Key Concepts for this section

1: Lorentz force law, Field, Maxwell's equation

2: Ion Transport, Nernst-Planck equation

3: (Quasi)electrostatics, potential function,

4: Laplace's equation, Uniqueness

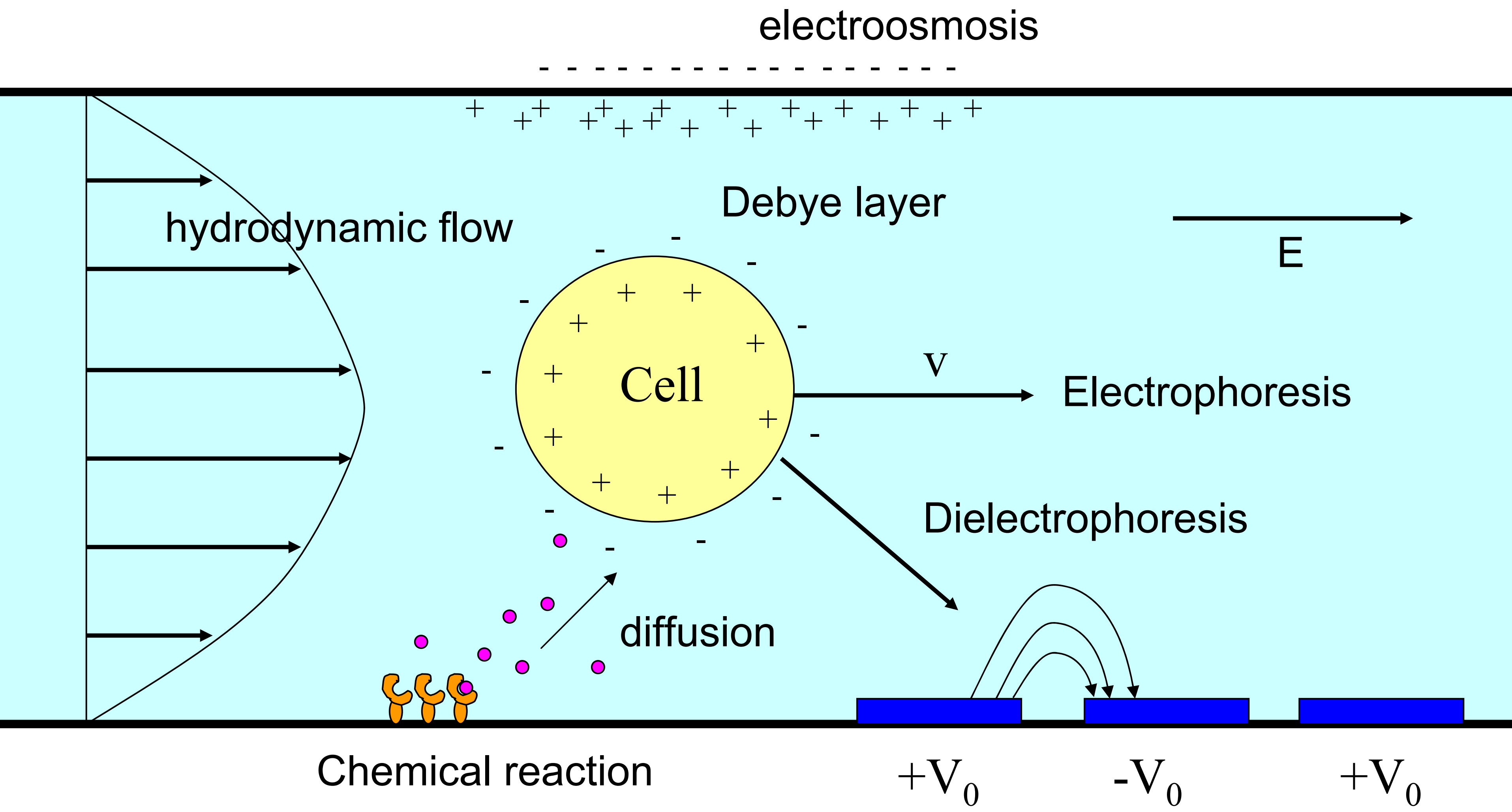
5: Debye layer, electroneutrality

Goals of Part II:

(1) Understand when and why electromagnetic (E and B) interaction is relevant (or not relevant) in biological systems.

