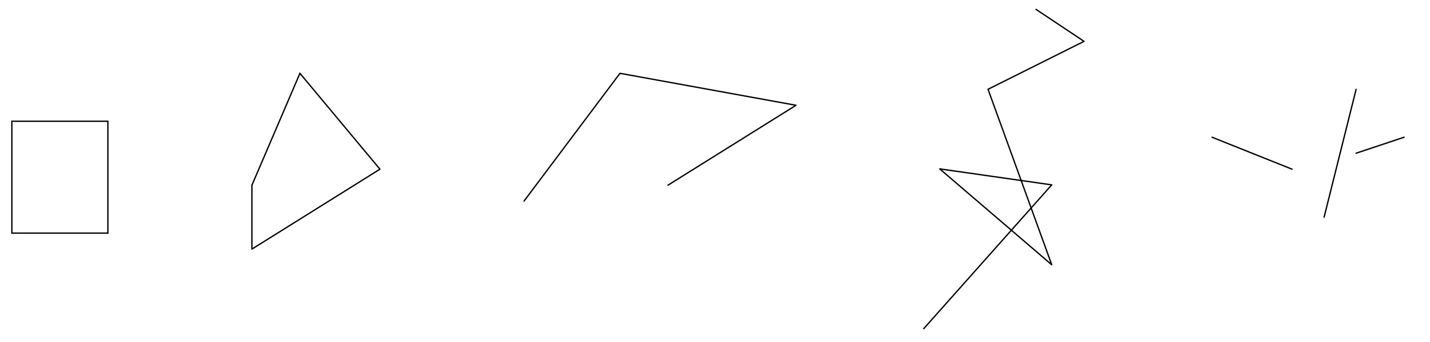
**shape**

arrangement of basic elements in space

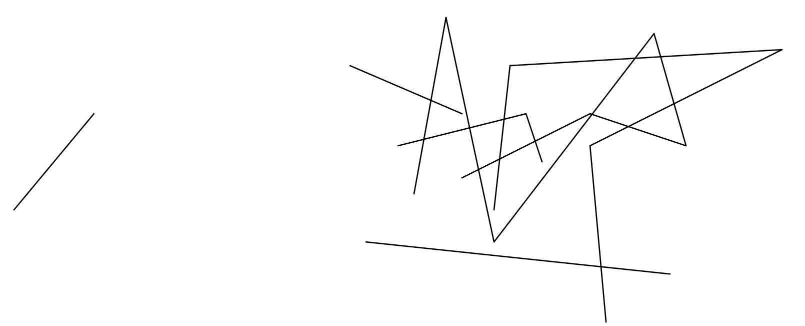
# shapes



# shapes made up of straight lines in 2D space (plane)



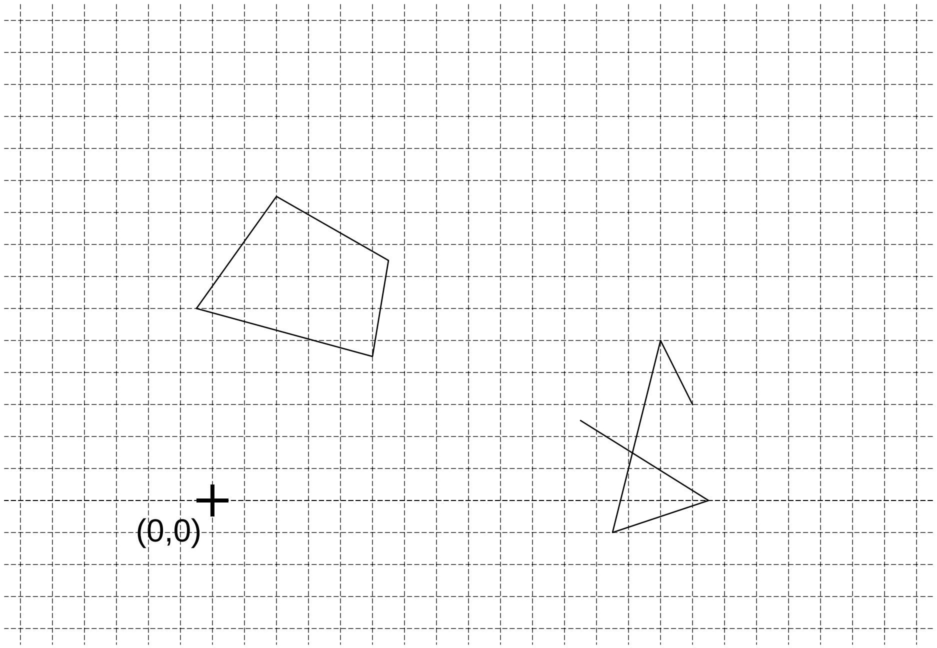
lines can be arranged in any way to make a shape



any number of lines can be arranged to make a shape

**empty shape**

**0**




shapes have a position, orientation, size in a coordinate system

## arithmetic of shapes

**relations**

part of (≤)
equality (=)

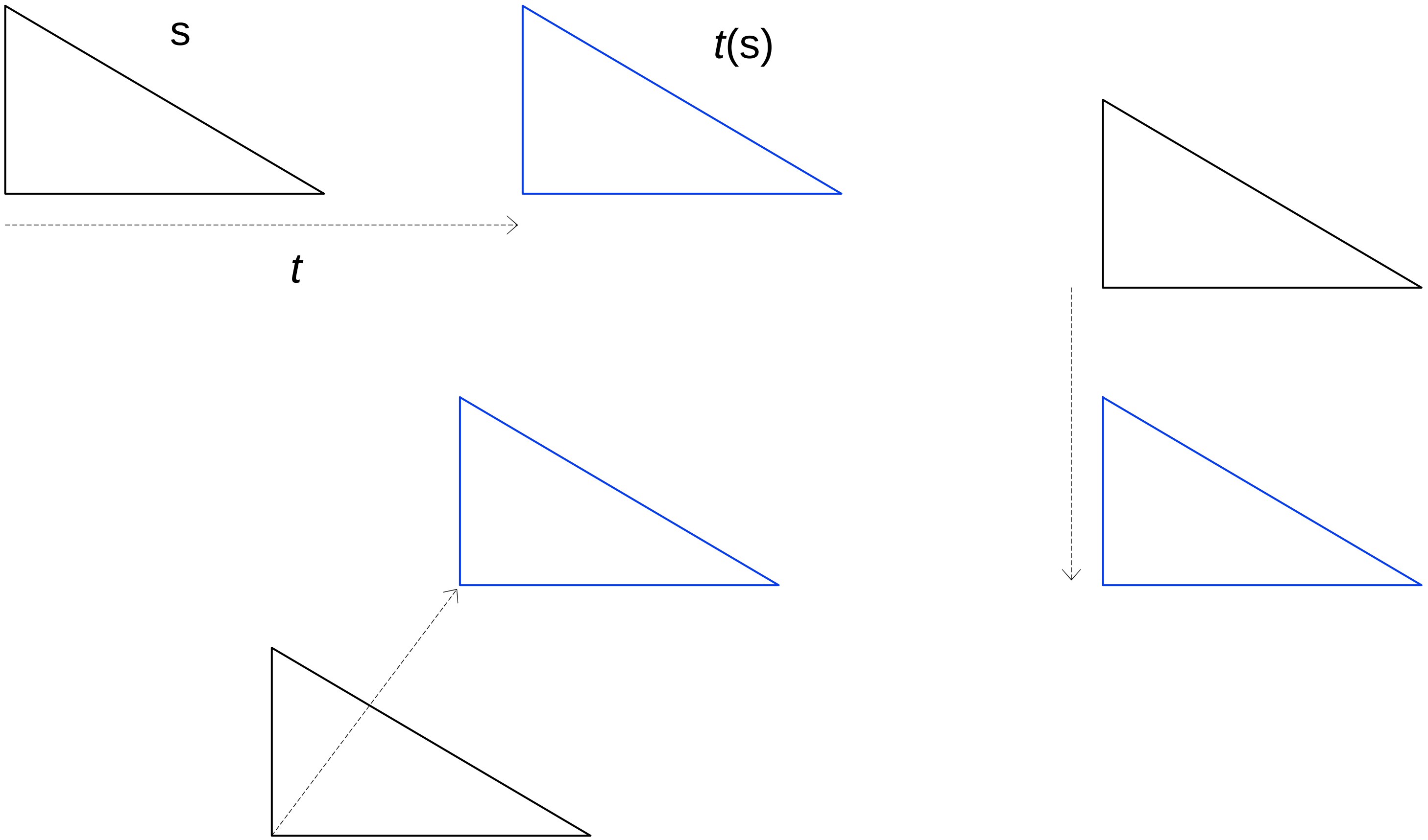
**operations**

addition (+)
subtraction (-)
transformations (t)

