Key Concepts for this section

1: Lorentz force law, Field, Maxwell's equation

2: Ion Transport, Nernst-Planck equation

3: (Quasi)electrostatics, potential function,

4: Laplace's equation, Uniqueness

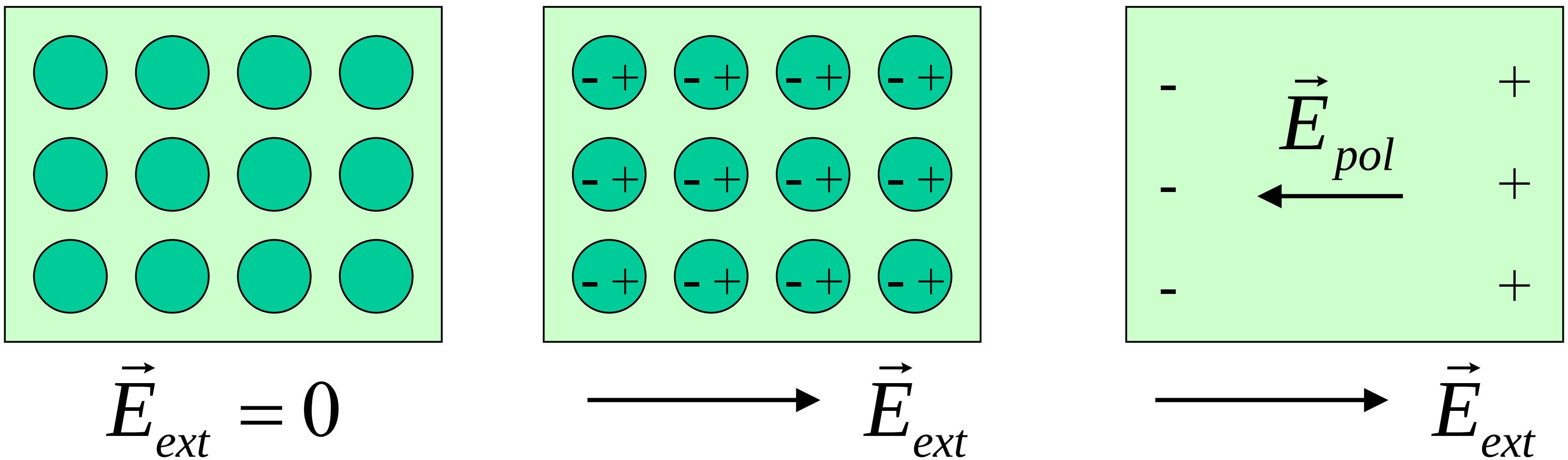
5: Debye layer, electroneutrality

Goals of Part II:

(1) Understand when and why electromagnetic (E and B) interaction is relevant (or not relevant) in biological systems.

(2) Be able to analyze quasistatic electric fields in 2D and 3D.

EM interactions in media - polarization (linear medium)

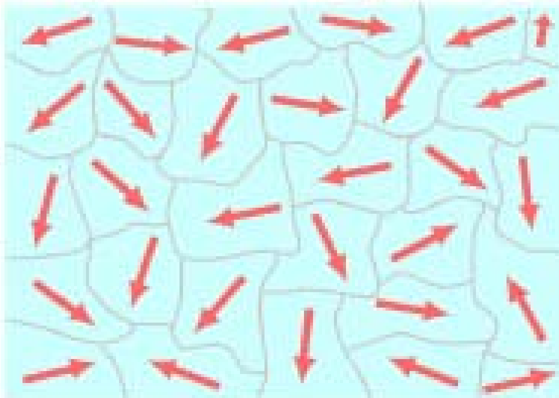


$$\vec{E}_{pol} \propto \vec{E}_{ext} \quad \Rightarrow \quad \vec{E}_{media} = \vec{E}_{ext} - \vec{E}_{pol} = \frac{1}{\varepsilon_r}\vec{E}_{ext}$$

ε_r : relative permittivity (dielectric constant) of the medium (=>1)

ε of various media

Medium	ε_r
Water (pure)	~80
0.9% NaCl solution	~60
Ethanol	24
Methanol	34
Acetic acid	15~16
Gases	~1
Glass	3~4
Plastics and rubbers	2~9



