# 1.1.1

## EXPRESSING SYSTEMS OF LINEAR ALGEBRAIC EQUATIONS AS: $A\,\underline{x} = \underline{b}$

We wish to solve systems of simultaneous linear algebraic equations of the general form:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\
&\vdots \\
&\vdots \\
a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= b_n \qquad \textbf{(1.1.1-1)}
\end{aligned}
$$

Where we have N equations for the N unknowns $x_1, x_2, \ldots, x_n$.

As a particular example, consider the following set of these three equations (N=3) for the three unknowns $x_1, x_2, x_3$:

$$x_1 + x_2 + x_3 = 4$$

$$2x_1 + x_2 + 3x_3 = 7$$

$$3x_1 + x_2 + 6x_3 = 2 \qquad \textbf{(1.1.1-2)}$$

$a_{ij}$ = constant coefficient (usually real) multiplying unknown $x_j$ in equation #i.
$B_i$ = constant "right-hand-side" coefficient for equation #i.

For the system (1.1.1-2) above,

| | | | |
|---|---|---|---|
| $a_{11} = 1$ | $a_{12} = 1$ | $a_{13} = 1$ | $b_1 = 4$ |
| $a_{21} = 2$ | $a_{22} = 1$ | $a_{23} = 3$ | $b_2 = 7$ |
| $a_{31} = 3$ | $a_{32} = 1$ | $a_{33} = 6$ | $b_3 = 2$ |

It is common to write linear systems in matrix/vector for as:

$$A\,\underline{x} = \underline{b} \qquad \textbf{(1.1.1-3)}$$

Where the vector of unknowns $\underline{x}$ is written as:

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ : \\ : \\ x_n \end{bmatrix} \qquad \textbf{(1.1.1-4)}$$

The vector of right-hand-side coefficients $\underline{b}$ is written:

$$\underline{b} = \begin{bmatrix} b_1 \\ b_2 \\ : \\ : \\ b_n \end{bmatrix} \qquad \textbf{(1.1.1-5)}$$

The matrix of coefficients A is written in a form with N rows and N columns,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ : & : & : & & : \\ : & : & : & & : \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \qquad \textbf{(1.1.1-6)}$$

We see that row 'i' contains the values $a_{i1}, a_{i2}, \dots, a_{iN}$ that are the coefficients multiplying each unknown $x_1, x_2, \dots, x_N$ in equation #i.

Rows ⇔ equations

Columns ⇔ coefficients multiplying a specific unknown in each equation.

$a_{ij}$ = element of A in ith row and jth column
= coefficient multiplying $x_j$ in equation #i.

After we will write the coefficients in matrix form explicitly, so that we may write A $\underline{x} = \underline{b}$ as:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2n} \\ : & : & : & & : \\ : & : & : & & : \\ a_{n1} & a_{n2} & a_{n3} & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2] \\ : \\ : \\ x_N \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ : \\ : \\ b_n \end{bmatrix} \qquad \textbf{(1.1.1-7)}$$

For the example system **(1.1.1-2)**:

$$x_1 + x_2 + x_3 = 4$$

$$2x_1 + x_2 + 3x_3 = 7$$

$$3x_1 + x_2 + 6x_3 = 2 \qquad \textbf{(1.1.1-2, repeated)}$$

We have:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 6 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 4 \\ 7 \\ 2 \end{bmatrix} \qquad \textbf{(1.1.1-8)}$$

As we will represent our linear systems as matrices “acting on” vectors, some review of basic vector notation is required.