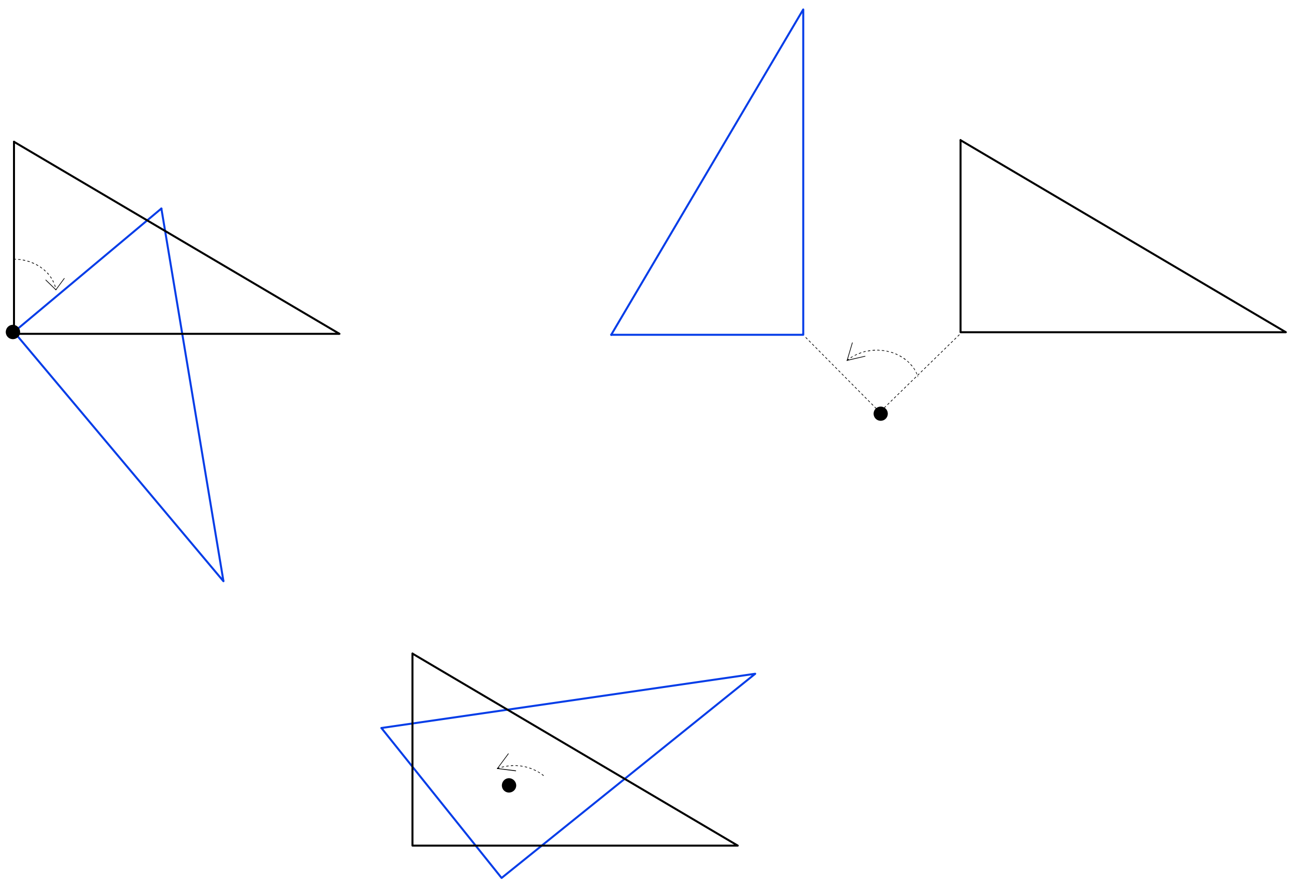
## 2D transformations

translation
rotation
reflection
glide reflection
scale

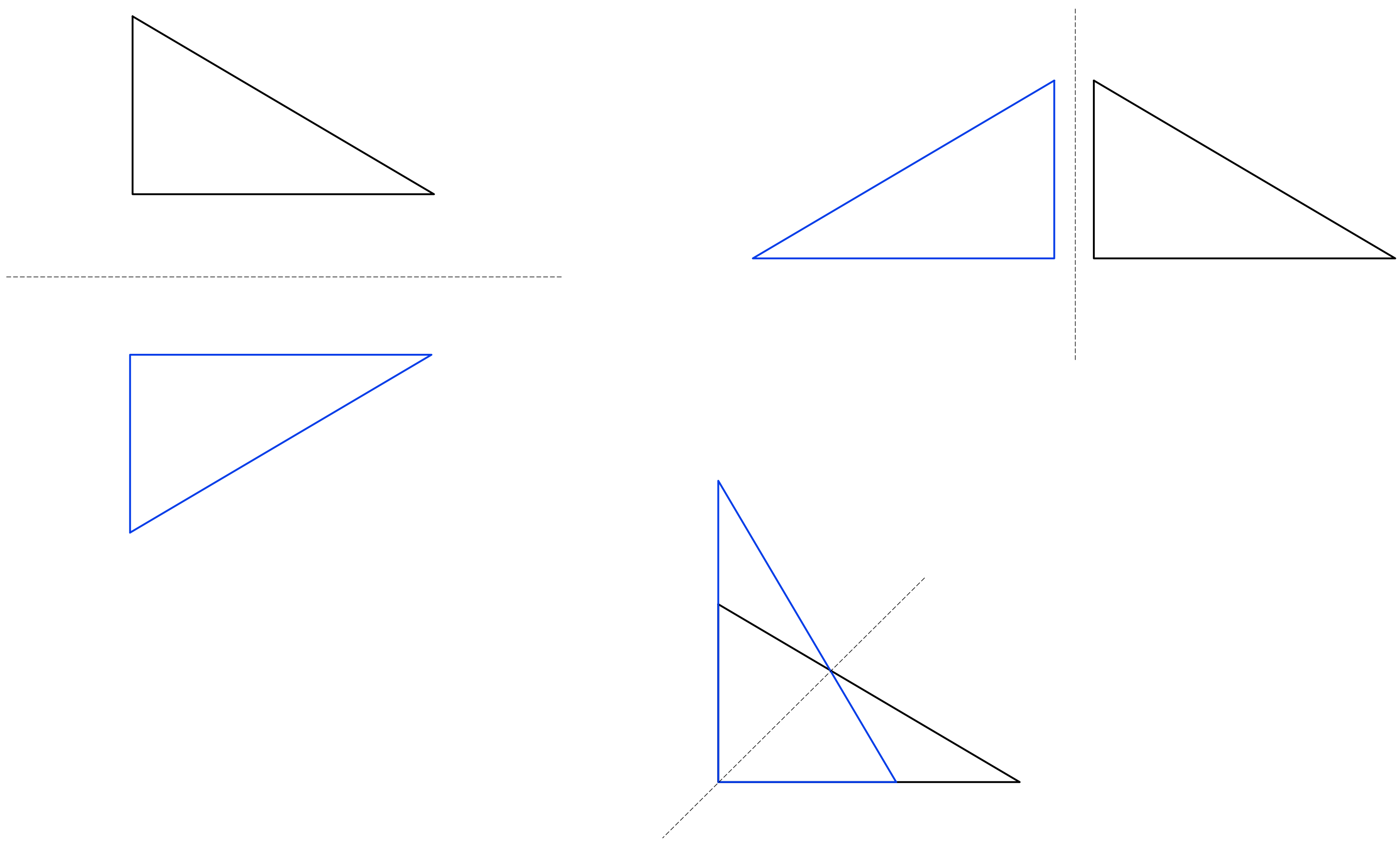
# translation



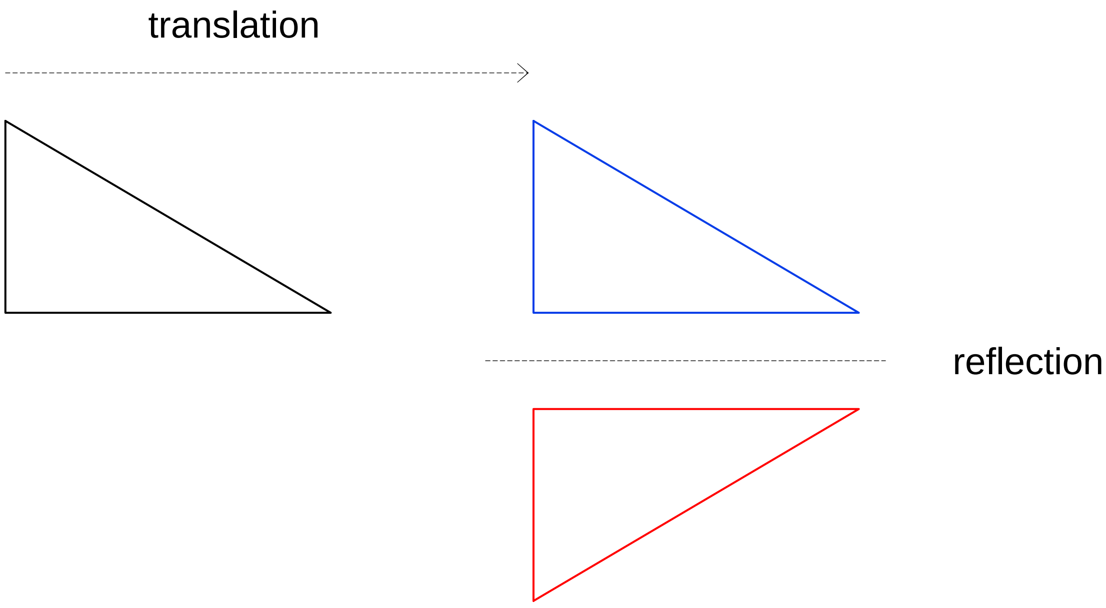
# rotation



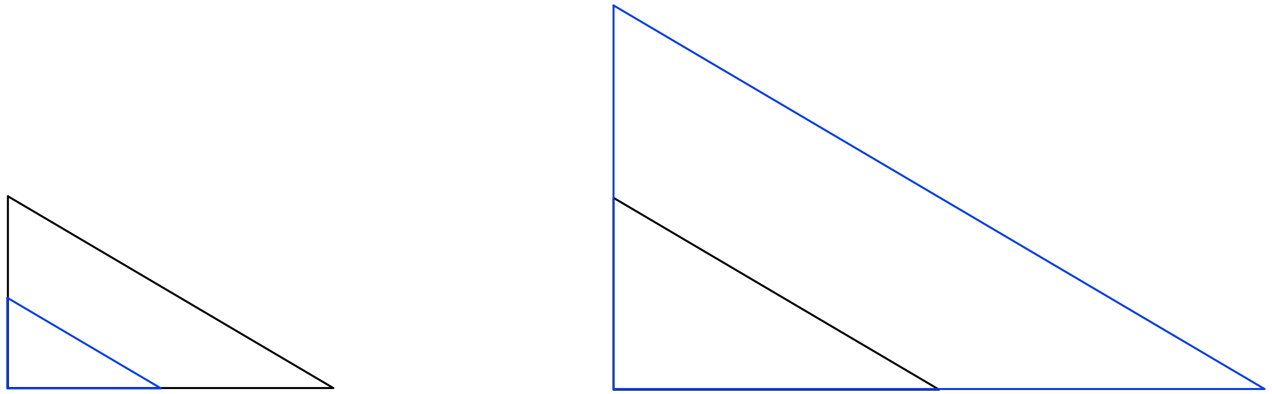
