A Very Brief Intro to Statistics: t-tests

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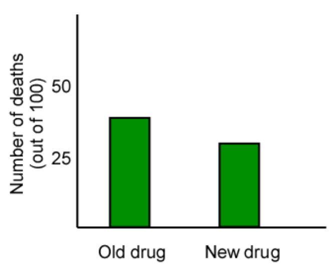
t Test at a glance

$$t = \frac{\text{Difference between groups (means)}}{\text{Normal variability within group(s)}}$$

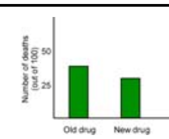
- If *t* is large, the difference between groups is **much bigger** than the normal variability within groups.
  - Therefore, two groups are significantly different from each other
- If *t* is small, the difference between groups is **much smaller** than the normal variability within groups.
  - Therefore, two groups are **not** significantly different from each other

Does a new drug cure cancer better than the old drug?

- The data:



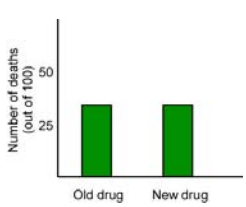
Does a new drug cure cancer better than the old drug?



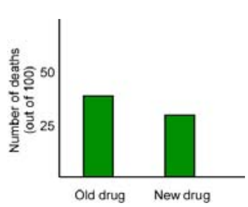
- There's an empirical difference between the old drug and the new drug.
- But is it due to a systematic factor (e.g. the new drug works better) or due to chance?
- If we gave the new drug to 100 more people, would we expect to continue to see improvement over the old drug? Do we expect this effect to *generalize*?

Alt: Is the difference between data & theory due to systematic factors + chance, or to chance alone?

- "Theory" = no difference between the drugs:



- Data:



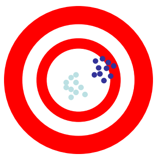
Also called the "null hypothesis"

Chance vs. systematic factors

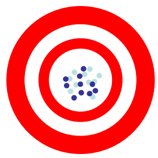
- A *systematic* factor is an influence that contributes a predictable advantage to a subgroup of our observations.
  - E.G. a longevity gain to elderly people who remain active.
  - E.G. a health benefit to people who take a new drug.
- A *chance* factor is an influence that contributes haphazardly (randomly) to each observation, and is unpredictable.
  - E.G. measurement error

Systematic + chance vs. chance alone: Is archer A better than archer B?

- Likely systematic + chance variation:



- Likely due to chance alone:



Observed effects can be due to:

A. Chance effects alone (all chance variation).
  - Often occurs. Often boring because it suggests the effects we're seeing are just random.
  - *Null hypothesis*

B. Systematic effects plus chance.
  - Often occurs. Interesting because there's at least some systematic factor.
  - *Alternative hypothesis*

C. Systematic effects alone (no chance variation).
  - We're interested in systematic effects, but this almost never happens!

An important part of statistics is determining whether we've got case A or B.

We have a natural tendency to over-estimate the influence of systematic factors

- The lottery is entirely a game of chance (no skill), yet subjects often act as if they have some control over the outcome. (Langer, 1975).
- We tend to feel that a person who is grumpy the first time we meet them is fundamentally a grumpy person. (The "fundamental attribution error," Ross, 1977.)

The purpose of statistics

- As researchers, we need a principled way of analyzing data, to protect us from inventing elaborate explanations for effects in data that could have occurred predominantly due to chance.

Example

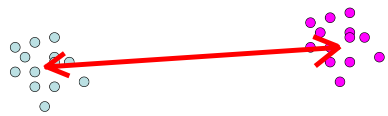
- You have subjects memorize lists of words, and record how many they can remember.
- Does the number they can remember depend upon word length?

	long words	short words
	4	4
	8	5
	9	6
	6	4
	6	5
	9	6
mean	7	5

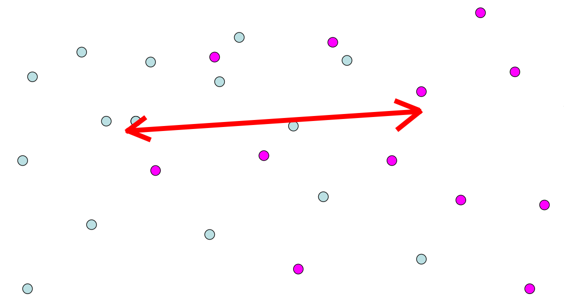
Today we'll test whether the difference in means is "significant," using a "t-test"

- "Significant" = a difference in means this big is unlikely to have occurred by chance
  - Thus there's likely to be a systematic, generalizable effect.
- Let's get some intuitions: what might determine whether or not we think a difference in means is "significant"?