(2) Be able to analyze quasistatic electric fields in 2D and 3D.

Example : BioMEMS systems



Differential form of Maxwell's equations

$$\oint_S \vec{A}\cdot d\vec{s} = \int_V (\nabla\cdot\vec{A})dV$$ Gauss' theorem

$$\oint_C \vec{A}\cdot\vec{dl} = \int_S (\nabla\times\vec{A})\cdot\vec{ds}$$ Stokes' theorem

$$\oint_S \varepsilon_0 \vec{E}\cdot d\vec{s} = \int_V \rho_e dV \quad \longrightarrow \quad \nabla\cdot(\varepsilon_0\vec{E}) = \rho_e$$

$$\frac{1}{\mu_o}\oint_C \vec{B}\cdot d\vec{s} = \int_S \vec{J}_e\cdot d\vec{a} + \frac{d}{dt}\int_S \varepsilon_o \vec{E}\cdot d\vec{a} \quad \longrightarrow \quad \frac{1}{\mu_0}\nabla\times\vec{B} = \vec{J}_e + \varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\oint_C \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{s} \quad \longrightarrow \quad \nabla\times\vec{\mathrm{E}} = -\frac{\partial\vec{\mathrm{B}}}{\partial \mathrm{t}}$$

$$\oint_S \vec{\mathrm{B}}\cdot \mathrm{d}\vec{\mathrm{s}} = 0 \quad \longrightarrow \quad \nabla\cdot\vec{\mathrm{B}} = 0$$

Maxwell's equation in source-free space

General solution for the Wave equation

$$\vec{E} \sim \sin(\omega t - \vec{k} \cdot \vec{r}) \; or \; \cos(\omega t - \vec{k} \cdot \vec{r}) \qquad \omega / k = c$$

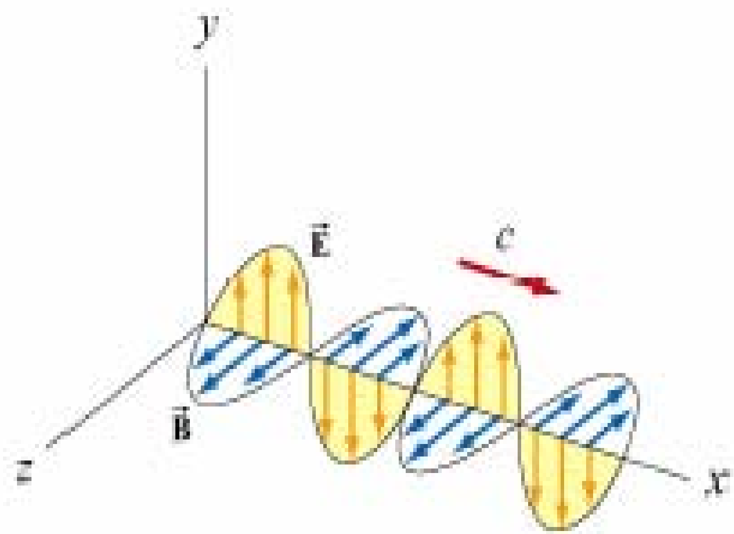
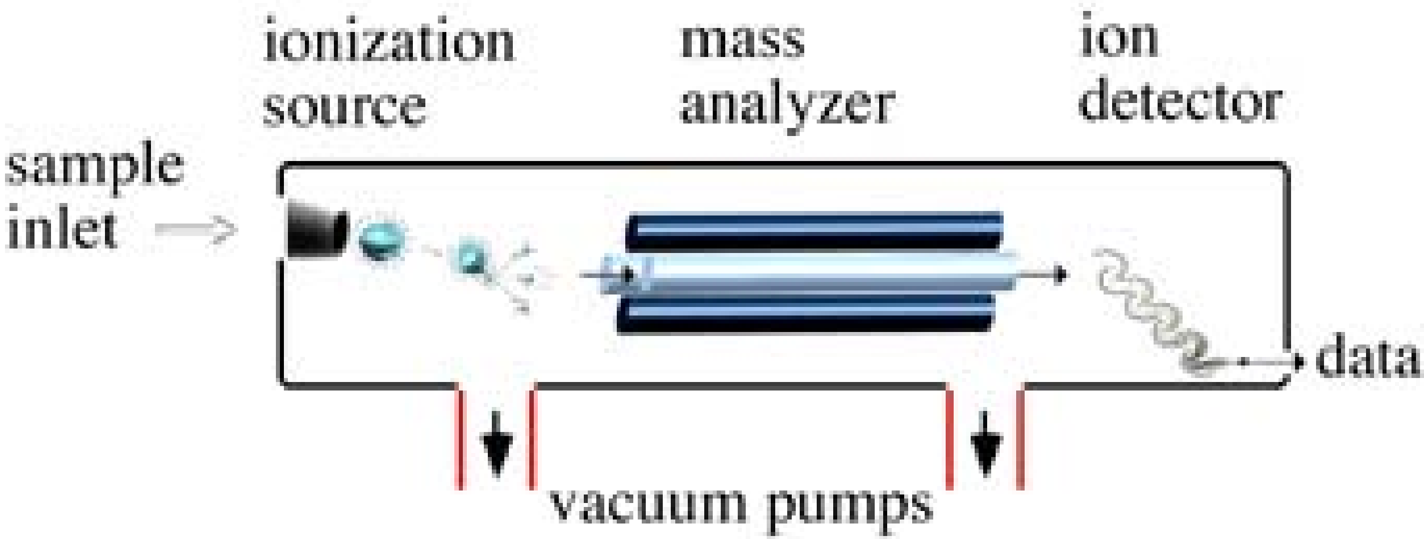