## reflection



# glide reflection

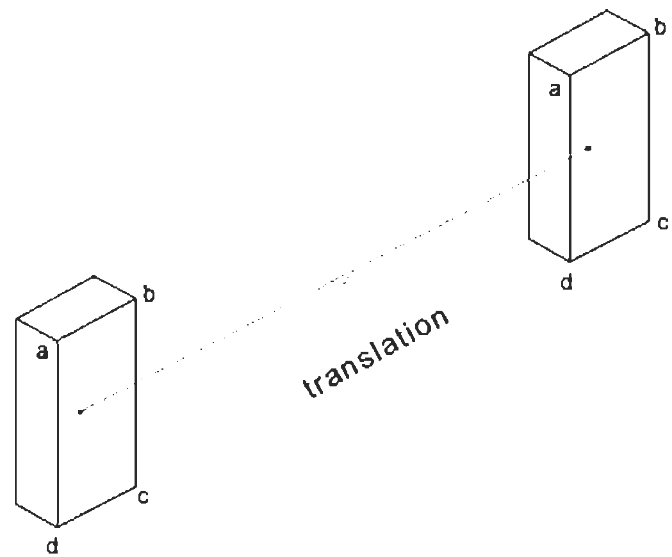


# scale

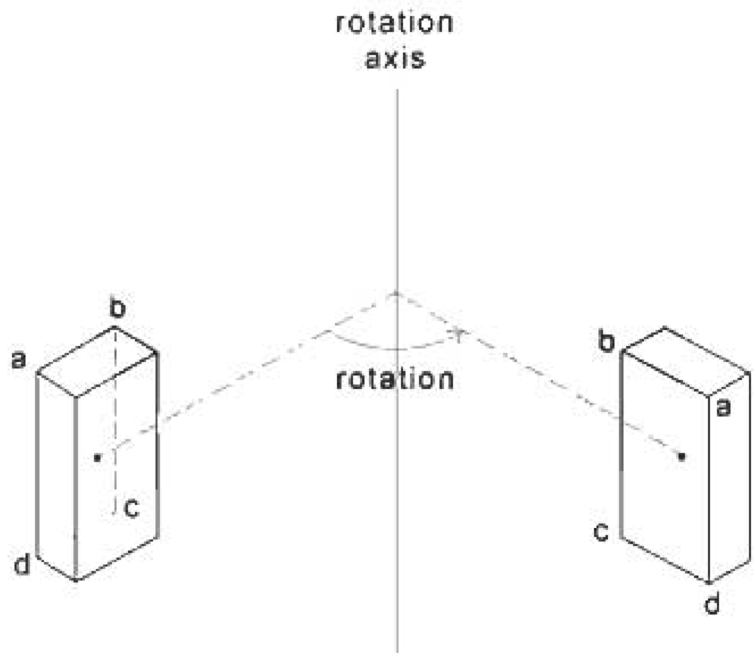


**3D transformations**

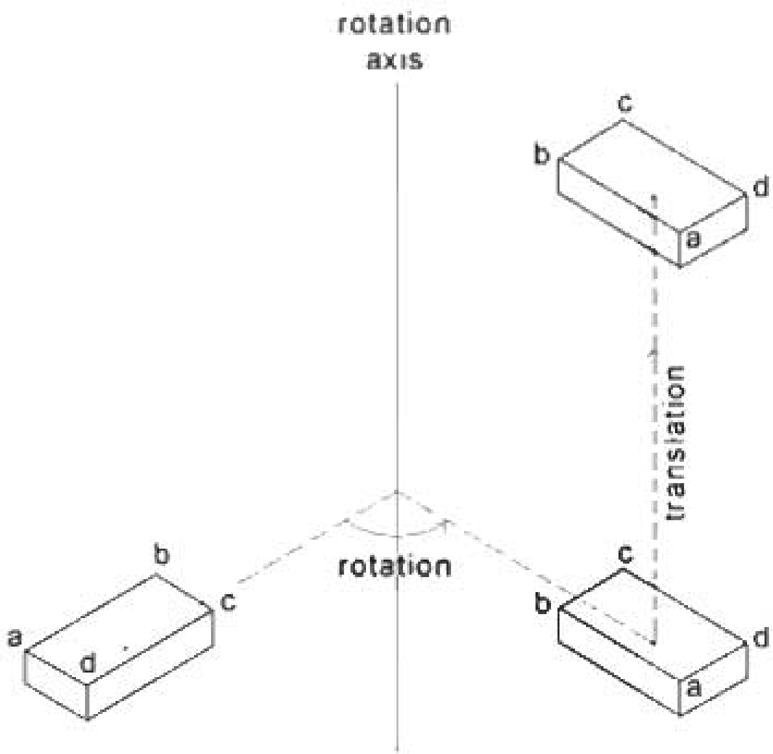
translation
rotation
screw rotation
reflection
glide reflection
rotor reflection
scale



