

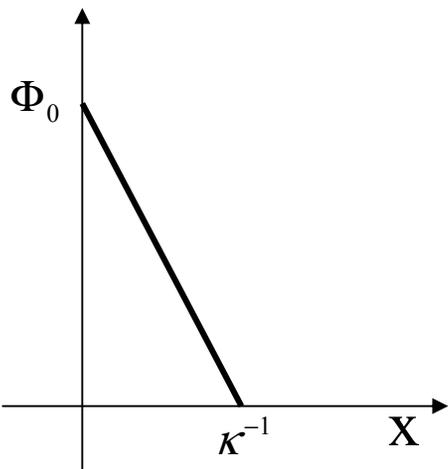
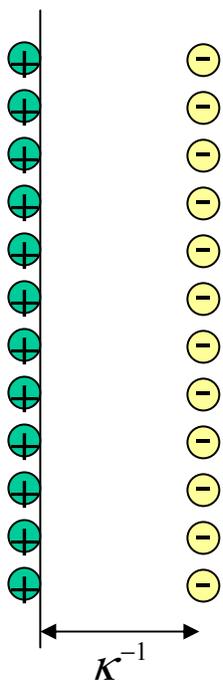
## Key Concepts for section IV (Electrokinetics and Forces)

- 1: Debye layer, Zeta potential, Electrokinetics
- 2: Electrophoresis, Electroosmosis
- 3: Dielectrophoresis
- 4: Inter-Debye layer force, Van-Der Waals forces
- 5: Coupled systems, Scaling, Dimensionless Number

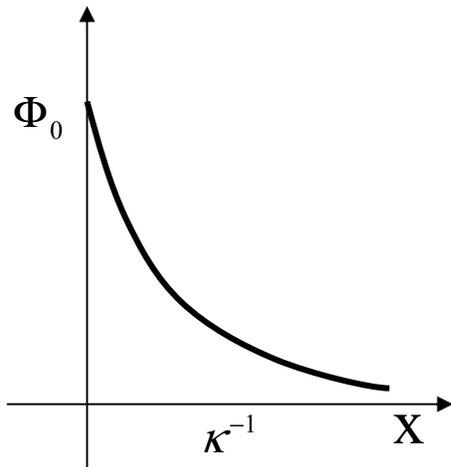
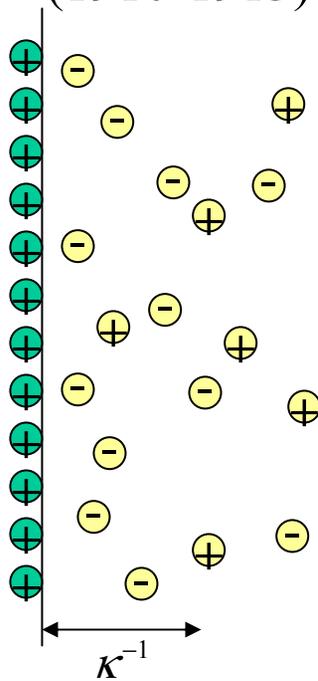
### **Goals of Part IV:**

- (1) Understand electrokinetic phenomena and apply them in (natural or artificial) biosystems**
- (2) Understand various driving forces and be able to identify dominating forces in coupled systems**

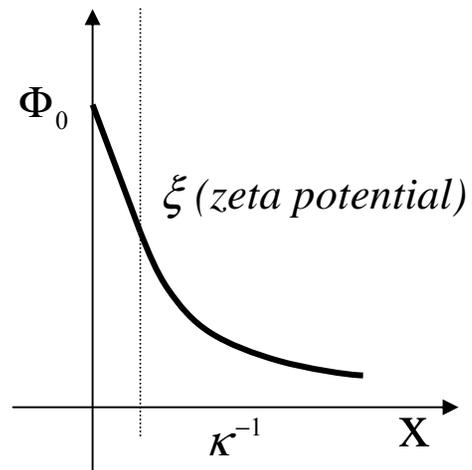
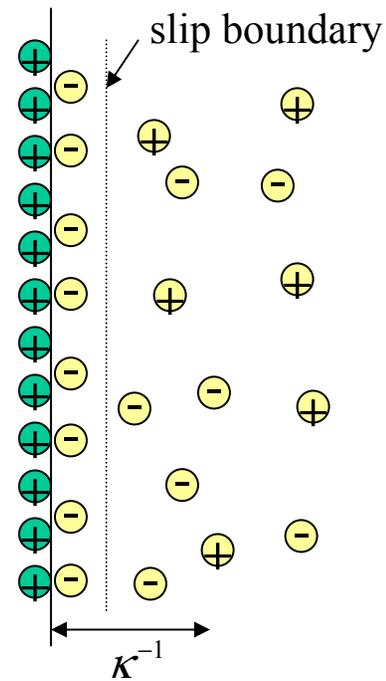
Helmholtz model  
(1853)



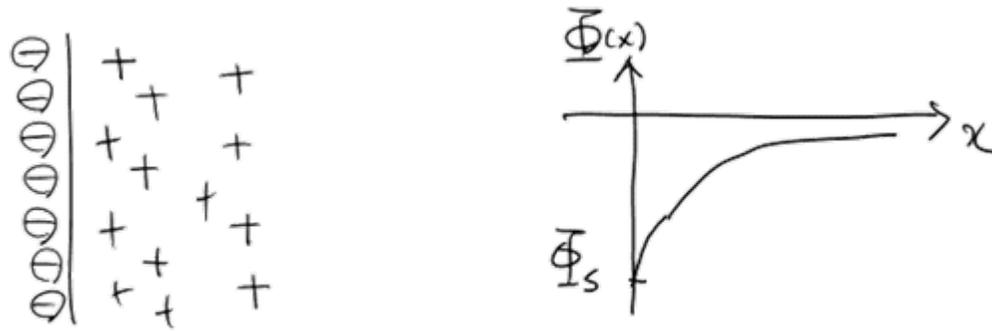
Guoy-Chapman model  
(1910-1913)



