

KALMAN FILTER: A REVIEW

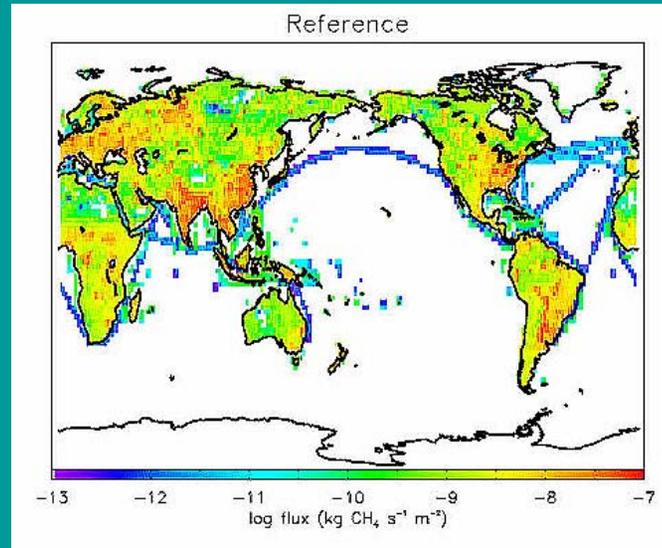
Table 1: Kalman Filter Equations*

Definition	Equation
Measurement equation (model)	$y_k^o = H_k x_k^t + \varepsilon_k; \quad y_k = H_k x_k^f$
System (state) equation (model)	$x_k = M_{k-1} x_{k-1} + \eta_{k-1}$
State update	$x_k^a - x_k^f = K_k (y_k^o - y_k)$
Error Update	$P_k^a = (1 - K_k H_k) P_k^f$
Kalman gain update	$K_k = P_k^f H_k^T (H_k P_k^f H_k^T + R_k)^{-1}$
State time extrapolation	$x_k^f = M_{k-1} x_{k-1}^a$
Error time extrapolation	$P_k^f = M_{k-1} P_{k-1}^a M_{k-1}^T + Q_{k-1}$
System random forcing covariance	$Q_k = E(\eta_k \eta_k^T)$
Measurement error covariance	$R_k = E(\varepsilon_k \varepsilon_k^T)$
Estimation error covariance	$P_k = E(v_k v_k^T)$
Input measurement matrix	$= H_k = \partial y_k / \partial x_k$
Input system random forcing covariance	$= Q_k$
Input state extrapolation	$= M_k$
Input measurement	y_k^o
Input measurement error covariance	$= R_k$
Filter iteration	$--- \rightarrow (k-1)^f, \rightarrow$ estimate $\rightarrow (k-1)^a, \rightarrow$ extrapolate $\rightarrow (k)^f, \rightarrow ---$

*A superscript a or superscript f denotes respectively the value before (f) or after (a) an update of an estimate using measurements, and k denotes the measurement number. In general, errors are assumed random with zero mean and measurement and estimation errors are uncorrelated.

REVIEW OF THE CH₄ INVERSE PROBLEM

Emissions from seven seasonally varying (3 wetland, 3 burning, rice) & two steady sources (animals & waste, coal & gas) to be optimally estimated as amendments to the (a priori) reference



Rest of the images removed due to copyright considerations.

See Figure 2. Chen, Y.-H. and R.G. Prinn, Estimation of atmospheric methane emissions between 1996-2001 using a 3D global chemical transport model, *Journal of Geophysical Research*, 111, D10307, doi:10.1029/2005JD006058, 2006.

Image removed due to copyright considerations.

See Figure 7. Chen, Y.-H. and R.G. Prinn, Estimation of atmospheric methane emissions between 1996-2001 using a 3D global chemical transport model, *Journal of Geophysical Research*, 111, D10307, doi:10.1029/2005JD006058, 2006.

**CAPTURES EXPECTED SEASONAL CYCLES
(RICE PEAKS EARLIER)**

SIGNIFICANT YEAR-TO-YEAR EMISSION VARIATIONS

MONTHLY ANOMALIES
(from 5-year mean annual cycles) using data from: (a) high frequency in situ sites and flask sites (blue lines), and (b) high frequency sites only (red lines)

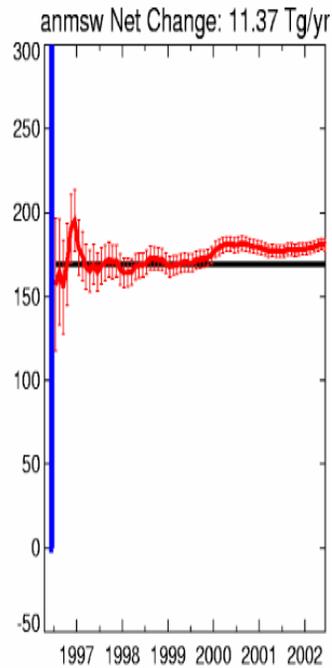
*Note different vertical scale for each process

Images removed due to copyright considerations.