Is the difference in mean of these 2 groups systematic, or just due to chance?



What about this difference in mean?



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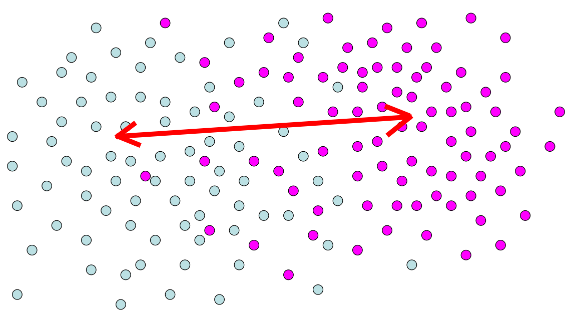

Normal distribution of data for two conditions

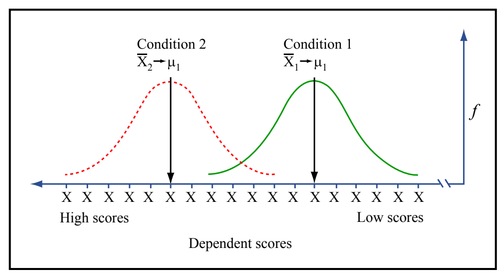

Figure by MIT OpenCourseWare.

Intuitions: Significant difference in means

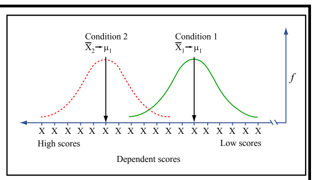

Figure by MIT OpenCourseWare.

- Occurs when the difference in means is large compared to the spread (e.g. variance s^2 or standard deviation s) of the data.
  - $t_{stat} \approx (m_1 - m_2) / s$
- Depends upon the number of samples.
  - With more samples, we're willing to say a difference is significant even if the variance is a bit larger compared to the difference in means.
    - $t_{stat} = (m_1 - m_2)$ / standard error (SE)
    - Standard error = something like s/sqrt(n)
    - Degree of freedom (df) describe the number of samples (n-1)

t-tests

- In general, we'll compute from our data some t_{stat}, of the form:

  $t_{stat} = (m_1 - m_2)/SE$

- t_{stat} is a measure of how reliable of a difference we're seeing between the two conditions.

- If this number is "big enough" we'll say that there is a *significant difference* between the two conditions.

The *t* Test

$$t = \frac{\text{Difference between groups (means)}}{\text{Normal variability within group (or standard error SE)}}$$

- If *t* is large, the difference between groups is **much bigger** than the normal variability within groups.
  - Therefore, two groups are significantly different from each other
- If *t* is small, the difference between groups is **much smaller** than the normal variability within groups.
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t-tests

- Would like to set a threshold, t_{crit}, such that $\mathbf{t_{stat} > t_{crit}}$ means the difference we see between the conditions is unlikely to have occurred by chance (and thus there's likely to be a real systematic difference between the two conditions).

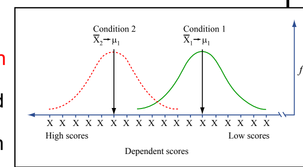

Figure by MIT OpenCourseWare.