Stern model (1924)



# < Mathematical model for Debye layer >



P-B equation. ①  $\frac{d^2\Phi}{dx^2} = -\frac{\rho_e}{\epsilon}$   
(Poisson's)

②  $\rho_e = zF(C_+(x) - C_-(x))$   
(Boltzmann Statistics) ③  $C_{\pm}(x) = C_0 e^{-\frac{\pm ze\Phi(x)}{kT}}$

④ P-B equation  
 $\frac{d^2\Phi}{dx^2} = -\frac{zFC_0}{\epsilon} \left( e^{-\frac{ze\Phi(x)}{kT}} - e^{\frac{ze\Phi(x)}{kT}} \right)$

If solved  $\Rightarrow$  Guoy-Chapmann Solution

If an approximation (Debye-Hückel approx.) is used.

$$e^{-\frac{ze\Phi(x)}{kT}} \approx 1 - \frac{ze\Phi(x)}{kT}$$

⑤ Linearized Poisson - Boltzmann

$$\frac{d^2\bar{\Phi}}{dx^2} = \kappa^2 \bar{\Phi}(x) \quad \kappa^2 = \frac{2z^2 F^2 C_0}{\epsilon RT}$$

$$\bar{\Phi}(x) = \bar{\Phi}_s e^{-\kappa x} \quad (\kappa^{-1} : \text{Debye length})$$

$\bar{\Phi}_s$  : surface potential

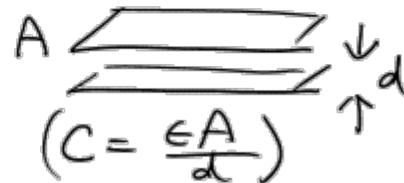
⑥ relation bet.  $\bar{\Phi}_s$  and surface charge density

$$\sigma_s = \epsilon \kappa \bar{\Phi}_s$$

$$= \frac{\epsilon \bar{\Phi}_s}{\kappa^{-1}} \quad \left( \begin{array}{l} \text{compare this with} \\ Q = CV \end{array} \right)$$

For area  $A$

$$A\sigma_s = \frac{\epsilon A}{\kappa^{-1}} \cdot \bar{\Phi}_s$$



capacitance of a parallel plate capacitor with  $d = \kappa^{-1}$

Debye layer is a extraordinary capacitor!

<http://www.nesscap.com/>

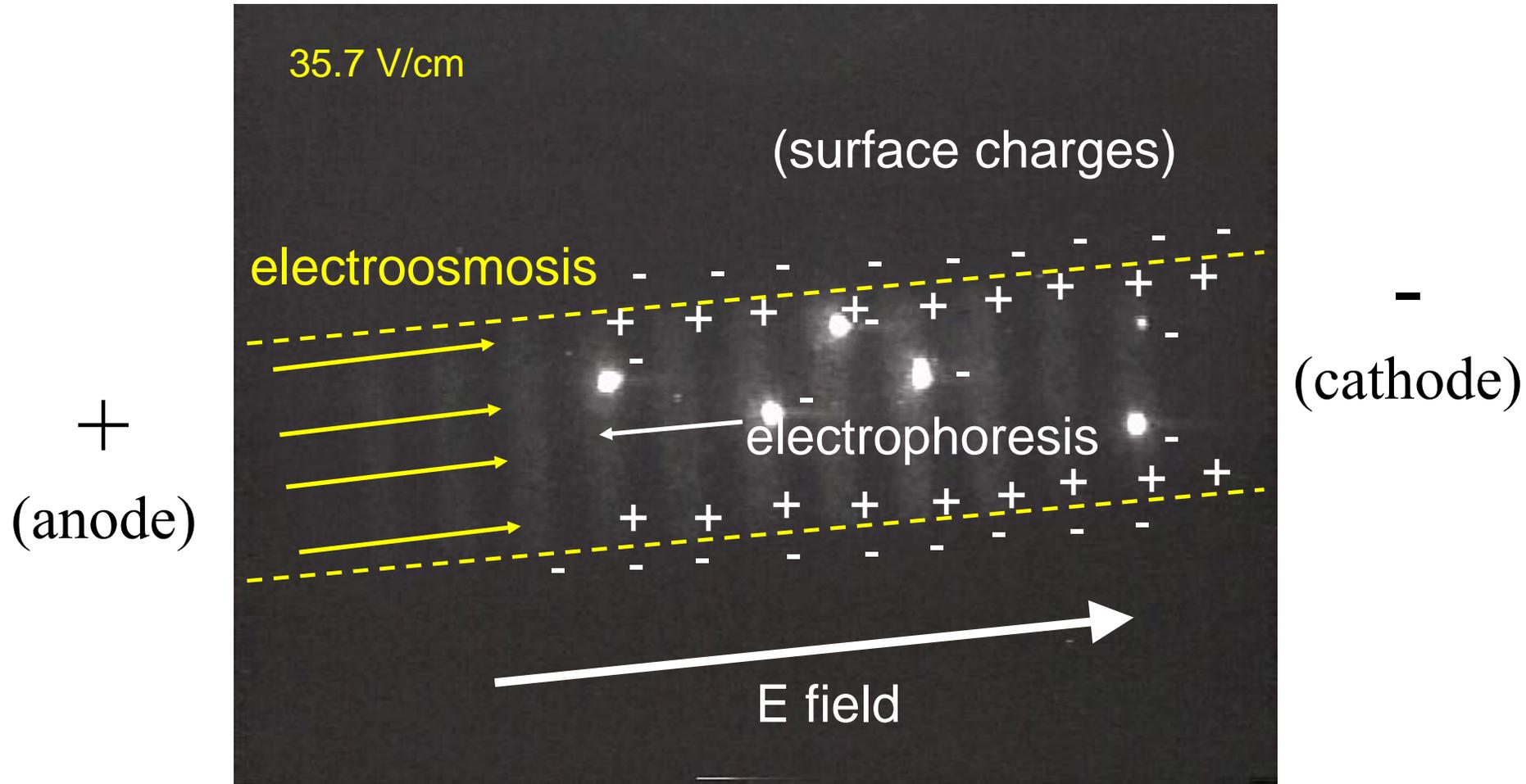
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Graph of power vs. energy characteristics of energy storage devices.