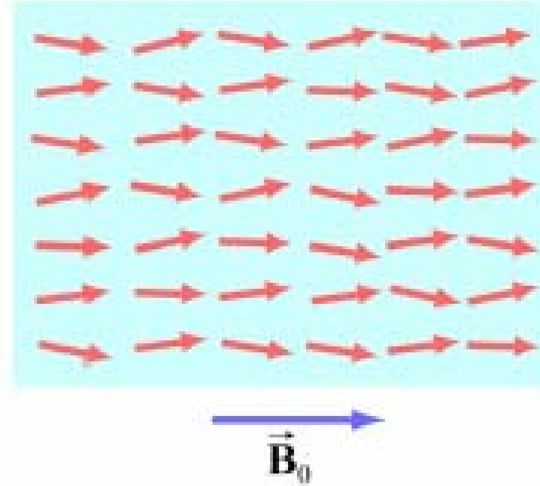


Figure 9.6.3 (a) Ferromagnetic domains. (b) Alignment of magnetic moments in the direction of the external field $\vec{B}_0$.

$$\vec{B}_{mag} \propto \vec{B}_{ext} \Rightarrow \vec{B}_{media} = \vec{B}_{ext} + \vec{B}_{mag} = \mu_r \vec{B}_{ext} \ (\mu_r \geq 1)$$

$$\nabla \times \vec{B} = \mu_0 \mu_r \vec{J} + \mu_0 \mu_r \varepsilon_0 \varepsilon_r \frac{\partial \vec{E}}{\partial t} = \mu \vec{J} + \mu \varepsilon \frac{\partial \vec{E}}{\partial t}$$

μ_r : relative magnetic permeability of the medium

μ_o : free space permeability ($4\pi \times 10^{-7}$ H/m)

μ of various media

Materials	Magnetic susceptibility χ_m	Relative permeability $\kappa_m = 1 + \chi_m$	Magnetic permeability $\mu_m = \kappa_m \mu_0$
Diamagnetic	$-10^{-5} \sim -10^{-9}$	$\kappa_m < 1$	$\mu_m < \mu_0$
Paramagnetic	$10^{-5} \sim 10^{-3}$	$\kappa_m > 1$	$\mu_m > \mu_0$
Ferromagnetic	$\chi_m \gg 1$	$\kappa_m \gg 1$	$\mu_m \gg \mu_0$

μ_r for water : very close to 1

μ_r (Ni)~600, μ_r (Fe)~5000

Mobility of various ions in water

Species	Mobility U_i ($cm^2/v/s$)	Diffusion coefficient D_i (cm^2/s)
Cations in H_2O (25^oC)		
H^+	36.30×10^{-4}	9.33×10^{-5}
K^+	7.62×10^{-4}	1.96×10^{-5}
Na^+	5.19×10^{-4}	1.33×10^{-5}
Li^+	4.01×10^{-4}	1.03×10^{-5}
Anions in H_2O (25^oC)		
OH^-	20.52×10^{-4}	5.27×10^{-5}
SO_4^{2-}	8.27×10^{-4}	1.06×10^{-5}
Cl^-	7.91×10^{-4}	2.03×10^{-5}
NO_3^-	7.40×10^{-4}	1.90×10^{-5}
Electrons in Si at 25^oC	1500	38.55
Holes in Si at 25^oC	600	15.42