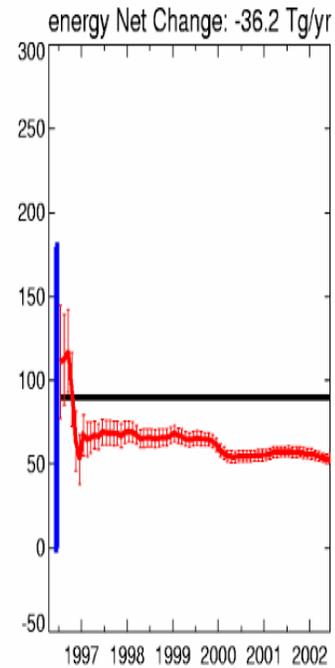
See the 8 sub-figures on the left in Figure 9. Chen, Y.-H. and R.G. Prinn, Estimation of atmospheric methane emissions between 1996-2001 using a 3D global chemical transport model, Journal of Geophysical Research, 111, D10307, doi:10.1029/2005JD006058, 2006.

**Estimation
of the
steady fluxes
and
errors**

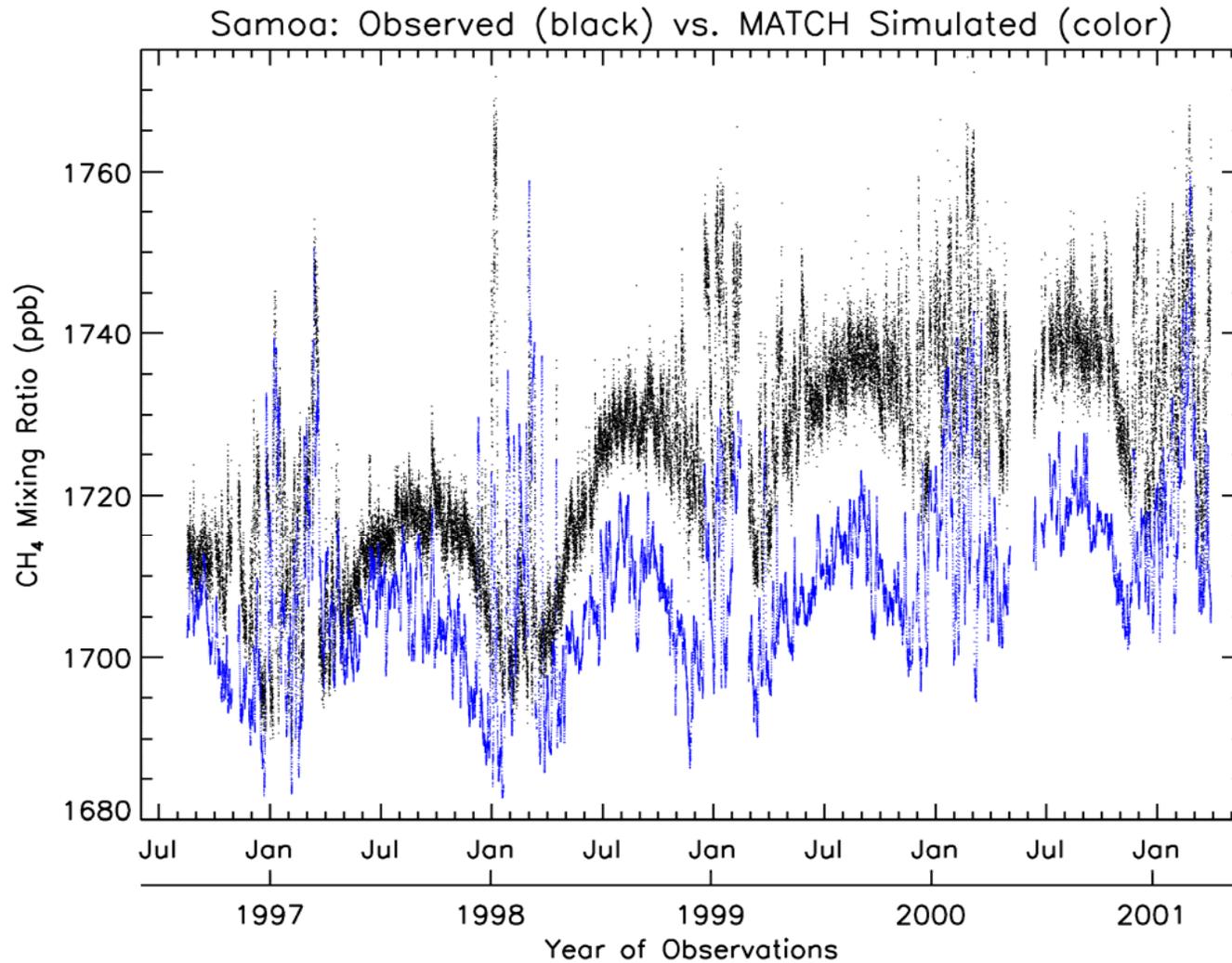
**ANIMALS &
WASTE**



**ENERGY
(COAL,GAS,etc.)**



Samoa observations versus model before inversion



Samoa observations versus model after inversion
Model with optimized emissions simulates observations
at almost all sites but Samoa is odd in 1997-1999