From NessCAP Inc. website

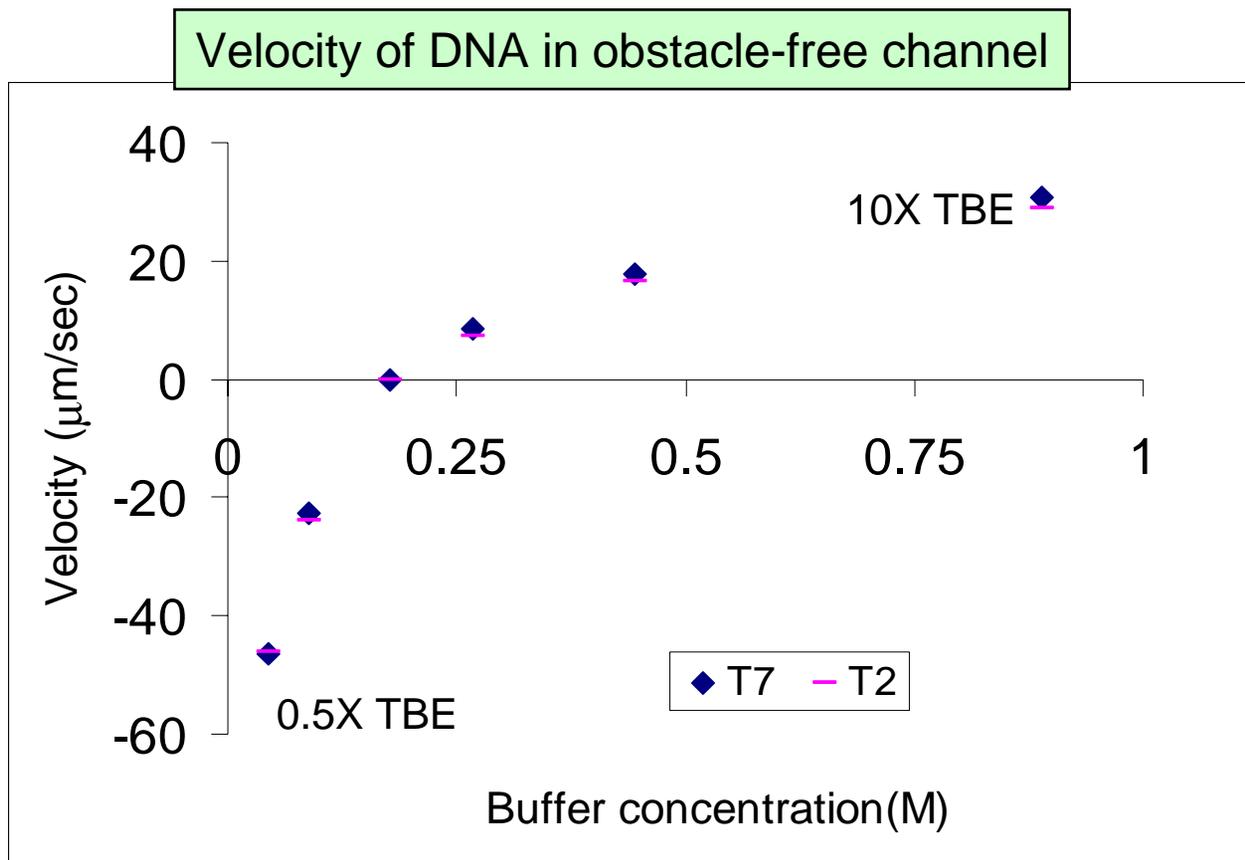
Image removed due to copyright restrictions.  
Datasheet for NESSCAP Ultracapacitor 3500F/2.3V

# Motion of DNA in Channels



Source: Prof. Han's Ph.D thesis

# DNA electrophoresis in a channel



# Electroosmosis

- The oxide or glass surface become unprotonated (pK ~ 2) when they are in contact with water, forming electrical double layer.
- When applied an electric field, a part of the ion cloud near the surface can move along the electric field.
- The motion of ions at the boundary of the channel induces bulk flow by viscous drag.