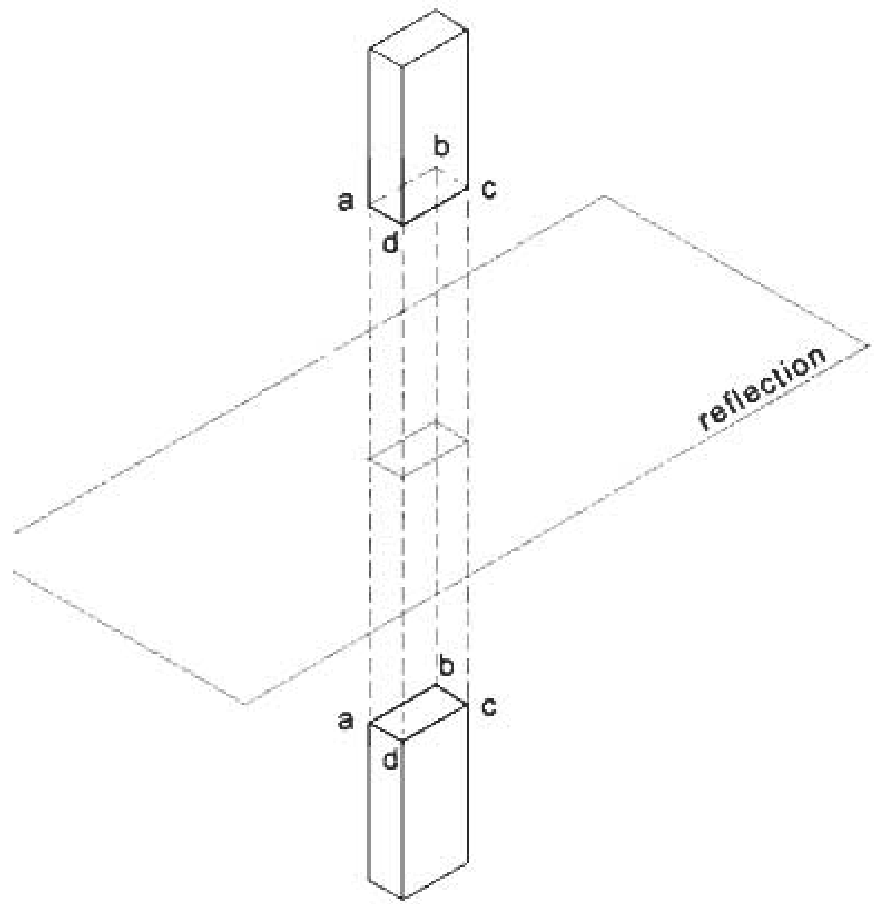

TRANSLATION



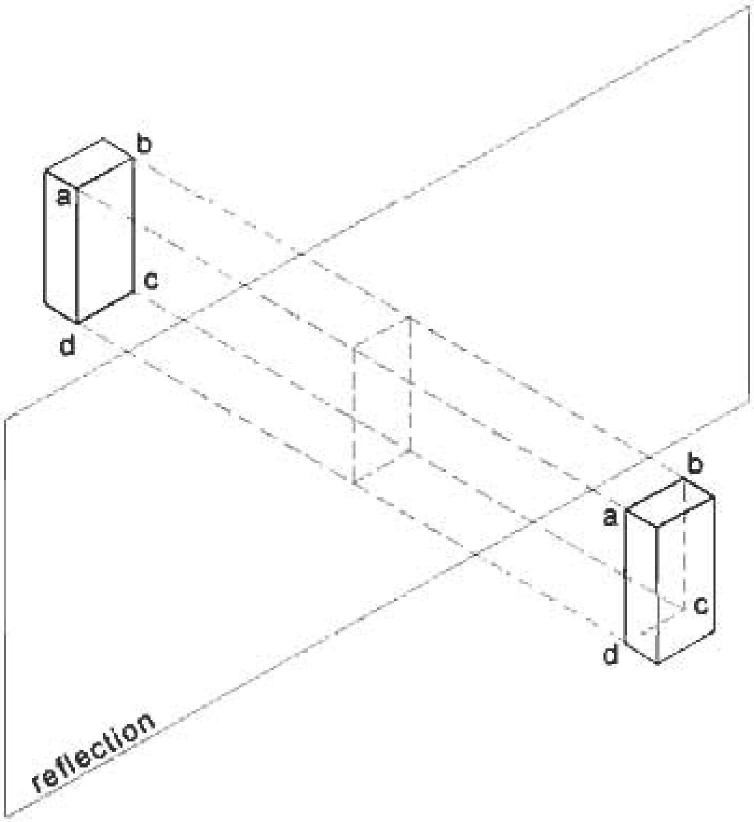

ROTATION



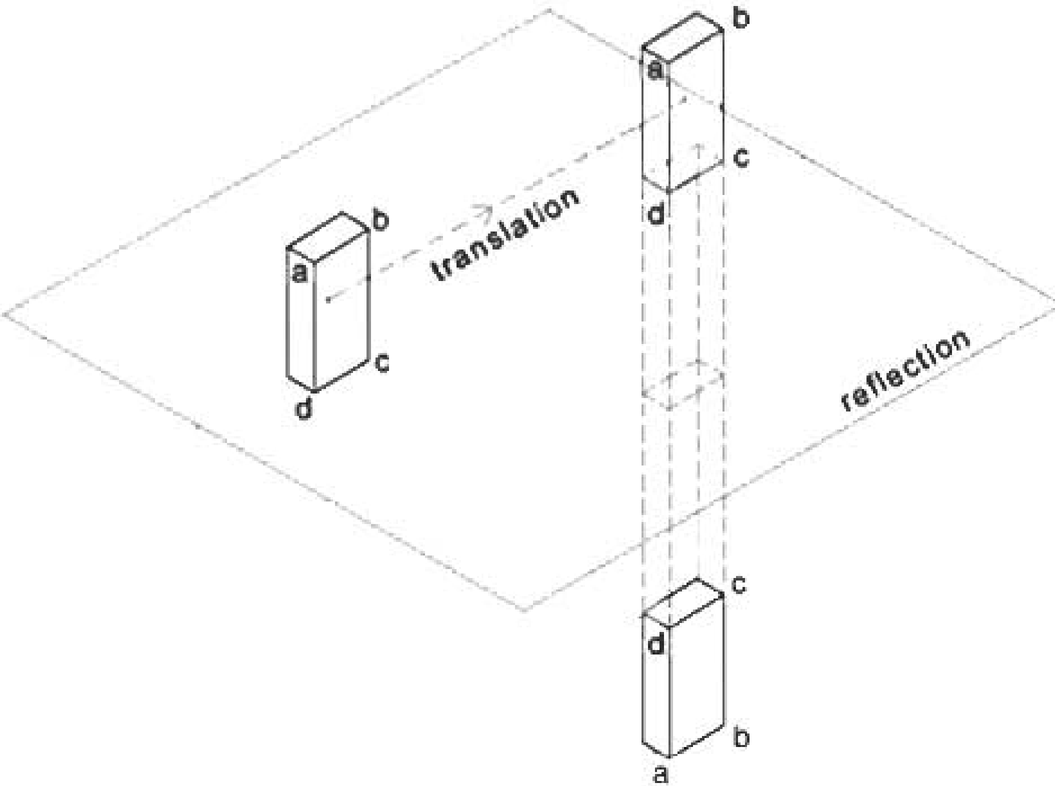

SCREW ROTATION



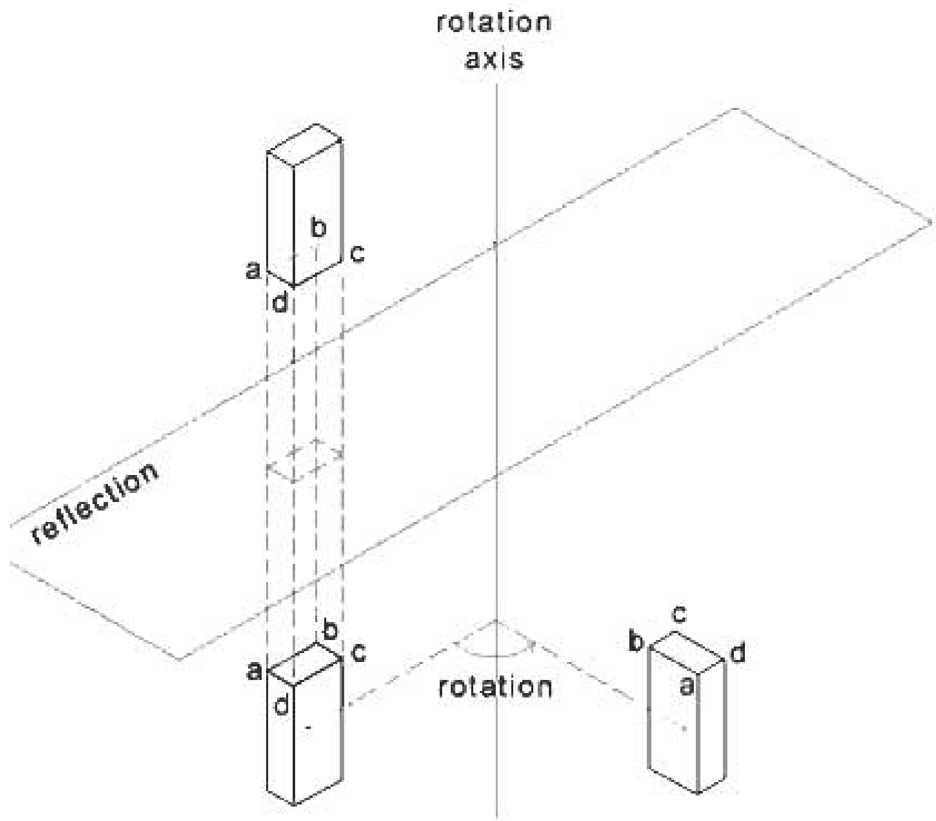

REFLECTION



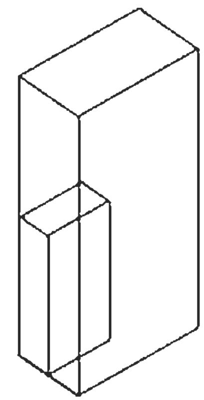

REFLECTION




GLIDE REFLECTION




ROTOR REFLECTION



scale (x2)

SCALE

**Question**

Given a vocabulary of shapes and operations of +, -, t, what designs can be created?