Comparative Number densities and Conductivities

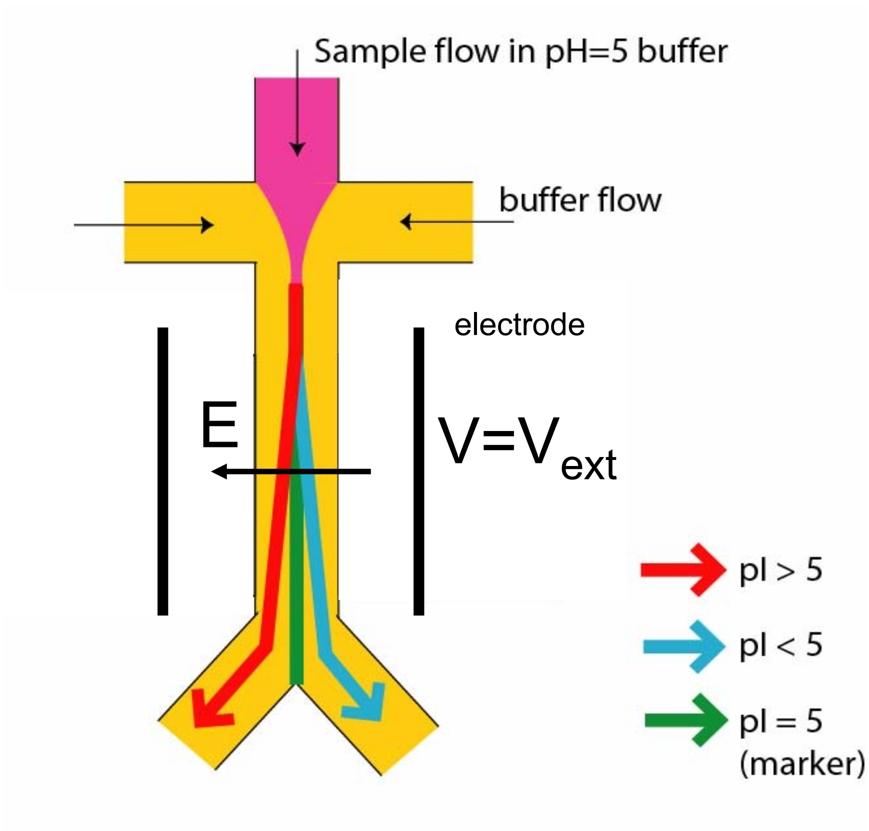
Material	n_i (#/cm^3)	σ ($m^{-1}\Omega^{-1}$)
DI water	$\sim 10^{17}$	4×10^{-6}
0.1M NaCl	6×10^{19}	1.07
Copper	$\sim 10^{22}$	5.8×10^{7}
Si (intrinsic)	$n=p\sim 10^{10}$	3.36×10^{-4}
Si (doped) $N_d=10^{16}$	$n_e=10^{16}$ $N_p=10^{4}$	2.4
Quartz		10^{-18}

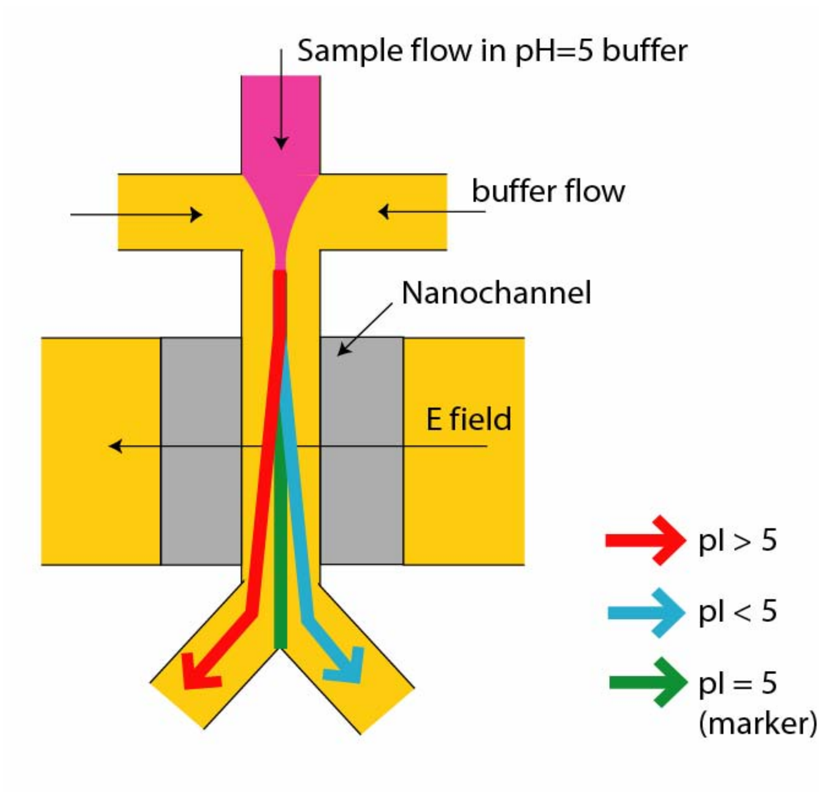
In silicon (semiconductor), $n \times p \sim 10^{20}$ (constant)