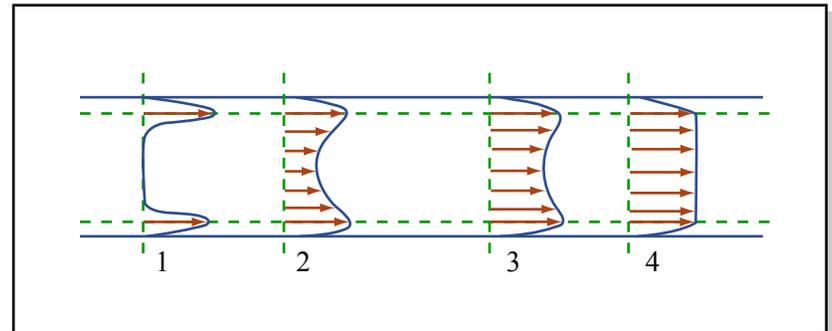
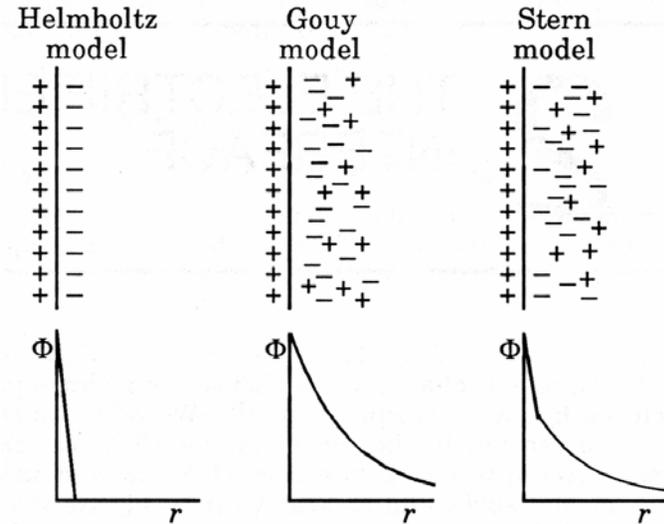


Figure by MIT OCW.

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Schematic of electroosmotic flow of water in a porous charged medium.

Figure 6.5.1 in Probst, R. F. *Physicochemical Hydrodynamics: An Introduction*. New York: NY, John Wiley & Sons, 1994.

<http://www.moldreporter.org/vol1no5/drytronic>



US006013164A

**United States Patent** [19]  
**Paul et al.**

[11] **Patent Number:** **6,013,164**  
[45] **Date of Patent:** **Jan. 11, 2000**

Gels and Tissues

Nanoporous materials

Extra-Cellular matrix

Microfluidic system

Electrokinetic pumping

<http://www.eksigent.com/>

- [54] **ELECTOKINETIC HIGH PRESSURE HYDRAULIC SYSTEM**
- [75] Inventors: **Phillip H. Paul**, Livermore; **David J. Rakestraw**, Fremont, both of Calif.
- [73] Assignee: **Sandia Corporation**, Livermore, Calif.
- [21] Appl. No.: **08/882,725**
- [22] Filed: **Jun. 25, 1997**
- [51] Int. Cl.<sup>7</sup> ..... **B01D 61/44**
- [52] U.S. Cl. .... **204/450; 204/600; 204/647; 204/648**
- [58] Field of Search ..... 204/450, 600, 204/647, 648

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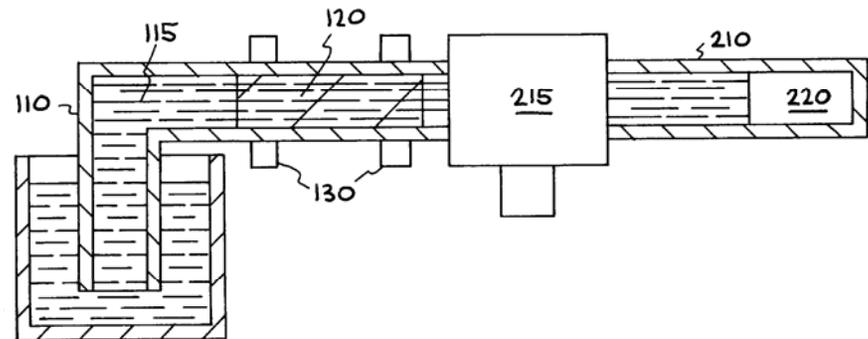
Pretorius, Victor; Hopkins, B.J. and J.D. Schieke. *Journal of Chromatography*, 99, 23-30. 1974. A new concept for high speed liquid chromatography.

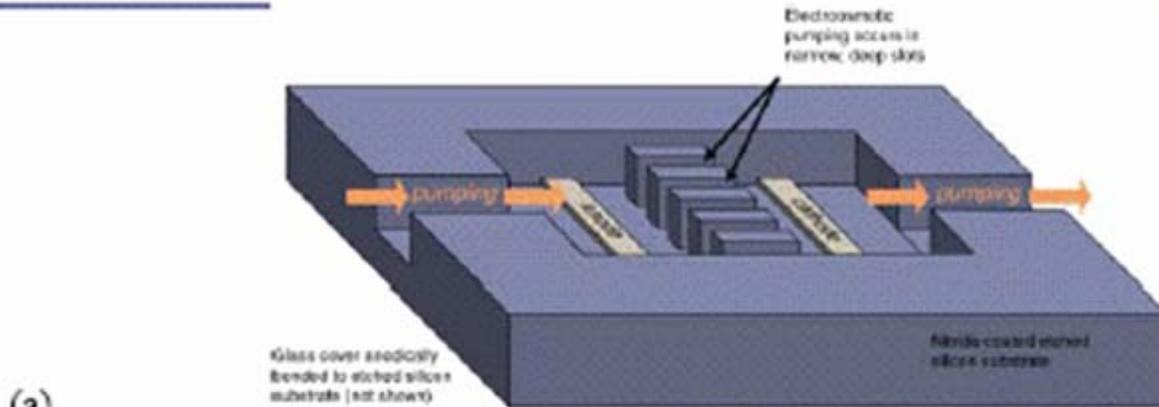
*Primary Examiner*—Arun S. Phasge  
*Attorney, Agent, or Firm*—Donald A. Nissen