# Recursion

A repetitive (seemingly circular) process with the results of each repetition depending on the results of the previous repetition.

**Base**
Something exists

**Recursion step**
If something exists, then other things can be produced from it in a clearly specified way.

**shape grammar**

| | |
|---|---|
| base: | **initial shape** |
| recursion: | **rules** |

**rule**

$$X \rightarrow Y$$

## computation
## (derivation)

$$s_1 \Rightarrow s_2 \Rightarrow s_3 \Rightarrow s_4 \Rightarrow \ \dots \ \Rightarrow s_n$$

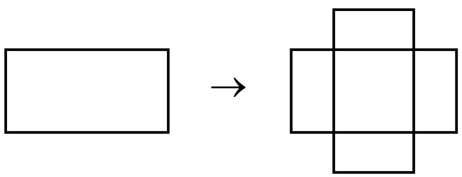
initial shape

rule applications

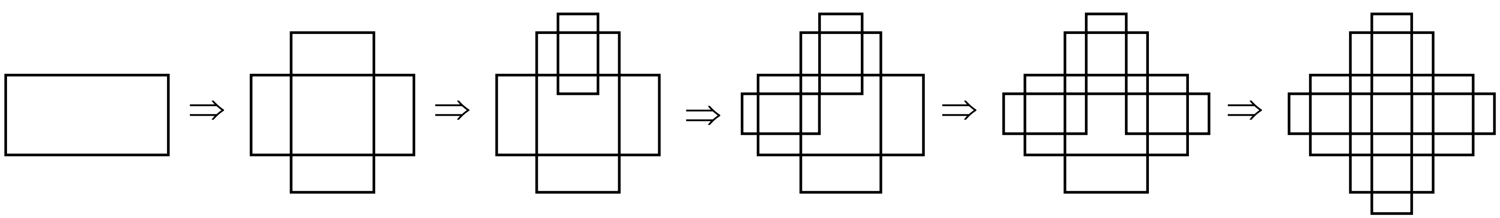
# nondeterminism

$$\begin{tikzcd}
 & & & s_4 & & \\
s_1 \arrow[r, Rightarrow] & s_2 \arrow[r, Rightarrow] & s_3 \arrow[r, Rightarrow] \arrow[ur, Rightarrow] \arrow[dr, Rightarrow] & s_5 \arrow[r, Rightarrow] & \cdots \arrow[r, Rightarrow] & s_n \\
 & & & s_6 & &
\end{tikzcd}$$

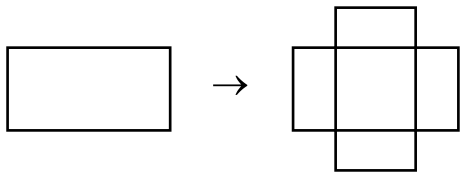
design idea



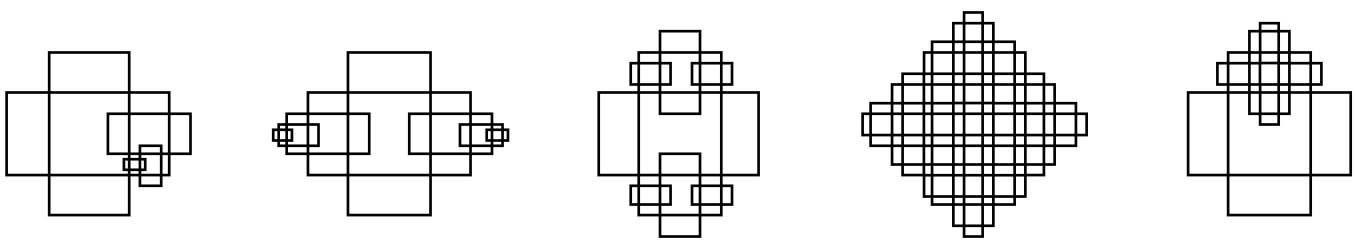
rule



computation



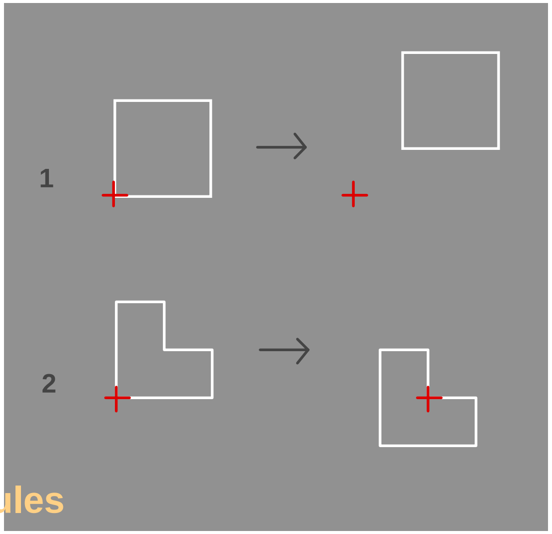
rule



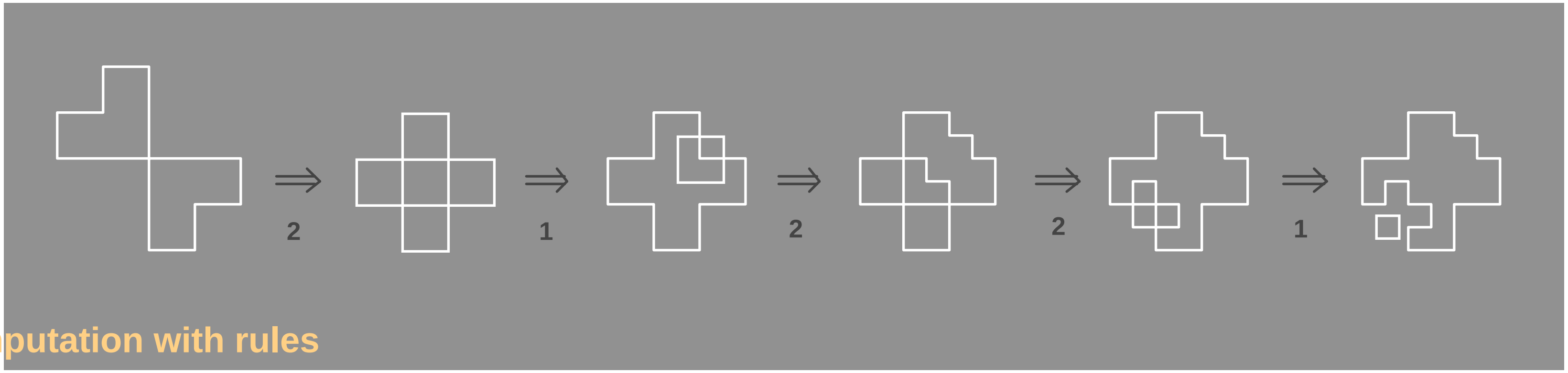
other designs generated by the rule

**shape emergence**

shape not predefined,
one that arises from the interaction of original shapes




rules




computation with rules

**shape rule: X $\rightarrow$ Y** **design**

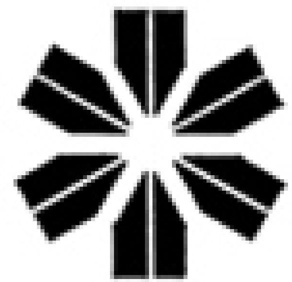
**A rule applies to a design:**
whenever there is a transformation *t* that makes the left-side X
a part of the design: $t(X) \leq$ design

**To apply the rule:**
first subtract the transformation *t* of the left-side X from the design,
and then add the same transformation *t* of the right-side Y to the design.