- How big is t_{stat} likely to be if there's actually *no difference between the two conditions?*

Read t-crit estimation in a table

In the word experiment

(cf. t-testdemo excel file):

df=5, level of significance is 0.05

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OK, so here's the general plan:

- Compute t_{stat} and df from your data (cf. T-TestDemo.xls)
- Decide upon a level of *confidence (significance)*. 99% and 95% are typical. => *significance level,* α = 0.01 or 0.05
- From this, and a t-table, find t_{crit}
- Compare t_{stat} to this threshold.
  - If $|t_{stat}|>|t_{crit}|$, "the difference is significant", there's likely an actual difference between the two conditions.
  - If not, the difference is "not significant."

3 kinds of t-tests

- Case 1: The two samples are *related*, i.e. not independent (e.g. the same subject did the 2 conditions of your experiment)
- Case 2: The samples are independent (e.g. different subjects), and the variances of the populations are *equal*.
- Case 3: The samples are independent, and the variances of the populations are *not equal*.

All tests are of the same form. We just need to know, for each case, how to compute SE (and thus t_{stat}), and what is df.

Case 1: When do you have related or paired samples?

- When you test each subject on both conditions.
  - E.G. You ask 100 subjects two geography questions: one about France, and the other about Great Britain. You then want to compare scores on the France question to scores on the Great Britain question.
  - These two samples (answer, France, & answer, GB) are not independent – someone getting the France question right may be good at geography, and thus more likely to get the GB question right.

Case 1: When do you have related or paired samples?

- When you have "matched samples".
  - E.G. You want to compare weight-loss diets A and B.
  - How well the two diets work may well depend upon factors such as:
    - How overweight is the dieter to begin with?
    - How much exercise do they get per week?
  - Match each participant in group A as nearly as possible to a participant in group B who is similarly overweight, and gets a similar amount of exercise per week.

Excel demo: Related samples t-test

- Let x_i and y_i be a pair in the experimental design
  - The scores of a matched pair of participants, or
  - The scores of a particular participant, on the two conditions of the experiment
- Let $D_i = (x_i – y_i)$
- Compute SE = stdev(D_i)/sqrt(n)
- $t_{stat} = (m_1 – m_2)/SE$,
- df = n-1 = # of pairs - 1

Case 2: Independent samples, equal variances

- Independent samples may occur, for instance, when the subjects in condition A are different from the subjects in condition B (e.g. most drug testing).
- Either the sample variances look very similar, or there are theoretical reasons to believe the variances are roughly the same in the two conditions.

Excel demo
Case 2: Independent samples, equal variances

- $t_{stat} = (m_1 – m_2)/SE$
- $SE = sqrt(s_{pool}^2\ (1/n_1 + 1/n_2))$
- $s_{pool}^2 = [(n_1 – 1)s_1^2 + (n_2 – 1)s_2^2]/(n_1 + n_2 – 2)$
- This is like an average of estimates s_1^2 and s_2^2, weighted by their degrees of freedom, $(n_1 – 1)$ and $(n_2 – 1)$, i.e. essentially by the number of samples used to compute s_1^2 and s_2^2.
- $df = n_1 + n_2 – 2$

Case 3: Independent samples, variances not equal

- The samples variances may be very different, or one may have theoretical reasons to suspect that the variances are not the same in the two conditions.
  - E.G. the response of healthy people to a drug may be more uniform than the response of sick people.
  - E.G. one high school may have students with a bigger range in the education of the students' parents, and one might thus expect a bigger range of test scores.

Excel demo: Case 3: Independent samples, variances not equal

- $t_{stat} = (m_1 – m_2)/SE$
- $SE = sqrt(s_1^2/n_1 + s_2^2/n_2)$
- For equal variances: d.f. = $n_1 + n_2 – 2$
- Unequal variances:

$$\text{d.f.} = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

How many subjects per level (condition) should you run?

- How many subjects to use depend on how much variability you expect in your data
- The more subjects you have, the less the means of the data will deviate from their true value
- The usual way of representing this error of measurement is called the **standard error of the mean (s.e.m)**
- Increasing the number of subjects does not decrease the error of measurement in a linear way.
- **Nb participants ? ~ 10 / condition, from 12-20 participants, results should be stable**

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How many subjects should you test?

- Doubling the number of subjects (from 10 to 20) reduces the s.e.m by only 30 % (theoretical case)

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