[57] **ABSTRACT**

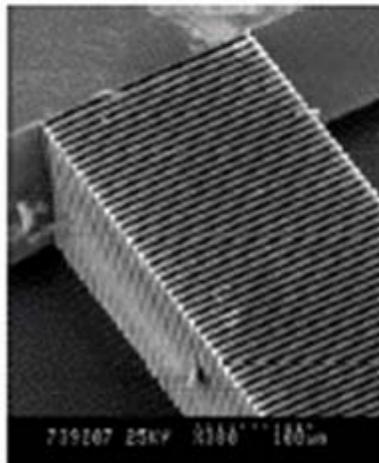
A compact high pressure hydraulic system having no moving parts for converting electric potential to hydraulic force and for manipulating fluids. Electro-osmotic flow is used to provide a valve and means to compress a fluid or gas in a capillary-based system. By electro-osmotically moving an electrolyte between a first position opening communication between a fluid inlet and outlet and a second position closing communication between the fluid inlet and outlet the system can be configured as a valve. The system can also be used to generate forces as large as 2500 psi that can be used to compress a fluid, either a liquid or a gas.

**22 Claims, 2 Drawing Sheets**





(a)



(b)

Figure 1. (a) Basic structure of the silicon electroosmotic micropump. An axial electric field generates electroosmotic pumping in deep, narrow slots plasma-etched into a silicon substrate. The silicon is electrically passivated. A cover (glass in the prototypes to allow optical access) is bonded onto the silicon to seal the micropump. (b) SEM showing the pumping structure formed by deep etching of silicon. Micropumps with slot widths ranging from 2-4  $\mu\text{m}$  have been fabricated.

Courtesy of Daniel J. Laser. Used with permission.

SNF Researcher: Daniel J. Laser

Principal Investigator: Kenneth E. Goodson, Department of Mechanical Engineering, Stanford University

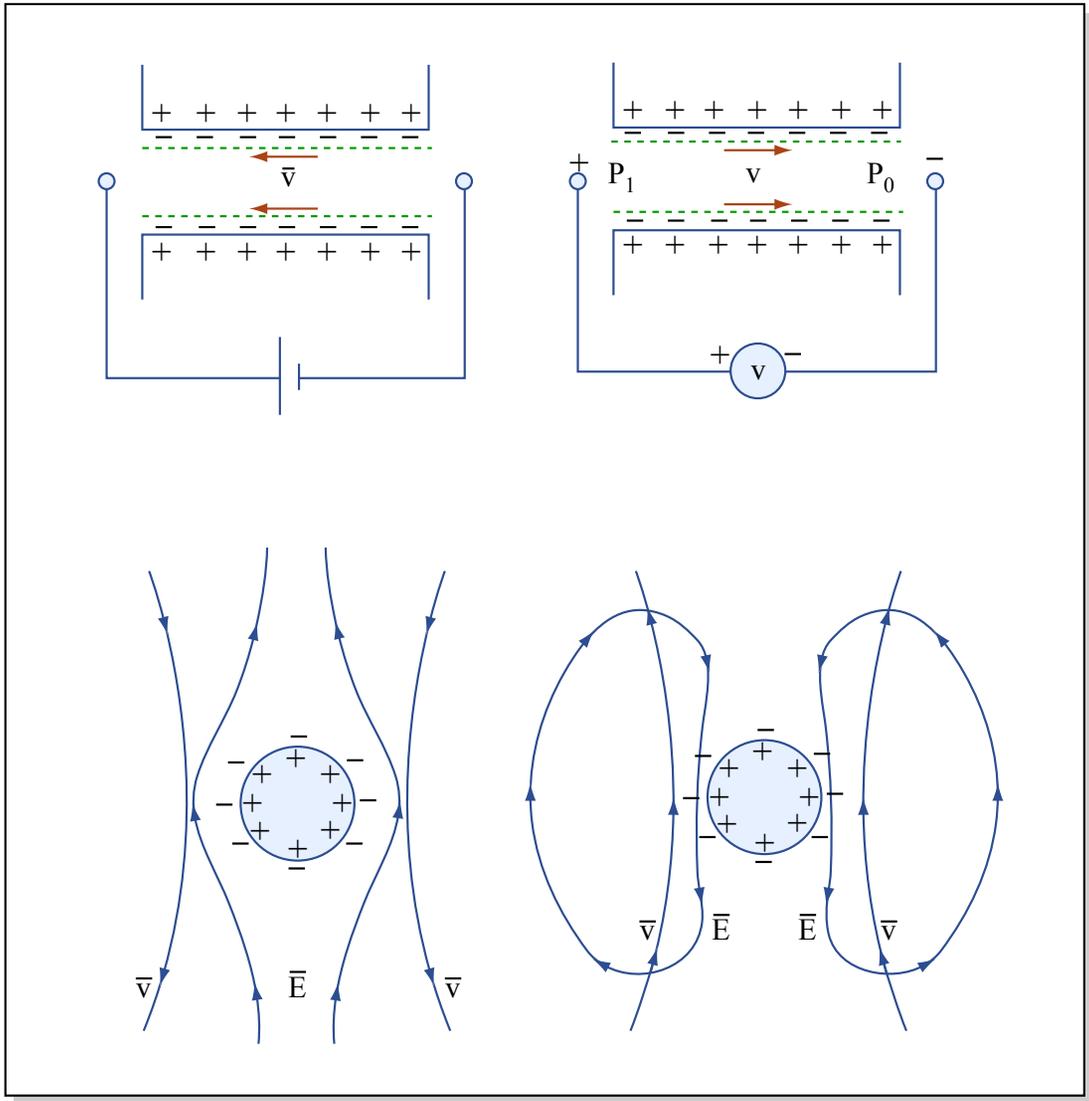
Collaborators: Juan G. Santiago and Thomas W. Kenny, Department of Mechanical Engineering, Stanford University