Figure 13.4.5 Plane electromagnetic wave propagating in the $+x$ direction.

Mass Spectrometry



Gary Suizdak’s tutorial page
(http://masspec.scripps.edu/MSHistory/whatisms.php)

Related MIT links :
http://web.mit.edu/toxms/www/links2.htm

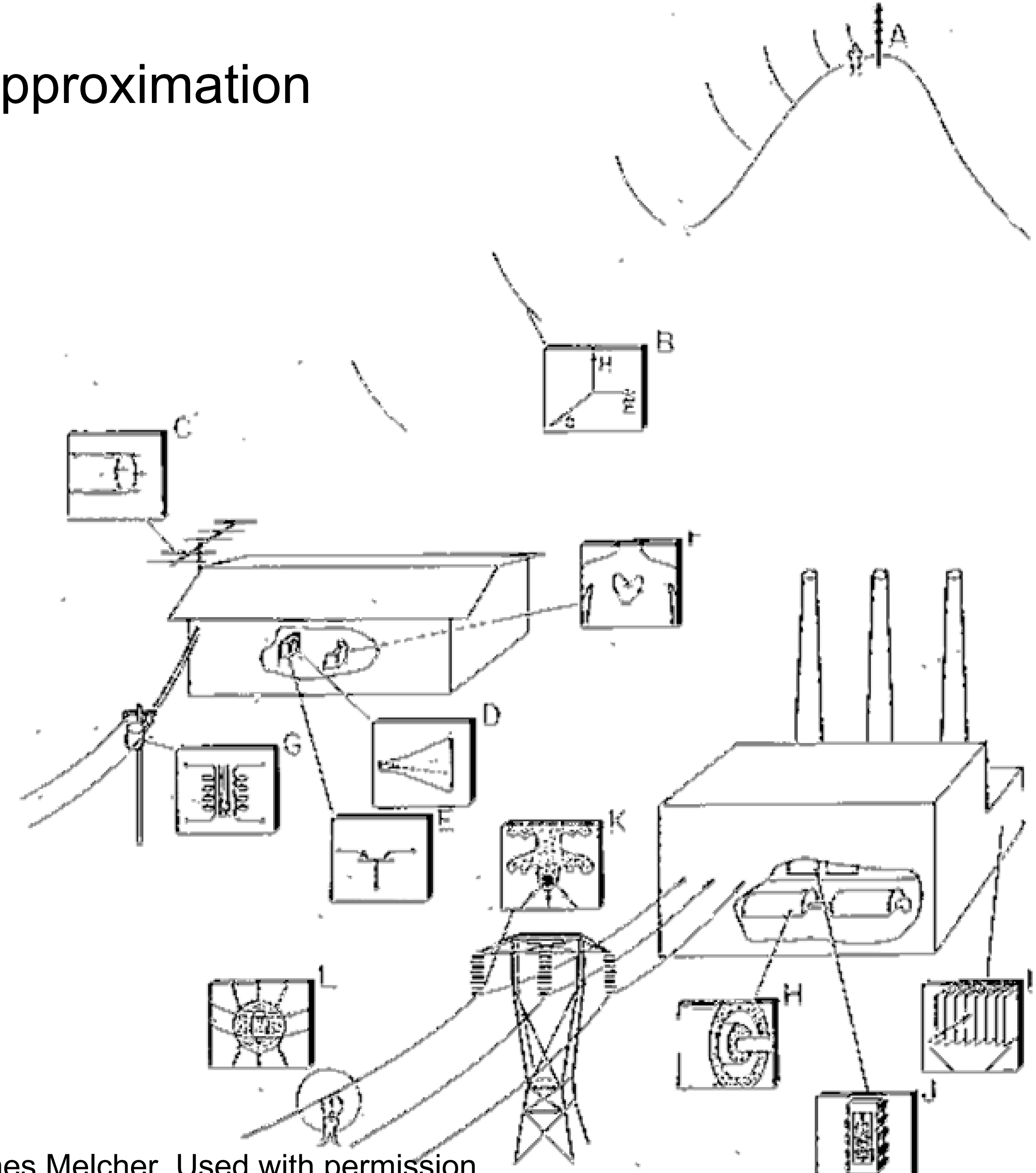
How good is this approximation?