In aqueous solutions, $[H^+][OH^-] = 10^{-14} = K_w$

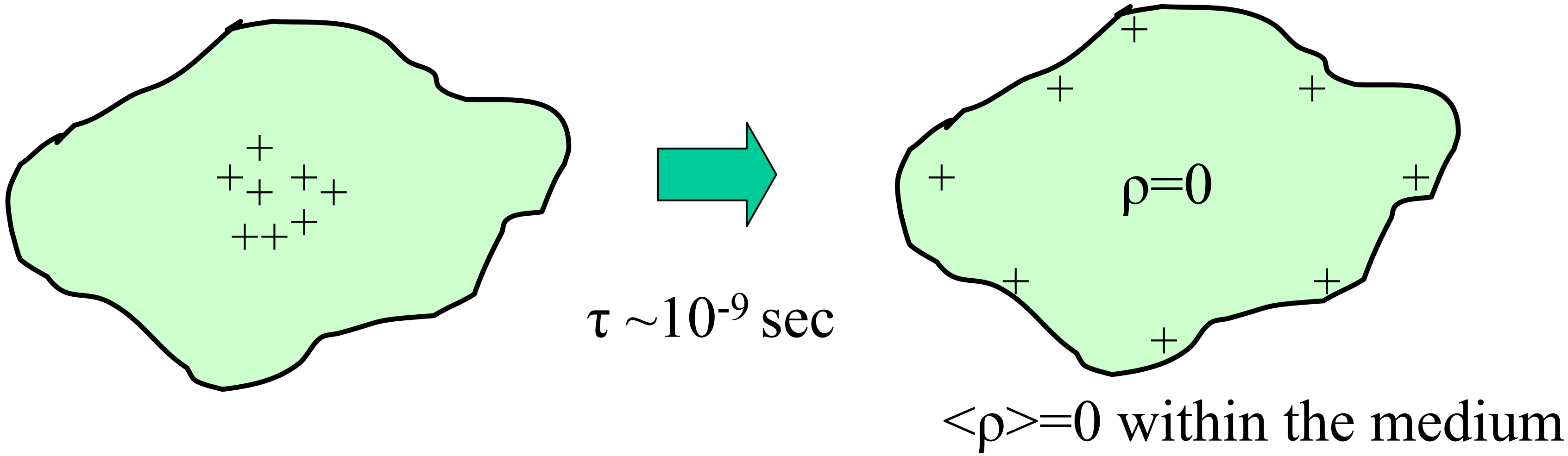
($pH = -\log_{10}[H^+]$)

Electrolytes (biological systems) are conductors.