WAS THERE A (tropics-weighted) OH DECREASE IN 1997-1999??

Image removed due to copyright considerations.

See Figure 12. Chen, Y.-H. and R.G. Prinn, Estimation of atmospheric methane emissions between 1996-2001 using a 3D global chemical transport model, Journal of Geophysical Research, 111, D10307, doi:10.1029/2005JD006058, 2006.

Global weighted average OH inferred from AGAGE CH_3CCl_3

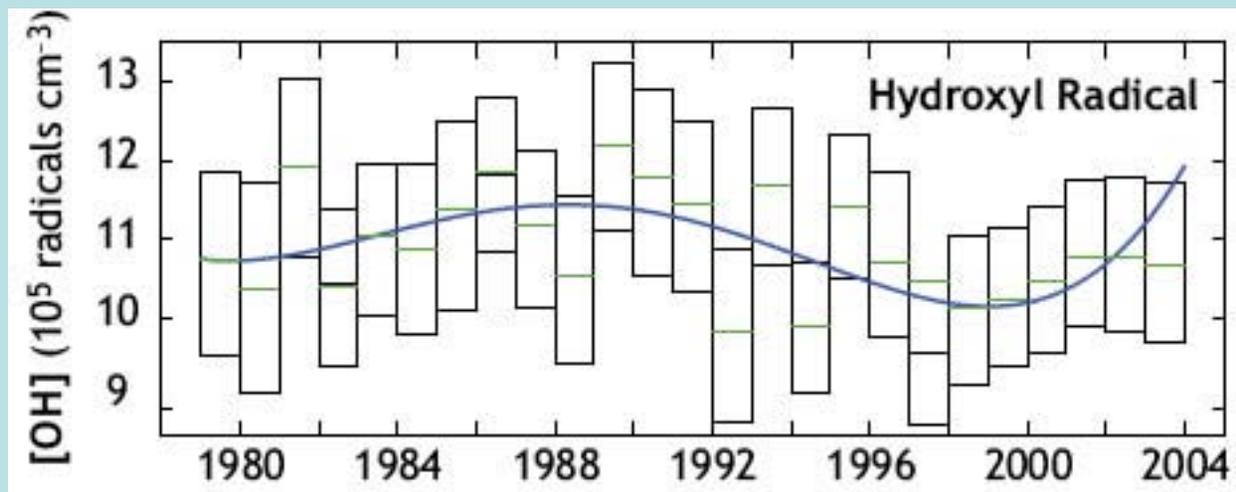


Image courtesy of AGAGE.

The inferred circa-1998 OH minimum coincides with massive global wildfires and a strong El Niño.

Summary: Interannual variability (Monthly Anomalies)

**32-33 Tg yr⁻¹ Total
Emission increase in 1998 with
8-17 Tg yr⁻¹ due to Rice regions !**

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copyright considerations.

See the 8 sub-figures on the
right in Figure 9. Chen, Y.-H.
and R.G. Prinn, Estimation of
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transport model, Journal of
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2006.

**Northern/Tropical Wetland
and Rice Region Emissions
dominate the total variability!**

1998 wetland Flux Anomalies

Fluxes in Tg yr ⁻¹	Northern Wetlands	Tropical Wetlands
Inversion	5-10	8.3-9.9
Bottom-up*	12	13

***wetland model driven by 1998 record
temperature and large precipitation
anomalies (Dlugokencky et al. (2001))**

**BUT Boreal Fires in Siberia may have
also contributed to our deduced
strong Northern wetlands increase!**

Summary: 5-year averages

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See Figure 8. Chen, Y.-H. and R.G. Prinn, Estimation of atmospheric methane emissions between 1996-2001 using a 3D global chemical transport model, Journal of Geophysical Research, 111, D10307, doi:10.1029/2005JD006058, 2006.

COMPARED TO PREVIOUS ESTIMATES:

- (1) ENERGY RELATED EMISSIONS SMALLER (RUSSIAN GAS LEAKS?)**
- (2) RICE RELATED EMISSIONS LARGER (PROXIMAL WETLANDS OR TROPICAL ECOSYSTEMS?)**