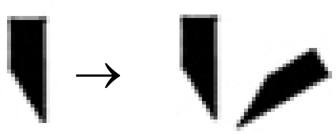
**The result of applying the rule is a new design:**
new design = [design - $t(X)$] + $t(Y)$

**rules for:**

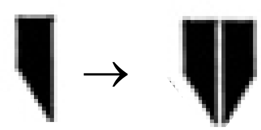
**kinds of designs**



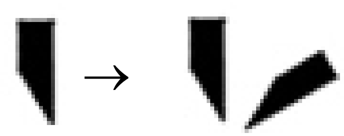
$X \rightarrow X + \textit{rotation}(X)$



$X \rightarrow X + \textit{reflection}(X)$

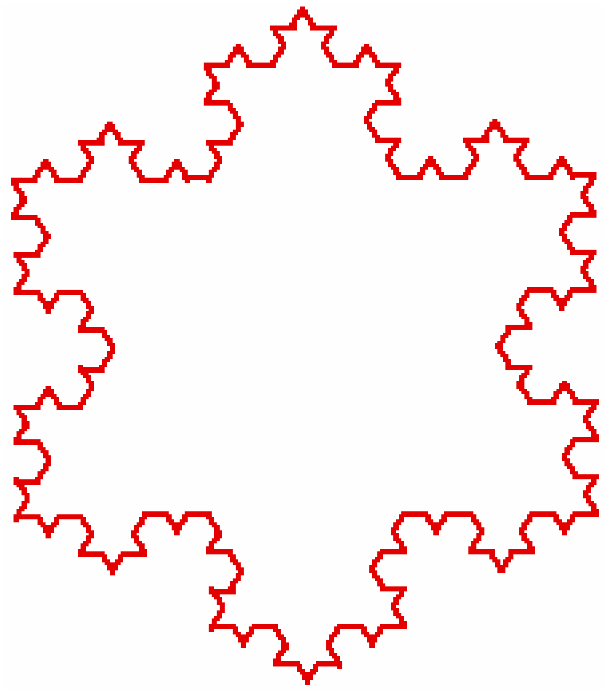


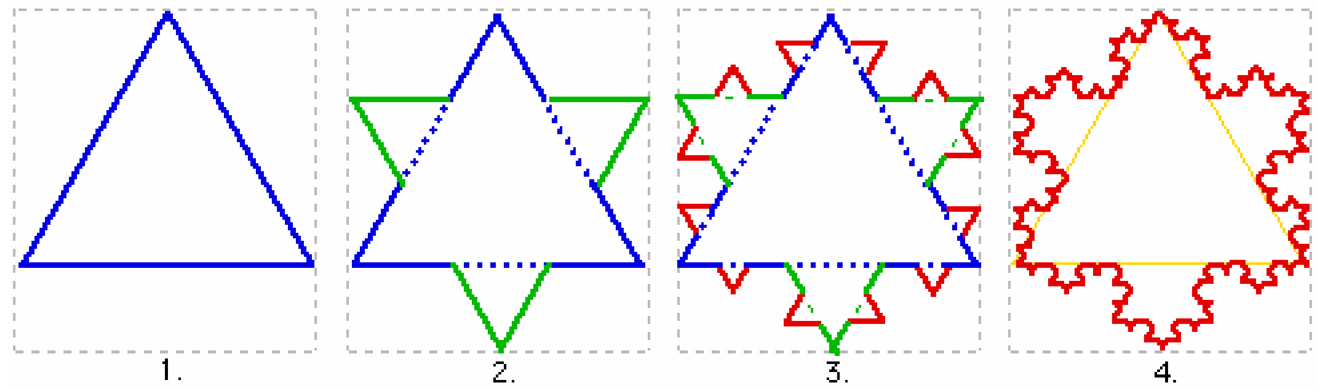
$X \rightarrow X + \textit{rotation}(X)$