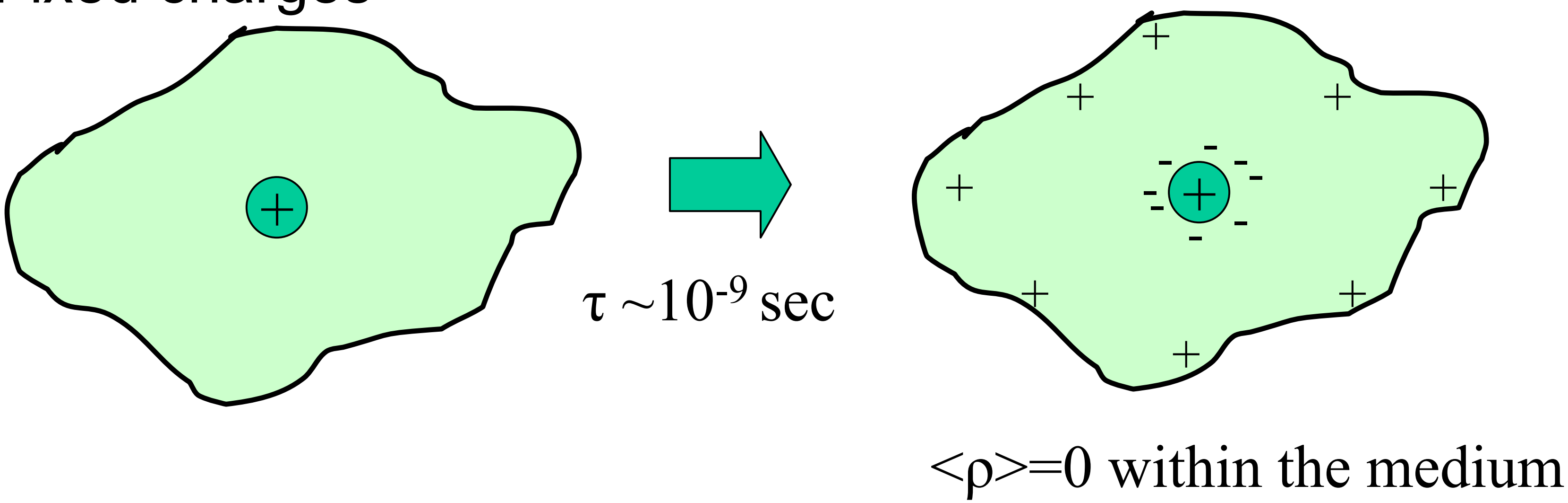




Charge Relaxation in electrolyte

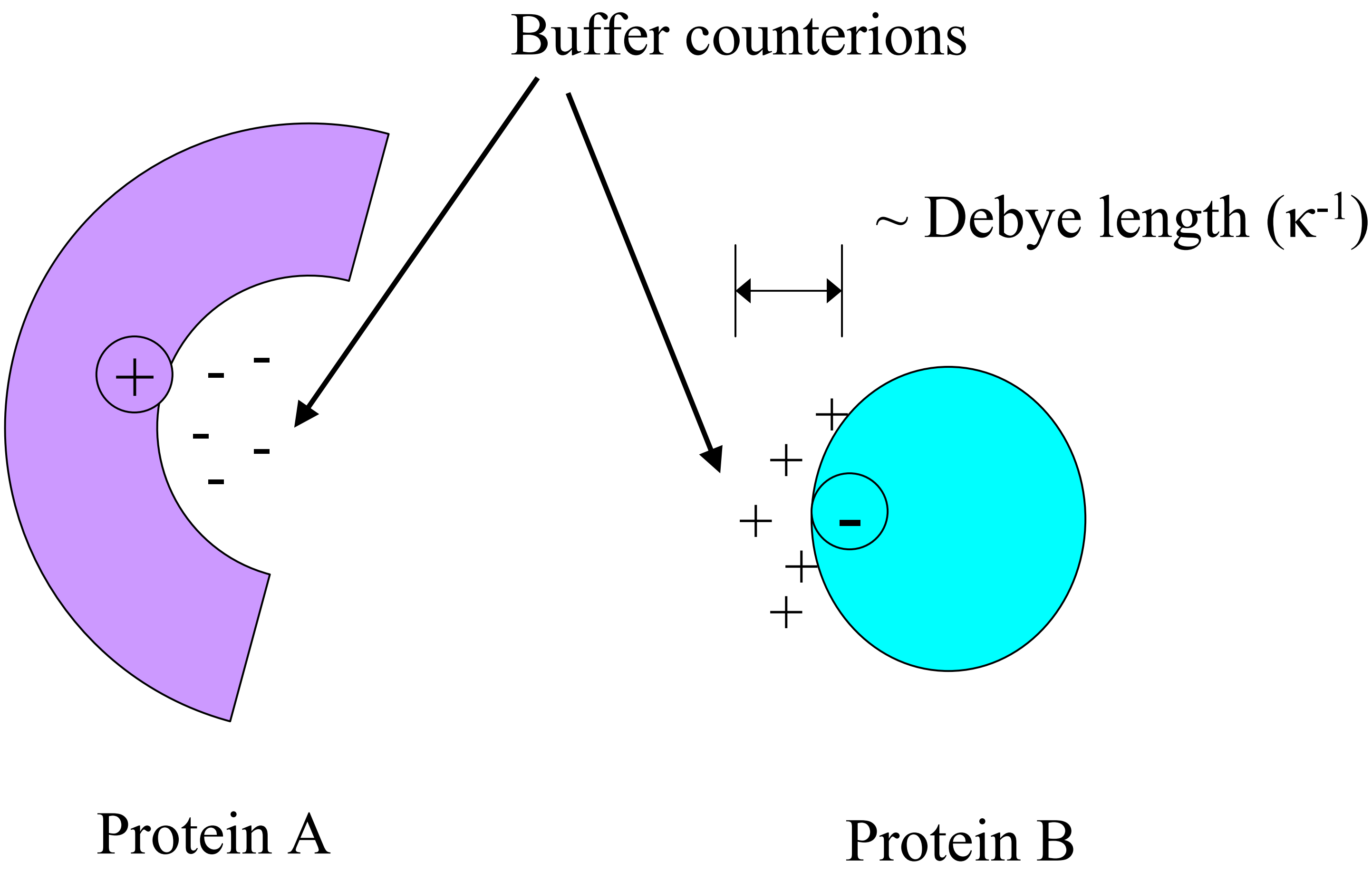


Fixed charges

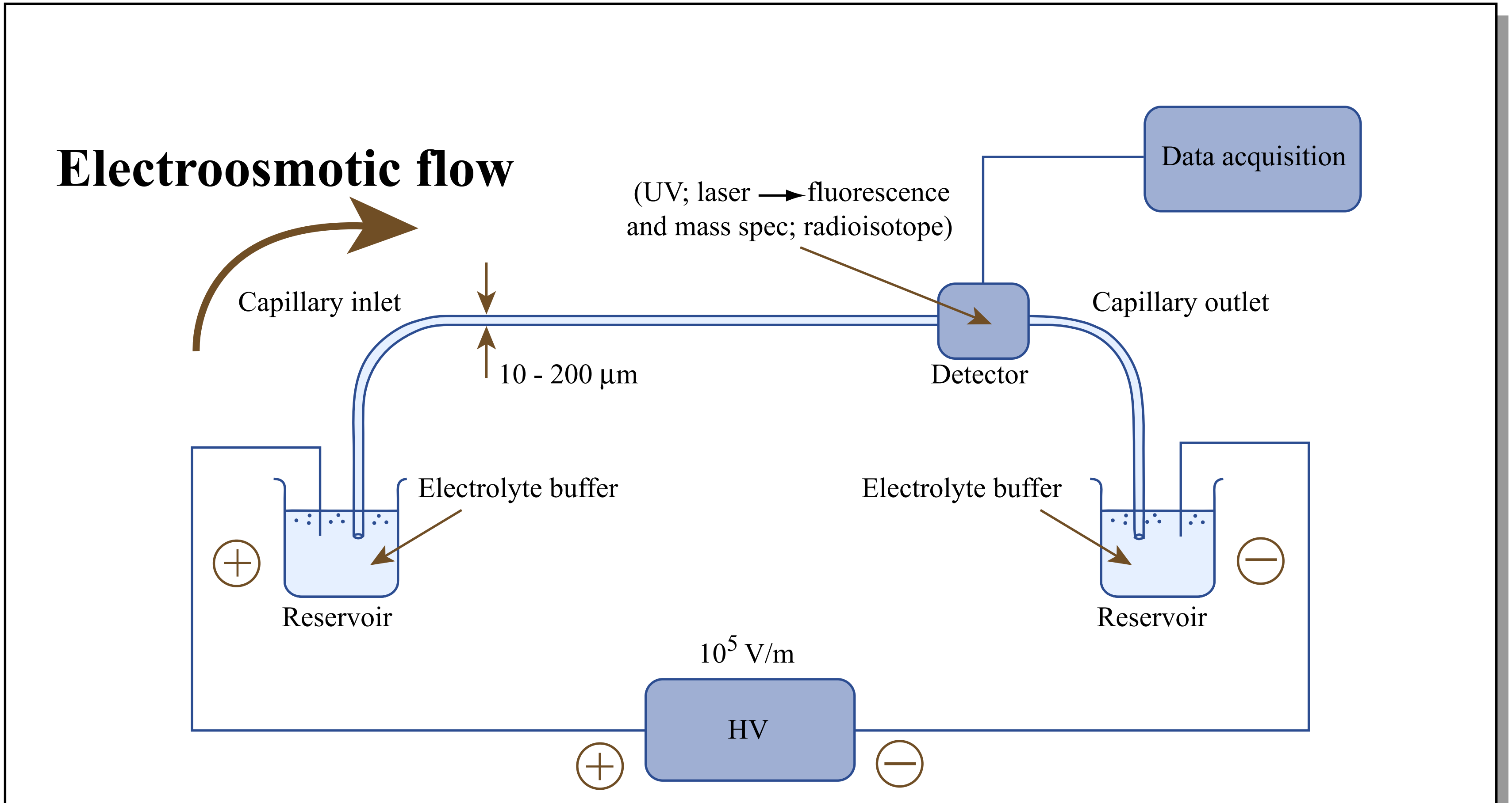


- (Quasi) Electroneutrality Approximation
  - It is an approximation.
    - Valid only after the relaxation time constant τ, and outside of the Debye length (κ^{-1}, to be discussed)
  - Not valid when there is a discontinuity
    - Valid only within a single medium (σ=constant)
    - Boundary of the medium could carry (surface) free charge
  - No inter-charge interaction in liquid media

Electroneutrality



Capillary Electrophoresis (1980s)



Generic diagram of a capillary electrophoresis system.

Figure by MIT OCW.

Micro Total Analysis System (microTAS): Parallelism

- 96~356 samples analyzed in a single chip simultaneously
- fluorescence detection of DNA at the center of the chip (rotating optical head)

Figure 1 removed due to copyright restrictions.

Yining Shi et al., Analytical Chemistry, **71**, 5354 (1999)