$$\frac{E_{error}}{E} \sim \frac{L^2}{c^2T^2} \sim \frac{L^2\omega^2}{c^2} \sim \frac{L^2}{\lambda^2} \quad (\lambda : \text{wavelength EM wave})$$

Frequency (f)	T ~ 1/f	λ ~cT
60 Hz	0.167 s	5000 km
1 MHz	1 μs	300 m
100 MHz	10 ns	3 m
10 GHz	0.1 ns	3 cm

EQS approximation

Figure 3.5.1
H&M



EM interactions in media - polarization

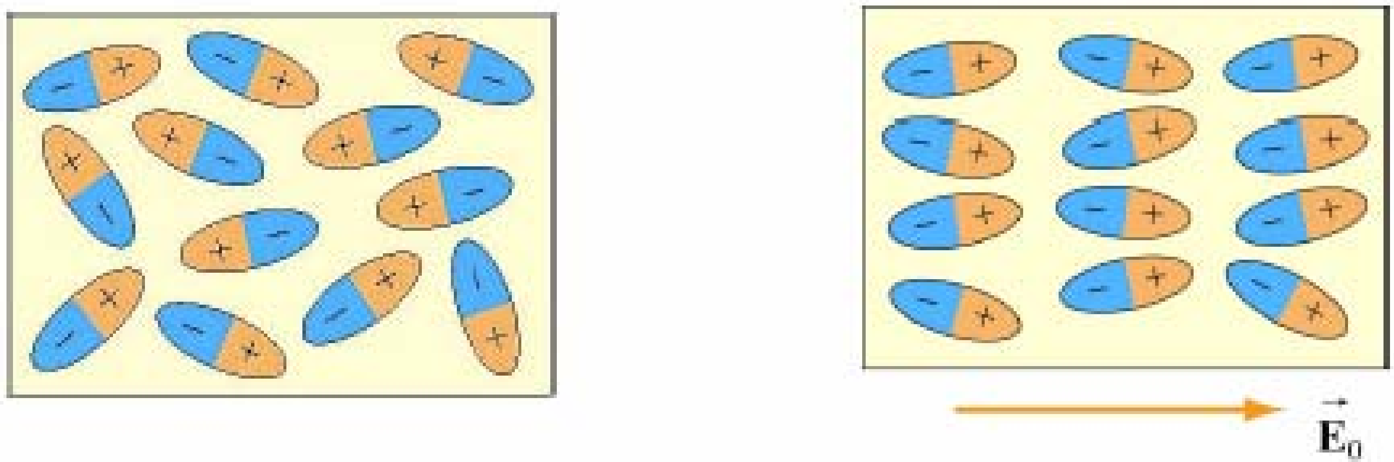
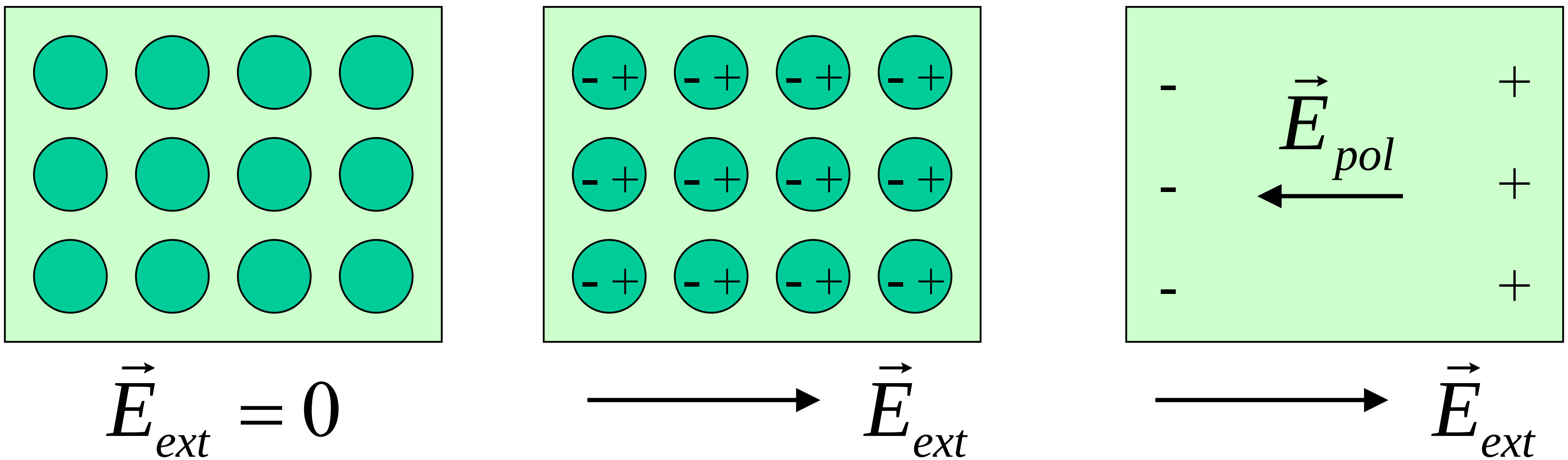