**symmetry rule**

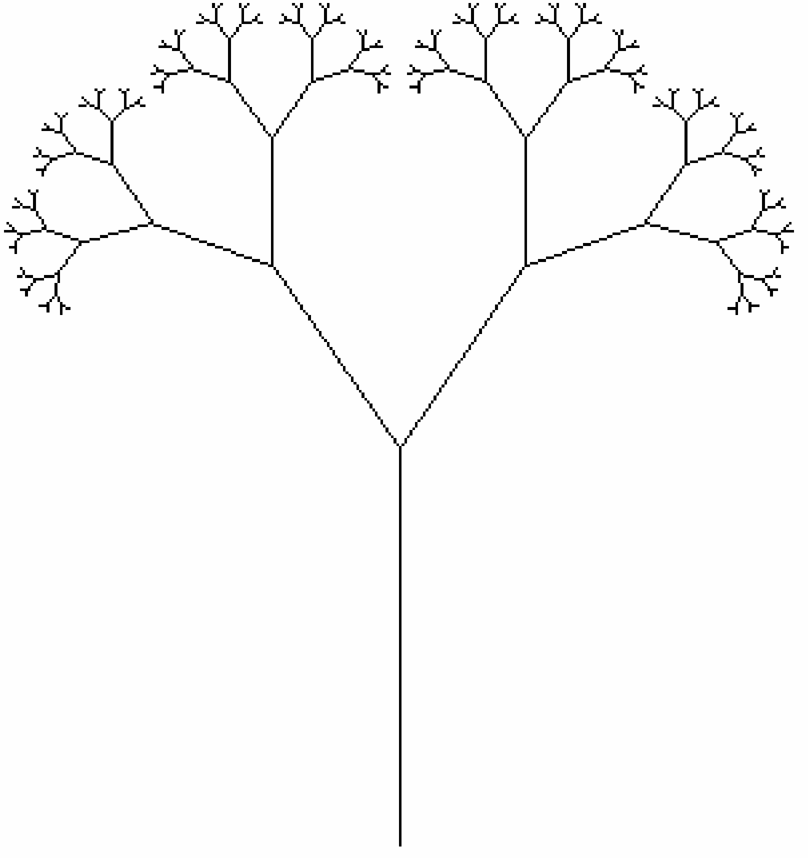
$$X \rightarrow X + t(X)$$



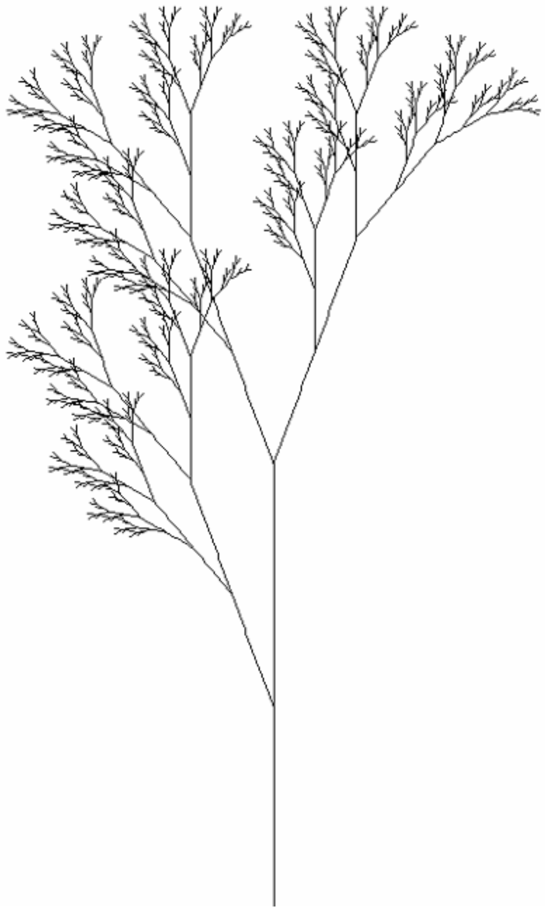



**Koch snowflake curve**

**Sierpinski triangle**



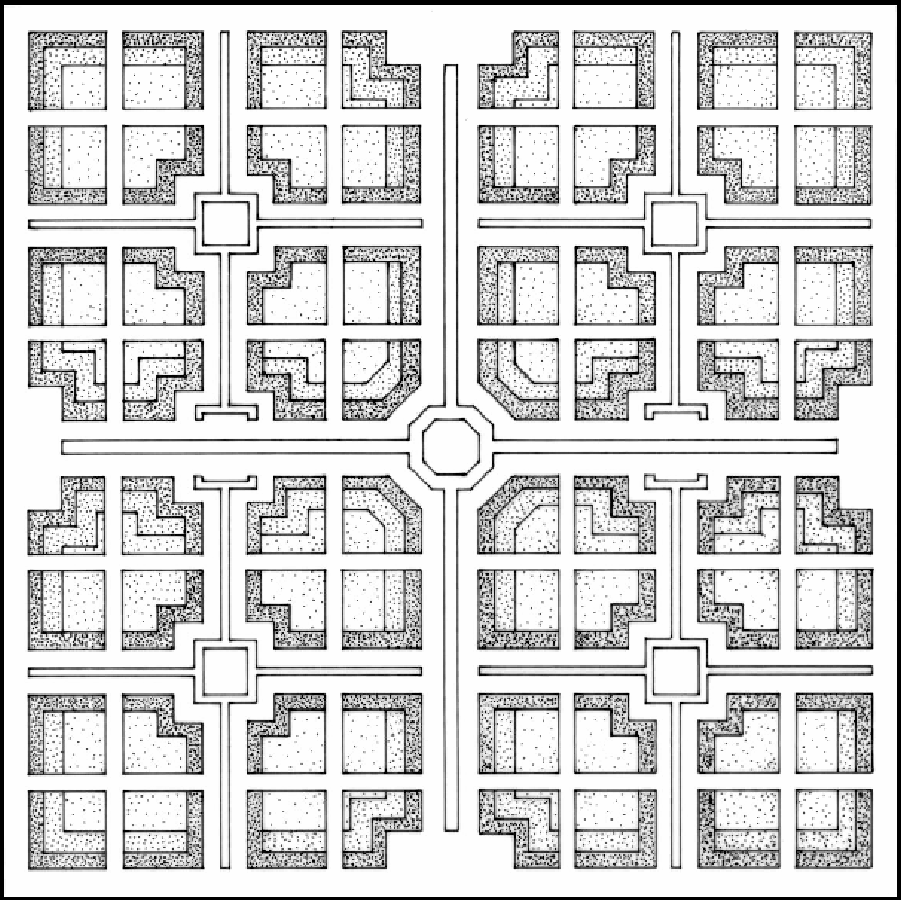
**fractal tree**



**parametric fractal tree**

**fractal rule**

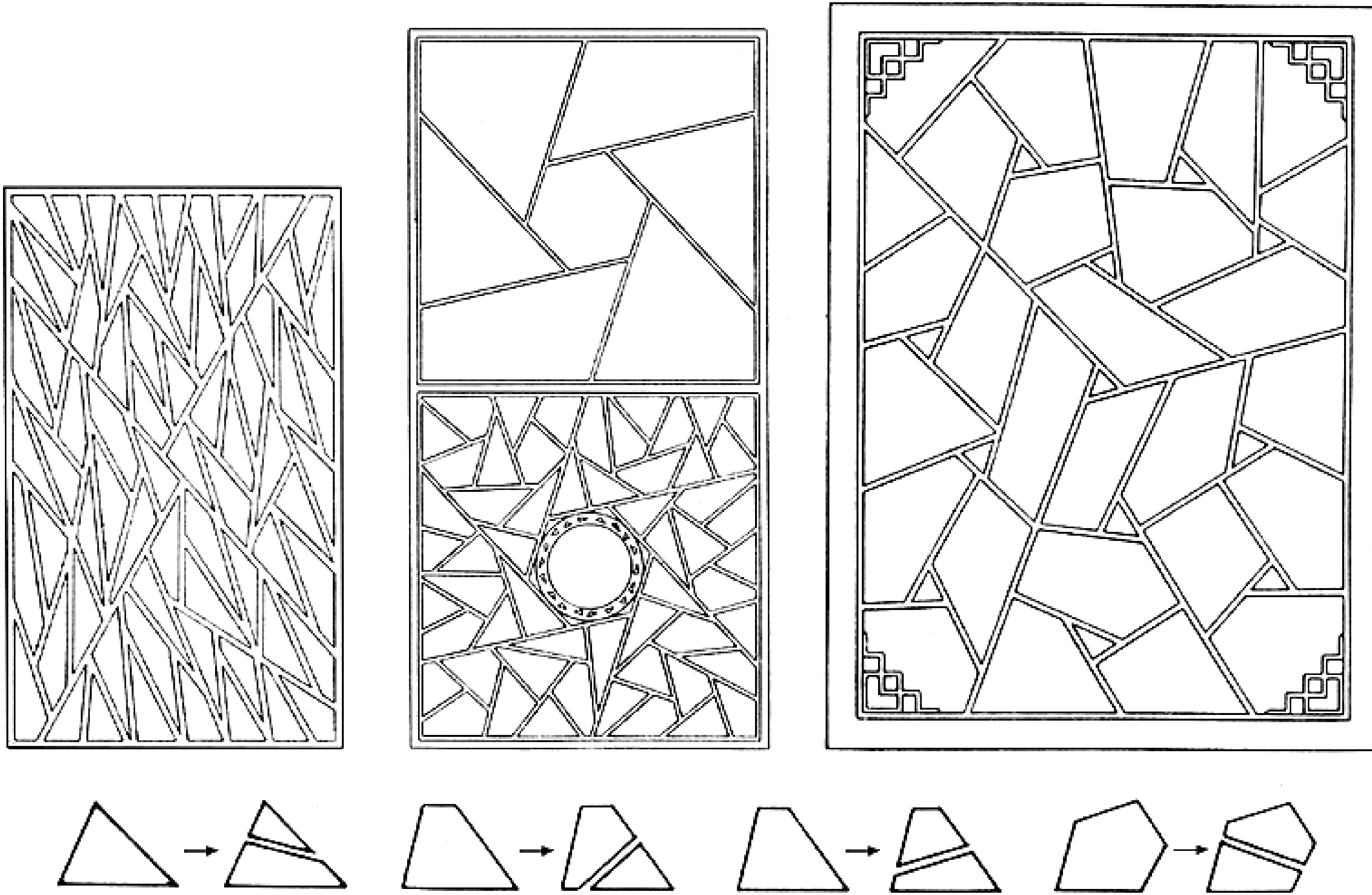
$$X \;\rightarrow\; t_1(X) + t_2(X) + \ldots + t_n(X)$$