[http://micromachine.stanford.edu/~dlaser/research\\_pages/silicon\\_eo\\_pumps.html](http://micromachine.stanford.edu/~dlaser/research_pages/silicon_eo_pumps.html)

# Electroosmosis

# Streaming potential

E field ( $\Delta\Phi$ )  
generates  
fluid flow (Q)



fluid flow (Q)  
generates  
E-field ( $\Delta\Phi$ )

E field ( $\Delta\Phi$ )  
generates  
particle  
motion (v)

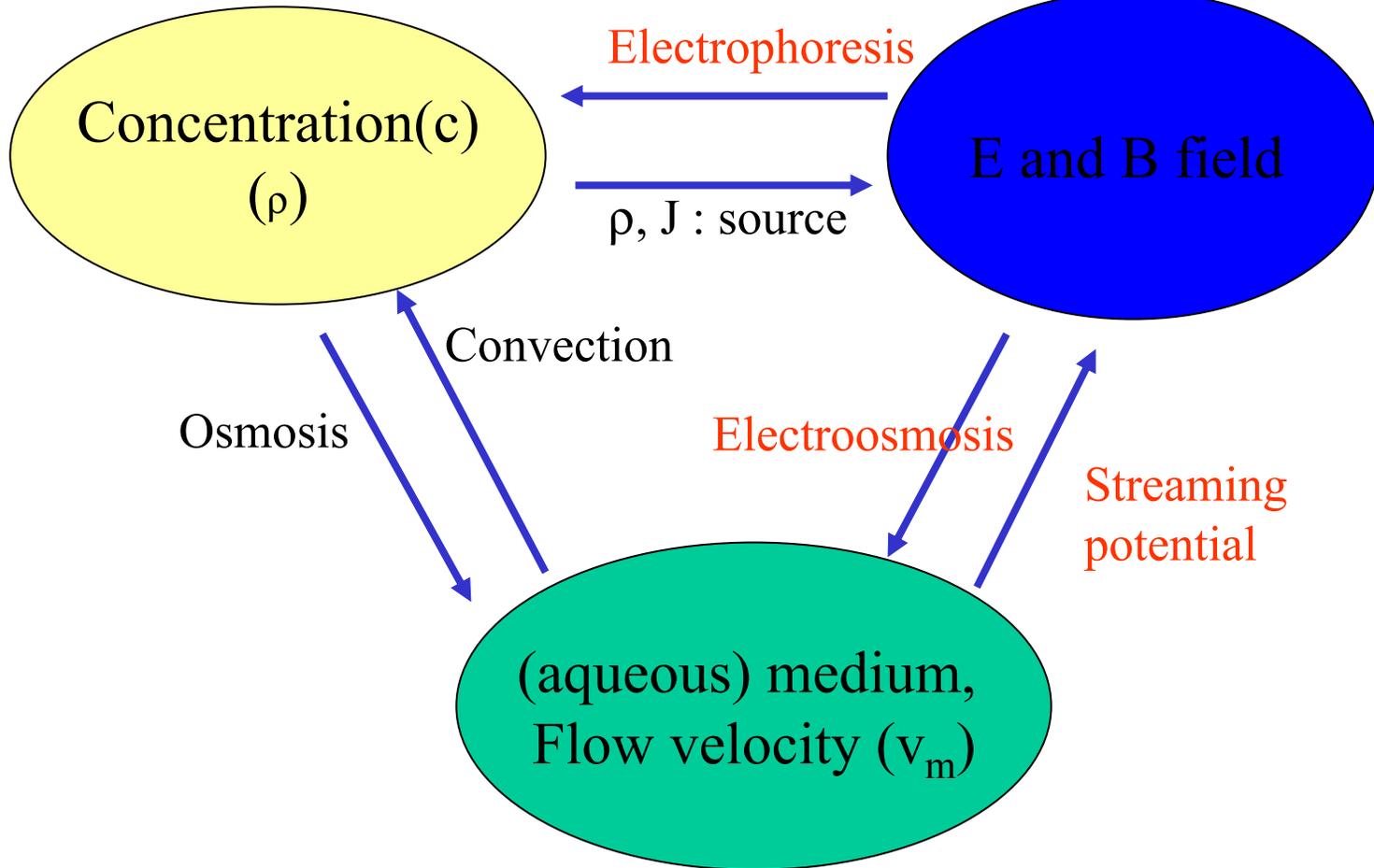
particle  
motion (v)  
generates  
E-field ( $\Delta\Phi$ )