Figure 5.5.1 Orientations of polar molecules when (a) $\vec{E}_0 = \vec{0}$ and (b) $\vec{E}_0 \neq 0$.

Figure 5.5.2 Orientations of non-polar molecules when (a) $\vec{E}_0 = \vec{0}$ and (b) $\vec{E}_0 \neq \vec{0}$.

EM interactions in media - polarization (linear medium)



$$\vec{E}_{pol} \propto \vec{E}_{ext} \quad \Rightarrow \quad \vec{E}_{media} = \vec{E}_{ext} - \vec{E}_{pol} = \frac{1}{\varepsilon_r}\vec{E}_{ext}$$

ε_r : relative permittivity (dielectric constant) of the medium (=>1)

ε of various media

Medium	ε_r
Water (pure)	~80
0.9% NaCl solution	~60
Ethanol	24
Methanol	34
Acetic acid	15~16
Gases	~1
Glass	3~4
Plastics and rubbers	2~9

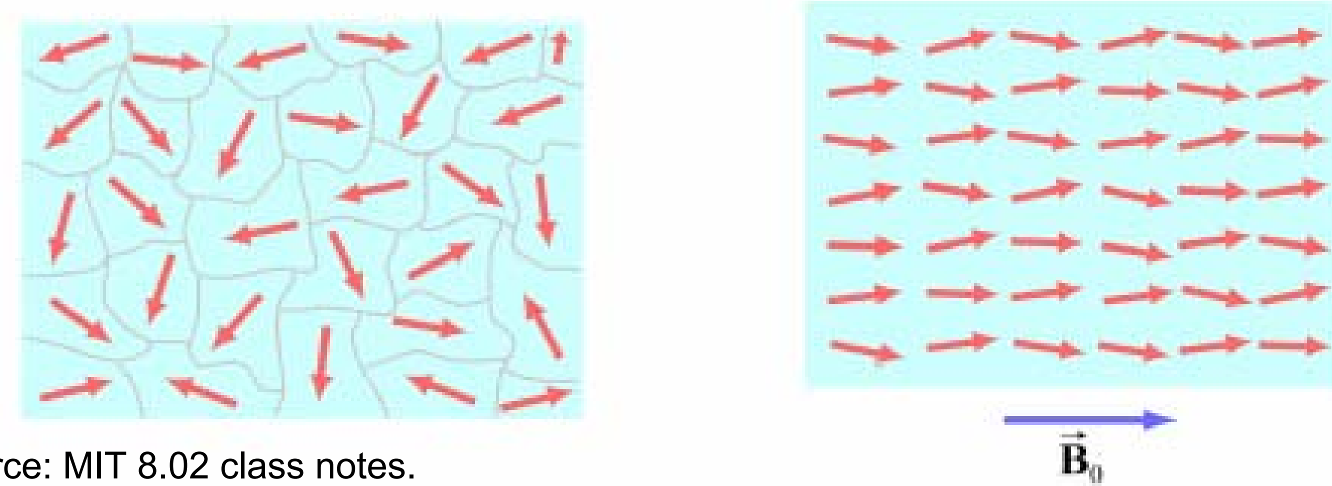


Figure 9.6.3 (a) Ferromagnetic domains. (b) Alignment of magnetic moments in the direction of the external field $\vec{\mathbf{B}}_0$.

$$\vec{B}_{mag} \propto \vec{B}_{ext} \Rightarrow \vec{B}_{media} = \vec{B}_{ext} + \vec{B}_{mag} = \mu_r \vec{B}_{ext} \; (\mu_r \geq 1)$$

$$\nabla \times \vec{B} = \mu_0 \mu_r \vec{J} + \mu_0 \mu_r \varepsilon_0 \varepsilon_r \frac{\partial \vec{E}}{\partial t} = \mu \vec{J} + \mu \varepsilon \frac{\partial \vec{E}}{\partial t}$$

μ_r : relative magnetic permeability of the medium

μ_o : free space permeability ($4\pi \times 10^{-7}$ H/m)

μ of various media