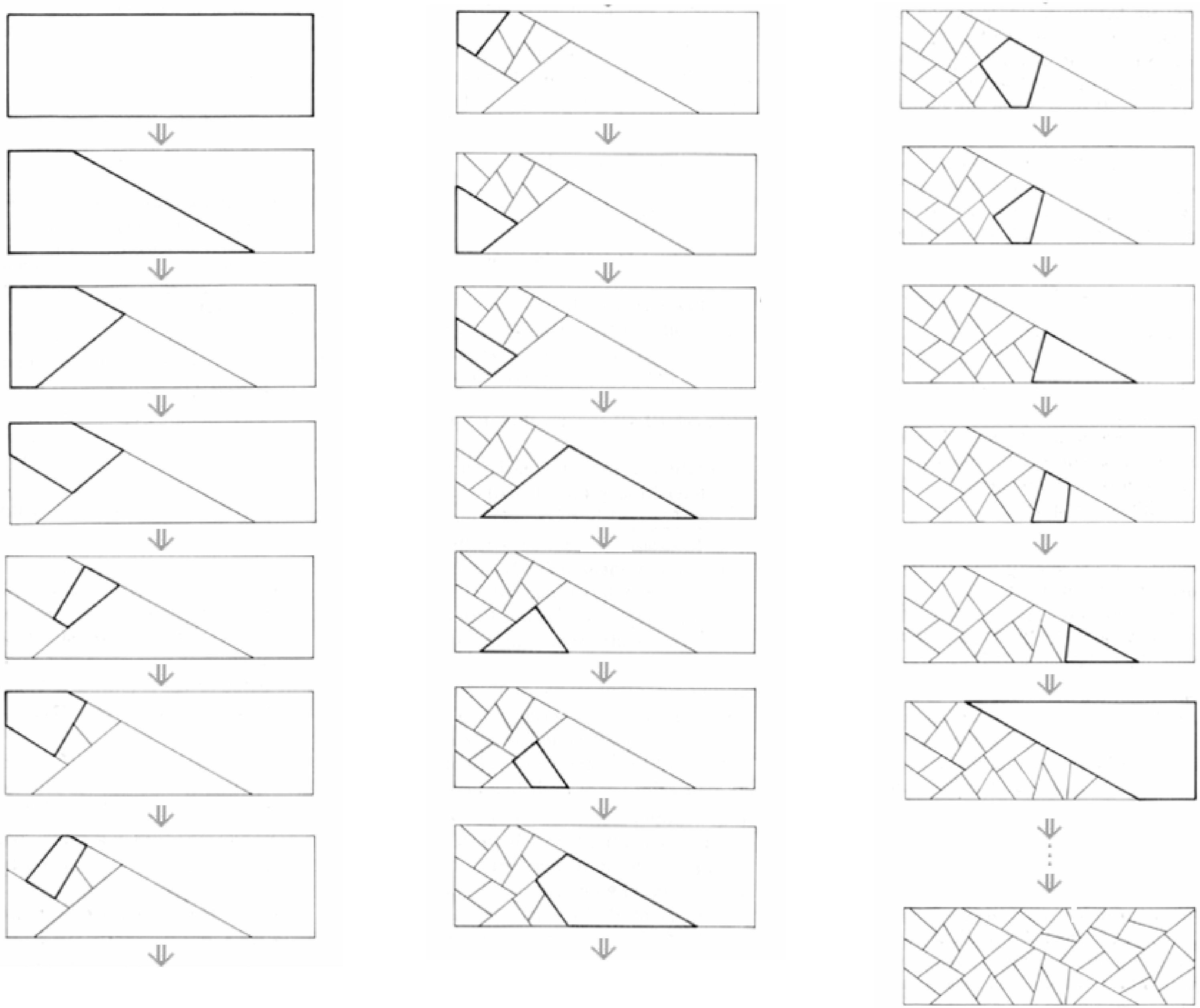
Mughul gardens

**rules for:**

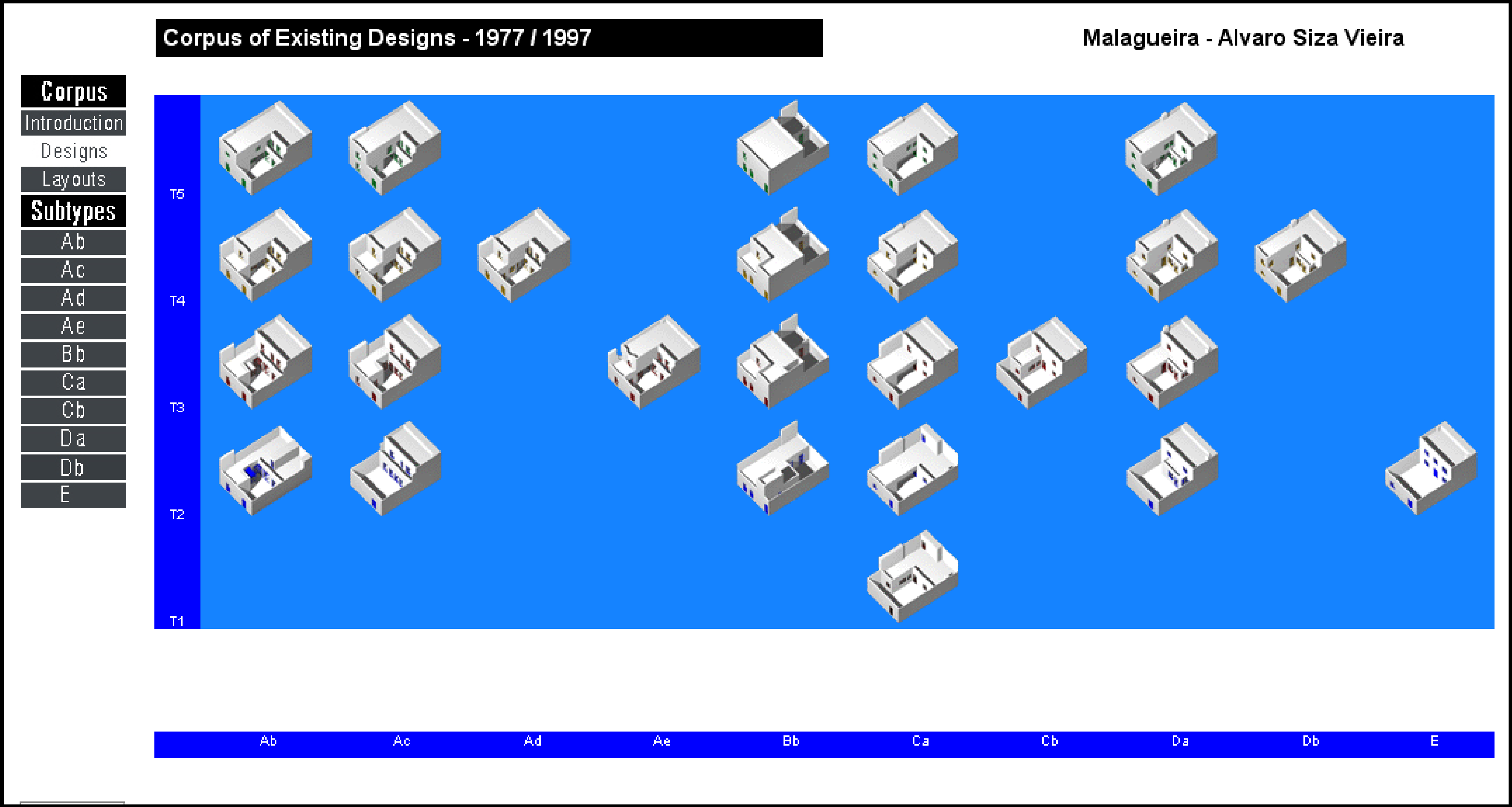
**design processes**



Chinese ice-ray shape grammar (Stiny, 1977)

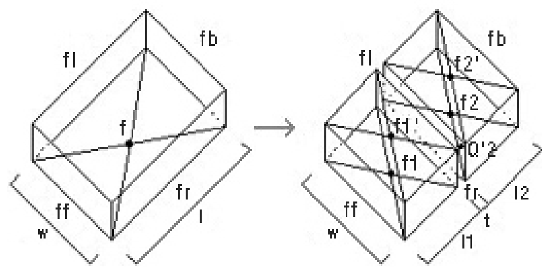


computation of an ice-ray design



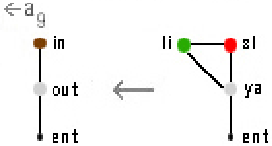

[Malagueira housing designs of Alvaro Siza (Jose Duarte)]

## Rule 9: dissecting the outside zone




$$R9: \langle F_1;\ f_b,\ f_r,\ f_f,\ li;\ o;\ Z \rangle \rightarrow \langle F_1;\ f_b,\ f_r,\ f_f,\ f_l;\ ya,\ sl;\ Z - \{ya, sl\} \rangle$$

**Context:** $g_4 : a_4 \leftarrow a_4$
**Housetype:** $g_5 : a_5 \leftarrow a_5$
**N. rooms:** $g_6 : a_6 \leftarrow a_6$
**Balconies:** $g_7 : a_7 \leftarrow a_7$
**Zones:** $g_8 : a_8 \leftarrow a_8 <$-(in, (x,y), w, l, a),
+(li, (x,y), w, l, a),
+(sl, (x,y), w, l, a),
**Room:** $g_9 : a_9 \leftarrow a_9$
**Adjacencies:** $g_{10}$:




rule from Siza grammar

**subdivision rule**

$$X \rightarrow X + D$$

where D is a dividing element,
a line or plane for example

**addition rule**

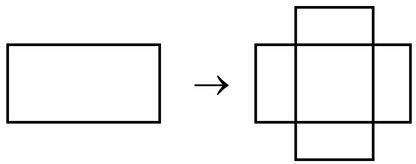
$$X \rightarrow X + Y$$

**addition rule**

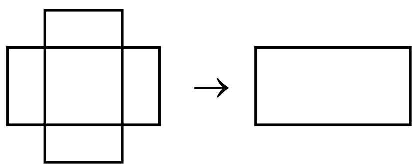
$$X \rightarrow X + Y$$

**subtraction rule**

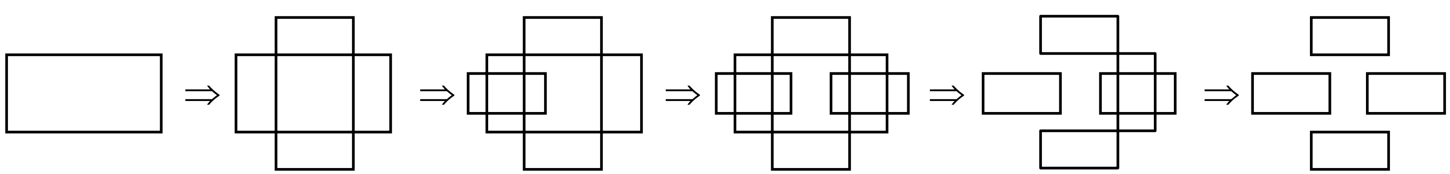
$$X + Y \rightarrow X$$



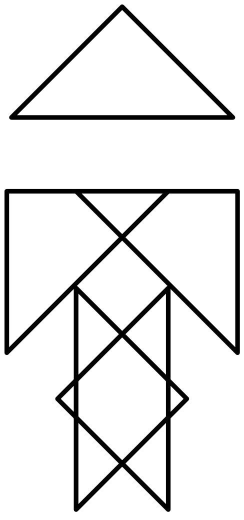
addition rule



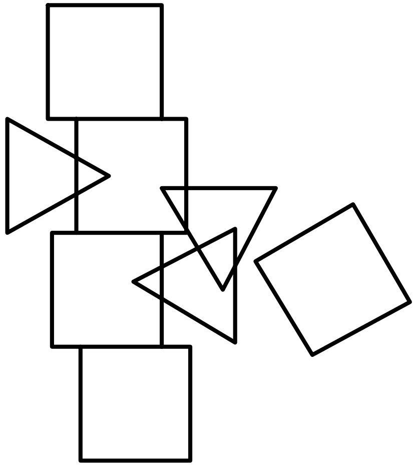
subtraction rule



computation



fellow



tumbling shapes

**rule $\mathbf{X \rightarrow Y}$**

addition rule: $X \leq Y$

subtraction rule: $Y \leq X$

add and subtract rule: $X \not\leq Y$ and $Y \not\leq X$