# Electrophoresis

# Sedimentation potential

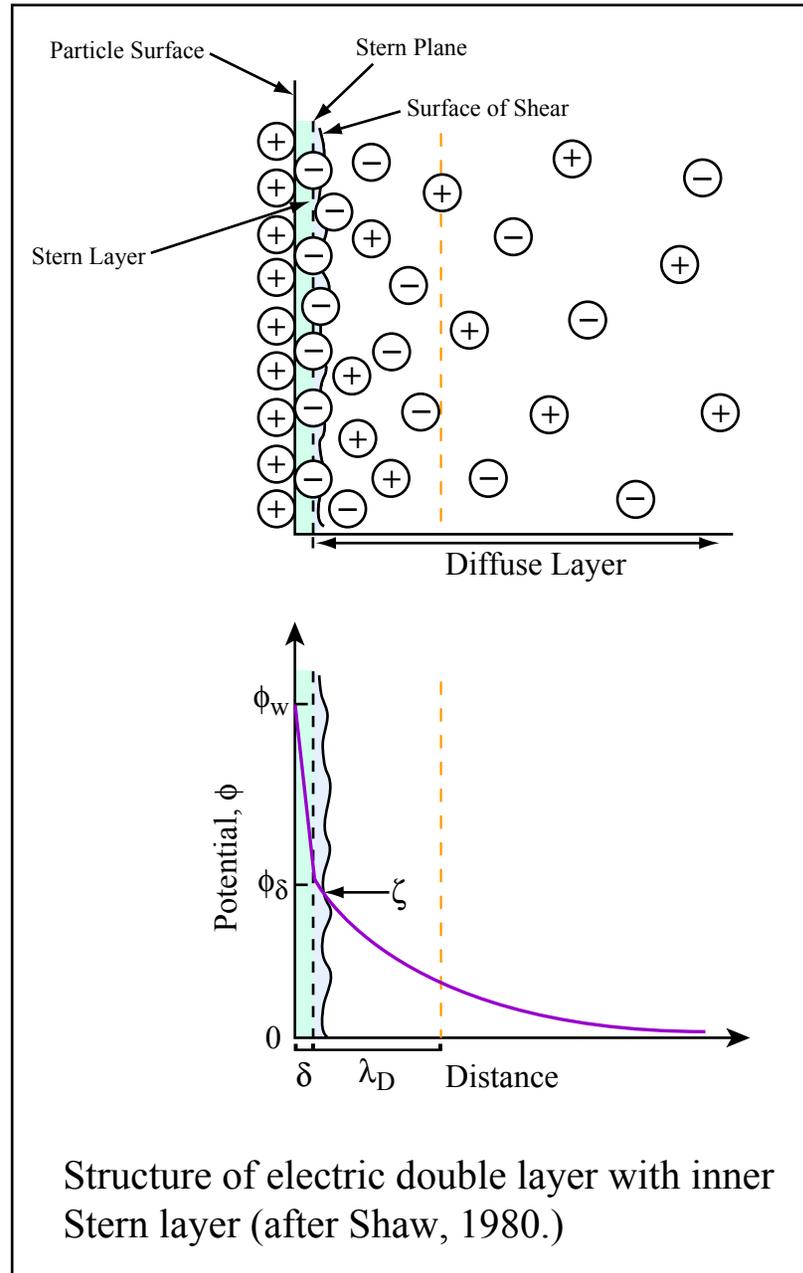
Fick's law of diffusion

Maxwell's equation

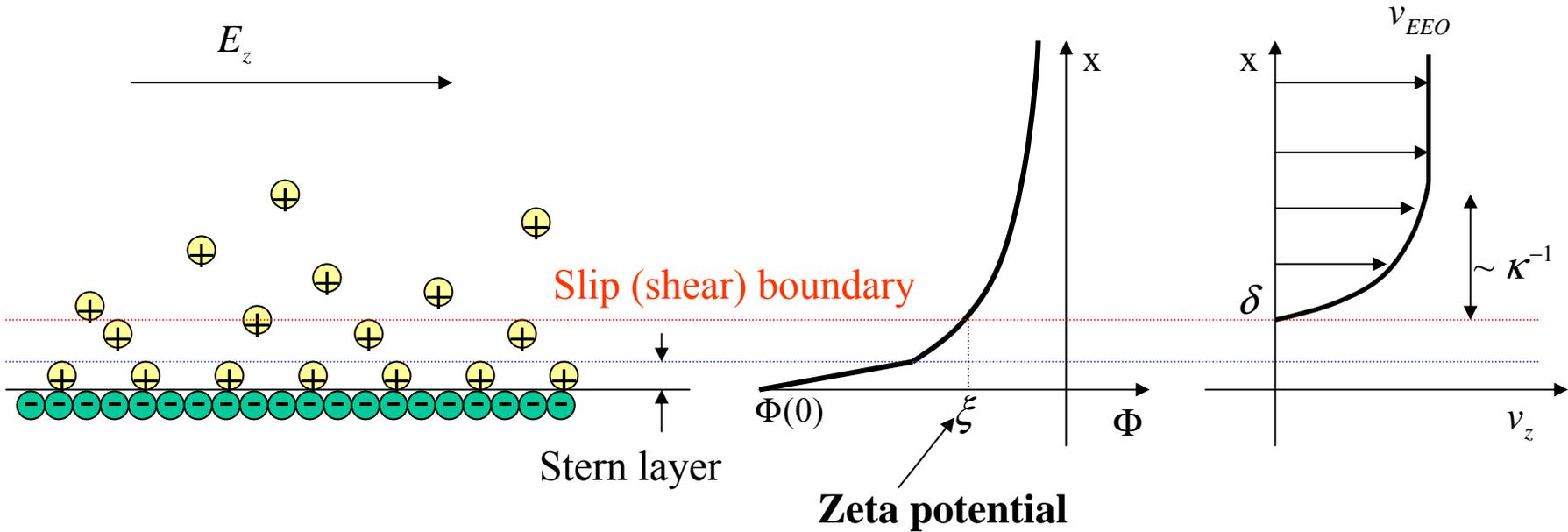


Navier-Stokes' equation

# Stern layer (Realistic Debye layer model)



# Sheer boundary, zeta potential



Stern layer : adsorbed ions, linear potential drop

Gouy-Chapman layer : diffuse-double layer

exponential drop

Shear boundary :  $v_z=0$

Navier-Stokes equation

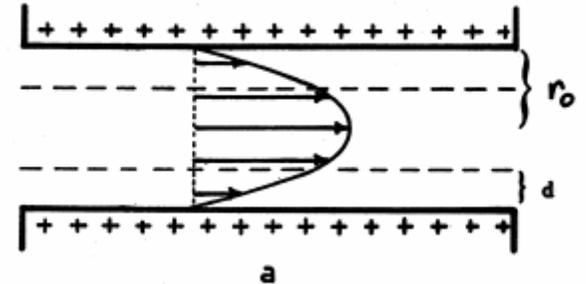
$$\rho \frac{d\vec{v}}{dt} = -\nabla p + \mu \nabla^2 \vec{v} + \rho_e \vec{E} \cong 0$$