Materials	Magnetic susceptibility χ_m	Relative permeability $\kappa_m = 1 + \chi_m$	Magnetic permeability $\mu_m = \kappa_m \mu_0$
Diamagnetic	$-10^{-5} \sim -10^{-9}$	$\kappa_m < 1$	$\mu_m < \mu_0$
Paramagnetic	$10^{-5} \sim 10^{-3}$	$\kappa_m > 1$	$\mu_m > \mu_0$
Ferromagnetic	$\chi_m \gg 1$	$\kappa_m \gg 1$	$\mu_m \gg \mu_0$

μ_r for water : very close to 1

μ_r (Ni)~600, μ_r (Fe)~5000

Mobility of various ions in water

Species	Mobility U_i ($cm^2/v/s$)	Diffusion coefficient D_i (cm^2/s)
Cations in H_2O (25^oC)		
H^+	36.30×10^{-4}	9.33×10^{-5}
K^+	7.62×10^{-4}	1.96×10^{-5}
Na^+	5.19×10^{-4}	1.33×10^{-5}
Li^+	4.01×10^{-4}	1.03×10^{-5}
Anions in H_2O (25^oC)		
OH^-	20.52×10^{-4}	5.27×10^{-5}
SO_4^{2-}	8.27×10^{-4}	1.06×10^{-5}
Cl^-	7.91×10^{-4}	2.03×10^{-5}
NO_3^-	7.40×10^{-4}	1.90×10^{-5}
Electrons in Si at 25^oC	1500	38.55
Holes in Si at 25^oC	600	15.42

Comparative Number densities and Conductivities

Material	n_i (#/cm^3)	σ ($m^{-1}\Omega^{-1}$)
DI water	$\sim 10^{17}$	4×10^{-6}
0.1M NaCl	6×10^{19}	1.07
Copper	$\sim 10^{22}$	5.8×10^{7}
Si (intrinsic)	$n=p\sim 10^{10}$	3.36×10^{-4}
Si (doped) $N_d=10^{16}$	$n_e=10^{16}$ $N_p=10^{4}$	2.4
Quartz		10^{-18}

In silicon (semiconductor), $n\times p\sim 10^{20}$ (constant)

In aqueous solutions, $[H^+][OH^-] = 10^{-14} = K_w$

($pH= -\log_{10}[H^+]$)