10.34 Quiz 1, October 4, 2006

A perfectly-stirred reactor (a CSTR) is loaded with cells which multiply at a rate:

$$Cell\ \ Multiplication = \frac{k_1 N_{cells}\left[Nutrients\right]}{\left(1+c_1\left[Nutrients\right]\right)\left(1+d\left[P\right]\right)}$$

So the net balance on cells in the CSTR is:

$$\frac{dN_{cells}}{dt} = Cell\ Multiplication - Rate\ at\ which\ cells\ \ flow\ out\ of\ \ reactor$$

A nutrient/water mixture with $[\text{Nutrients}]_{\text{in}}$ = 0.2 M is flowed into the 150 liter CSTR at a rate of 2.3 liter/min. The cells in the CSTR eat some of the nutrients:

$$Nutrient\ Consumption\ Rate = k_2 N_{cells} + c_2\left(Cell\ Multiplication\right)$$

and produce a desired pharmaceutical product P:

$$P\ production\ rate = \frac{k_3 N_{cells}\exp\left(-d\left[P\right]\right)}{\left(1+c_1\left[Nutrients\right]\right)}\cdot\left(\left[Nutrients\right]-0.01\right)^2$$

The parameters are :

$k_1 = 0.5 \quad M^{-1}s^{-1}$	$c_1 = 0.1 \quad M^{-1}$
$k_2 = 1\times10^{-7}\ \ moles\ cell^{-1}\ s^{-1}$	$c_2 = 1\times10^{-5}\ moles\ cell^{-1}$
$k_3 = 1\times10^{-6}\ \ moles\ cell^{-1}\ s^{-1}\ M^{-1}$	$d = 0.01 \quad M^{-1}$

(a) Write a couple of Matlab functions that together compute the concentrations [P] and [Nutrients] (units: M = moles/liter), as well as the number of cells per liter, in the output stream when the system is operated at steady-state. Give numerical values for all the inputs. Do you think that scaling will be a problem? Explain and give an appropriate scaling factor if necessary.

(b) If your program from part (a) works correctly, how would you test whether the solution found is physical and achievable (i.e. stable)? (Explain in words; bonuses for giving correct relevant equations and/or Matlab functions).

(c) If your program from part (a) converges to an unphysical or unstable solution, what would you do next to try to find an experimentally-relevant steady-state solution? (Explain in words; bonuses for giving correct relevant equations and/or Matlab functions.)

For your convenience, on the next page we have supplied the Matlab functions `Cell_Multiplication.m`, `Nutrient_Consumption.m`, `P_production.m`, and `param_set.m`, which you can call with the functions you write yourself. You can also call any built-in functions in Matlab and any functions presented as examples in class.

```
function CMrate = Cell_Multiplication(Ncells,Nutrients,P,params)
% computes the Cell Multiplication rate (cells/s) in the CSTR
% inputs:
%   Ncells      number of cells in the CSTR
%   Nutrients   concentration of Nutrients in the CSTR (moles/liter)
%   P           concentration of Product in CSTR, [=] moles/liter
%   params      values of [k1,k2,k3,c1,c2,d] as listed in problem statement.
k1 = params(1); c1=params(4); d=params(6);
CMrate = k1.*Ncells.*Nutrients./((1+c1.*Nutrients).*(1+d.*P));
```

```
function NCrate = Nutrient_Consumption(Ncells,Nutrients,P,params)
% computes the nutrient consumption rate in the CSTR (moles/s)
% inputs:
%   Ncells       number of cells in the CSTR
%   Nutrients    concentration of Nutrients in the CSTR (moles/liter)
%   P            concentration of Product in CSTR, [=] moles/liter
%   params       values of [k1,k2,k3,c1,c2,d] as listed in problem
statement.
k2 = params(2);    c2 = params(5);
NCrate = k2*Ncells + c2*Cell_Multiplication(Ncells,Nutrients,P,params);
```

```
function Prate = P_production(Ncells,Nutrients,P,params)
% computes the product production rate in the CSTR (moles/s)
% inputs:
%   Ncells       number of cells in the CSTR
%   Nutrients    concentration of Nutrients in the CSTR (moles/liter)
%   P            concentration of Product in CSTR, [=] moles/liter
%   params       values of [k1,k2,k3,c1,c2,d] as listed in problem
statement.
k3 = params(3); d=params(6); c1=params(4);
Prate = k3.*Ncells.*exp(-d.*P).*((Nutrients-0.01).^2)./(1+c1.*Nutrients)
```

```
function params = param_set
% sets params as in the problem statement
% [k1  k2  k3  c1  c2  d]
params = [0.5 1e-7 1e-6 0.1 1e-5 0.01];
```