Handout: **One Sample Hypothesis Testing And Inference For the Mean**

Hypothesis Testing

I. *A sample from Normal distribution.*

Suppose $X_1, ..., X_n$ is an independent sample from $Normal(\mu, \sigma^2)$ distribution.

Mean	μ	Unknown
Variance	σ^2	Either
Data	$X_1, ..., X_n$	Known
Sample size	n	Known
Sample Mean	$m = \bar{X}$	Known
Standard Deviation	SD	Known
Distribution	$Normal(\mu, \sigma^2)$	Assumed

Reasonable estimate for the population mean μ is

$$m = \bar{X}.$$

Note that

$$E(m) = \mu$$

and

$$SE(m - \mu_0) = \sqrt{Var(m - \mu_0)} = \sqrt{Var(m)}(= SE(m)) = \sqrt{\sigma^2/n} = \sigma/\sqrt{n}.$$

For testing

$$\begin{aligned} H_0 : \quad \mu =& \mu_0 \qquad (1) \\ &\text{against} \\ H_1 : 1)\mu \neq& \mu_0 \text{ or} \\ 2)\mu <& \mu_0 \text{ or} \\ 3)\mu >& \mu_0 \end{aligned}$$

use

$$\text{test statistics } d^{\star}_{obt} = \frac{m - \mu_0}{SE(m)}$$

which follows some distribution d^*.

Theorem. Under the above assumptions about the sample $X_1, ..., X_n$, for testing test H_0: $\mu - \mu_0 = 0$ at α - significance level

vs

1) H_1: $\mu - \mu_0 \neq 0$. Reject H_0 if $|d^*_{obt}| \geq d^*_{crit}(\alpha/2)$

2) H_1: $\mu - \mu_0 < 0$. Reject H_0 if $d^*_{obt} \leq -d^*_{crit}(\alpha)$

3) H_1: $\mu - \mu_0 > 0$. Reject H_0 if $d^*_{obt} \geq d^*_{crit}(\alpha)$

Computation of $SE(m)$ and choice of distribution d^*:

1. σ is known
   $SE(m) = \sigma/\sqrt{n}$
   Test statistics $d^*_{obt} = z_{obt} = \frac{m-\mu_0}{\sigma/\sqrt{n}}$ follows standard Normal distribution z.

2. σ is unknown, but n is large (≥ 30)
   In this case can omit Normality assumption.
   $SE(m) = \sigma/\sqrt{n} \approx SD/\sqrt{n}$ and
   test statistics $d^*_{obt} = z_{obt} = \frac{m-\mu_0}{SD/\sqrt{n}}$ approximately follows standard Normal distribution z.

3. σ is unknown, and n is not large enough (≤ 30)
   $SE(m) = \sigma/\sqrt{n} \approx SD/\sqrt{n}$ and
   test Statistics $d^*_{obt} = t_{obt} = \frac{m-\mu_0}{SD/\sqrt{n}}$ follows t distribution with $df = n - 1$ degrees of freedom.

II. *Proportions*

For a random variable X drawn from a $Binomial(n, p)$ distribution, let $\bar{p} = X/n$. For testing

$$
\begin{aligned}
H_0 : \quad p =& p_0 \\
&\text{against} \\
H_1 : 1) p \neq& p_0 \text{ or} \\
2) p <& p_0 \text{ or} \\
3) p >& p_0
\end{aligned} \tag{2}
$$

use test statistics $d^{\star}_{obt} = \frac{\bar{p}-p_0}{SE(\bar{p}-p_0)}$.

Since for a $Binomial(n, p)$ random variable X and $\bar{p} = X/n$, $Var(\bar{p} - p_0) = Var(\bar{p}) = \frac{p(1-p)}{n}$, for H_0 true (that is, $p = p_0$) have

$$SE(\bar{p} - p_0) = \sqrt{Var(\bar{p} - p_0)} = \sqrt{\frac{p_0(1 - p_0)}{n}}$$

Then

$$\text{test statistics } d^{*}_{obt} = z_{obt} = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

has approximately Normal z distribution if np, $n(1 - p) \geq 10$.

Confidence Intervals

For a test statistics $d^*_{obt} = \frac{m-\mu_0}{SE(m)}$ we reject H_0 if $|d^*_{obt}| \geq d^*_{crit}(\alpha/2)$. If

$$-d^*_{crit}(\alpha/2) < d^*_{obt} < d^*_{crit}(\alpha/2)$$

we conclude that evidence against H_0 is not statistically significant at α - significance level.

Conficence interval for μ is computed by inverting non-rejection region

$$-d^*_{crit}(\alpha/2) < \frac{m-\mu_0}{SE(m)} < d^*_{crit}(\alpha/2)$$

$$-d^*_{crit}(\alpha/2)SE(m) < m-\mu_0 < d^*_{crit}(\alpha/2)SE(m)$$

with $(1-\alpha)100\%$ confidence interval for μ:

$$(m - d^*_{crit}(\alpha/2)SE(m);\ m + d^*_{crit}(\alpha/2)SE(m))$$