New term

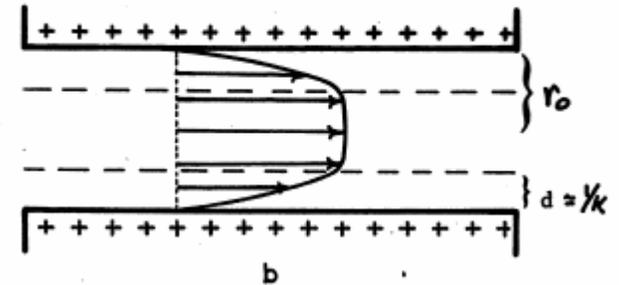
$$\nabla \cdot \vec{v} = 0 \text{ (incompressible)}$$

$$v_z(r) = \underbrace{-\frac{\varepsilon}{\mu}(\zeta - \Phi(r))E_{z0}}_{\text{electroosmotic flow}} - \underbrace{\left(\frac{R^2 - r^2}{4\mu}\right)\frac{\Delta P}{L}}_{\text{Poiseuille flow}}$$

$\Delta P \neq 0, E_z = 0$ : Poiseuille flow  
parabolic flow profile



$\Delta P = 0, E_z \neq 0$ : Electroosmotic flow  
flat (plug-like) profile



$$v_{EEO} = -\frac{\varepsilon\zeta}{\mu}E_z = \mu_{EEO}E_z \quad (\text{outside of the Debye layer})$$

$\mu_{EEO}$ : electroosmotic 'mobility'

## Paul's experiment

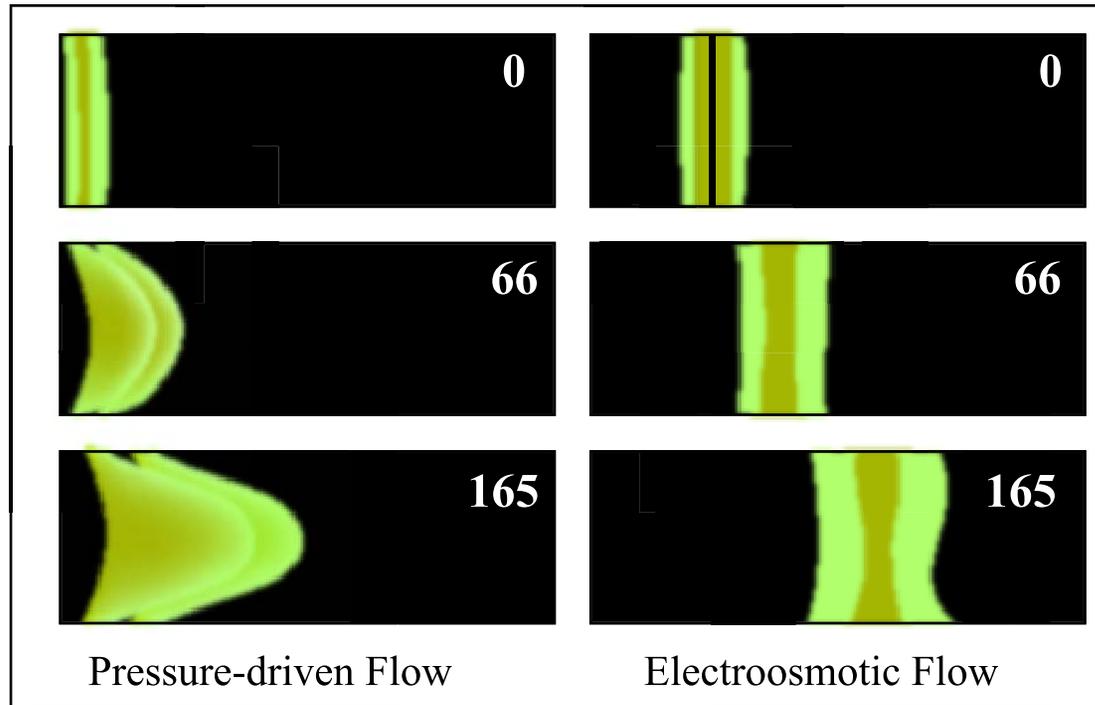


Figure by MIT OCW. After Paul, et al. *Anal Chem* 70 (1998): 2459.

Pressure-driven flow vs. Electroosmotic flow. These images were taken by caged dye techniques (Paul et al. *Anal. Chem.* 70, 2459 (1998)). At  $t=0$ , an initial flat fluorescent line is generated in microchannel by pulse-exposing and breaking the caged dye, rendering them fluorescent. These dyes were transported by the fluid flow generated by pressure (left column) or electroosmosis (right column), demonstrating the flow profile.

$$v_{EEO} = -\frac{\varepsilon\zeta}{\mu} E_z$$

Electroosmotic flow velocity is independent of the size (and shape) of the tube.



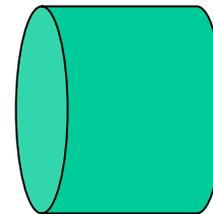
$$R=1\mu\text{m}$$

$$v_{EEO} = -\frac{\varepsilon\zeta}{\mu} E_z$$



$$R=10\mu\text{m}$$

$$v_{EEO} = -\frac{\varepsilon\zeta}{\mu} E_z$$



$$R=1\text{m}$$

$$v_{EEO} = -\frac{\varepsilon\zeta}{\mu} E_z$$

?

$$v_{Poiseuille} = -\frac{R^2 \Delta P}{8\mu L}$$

(averaged over r)

Pressure-driven flow velocity is a strong function of R.