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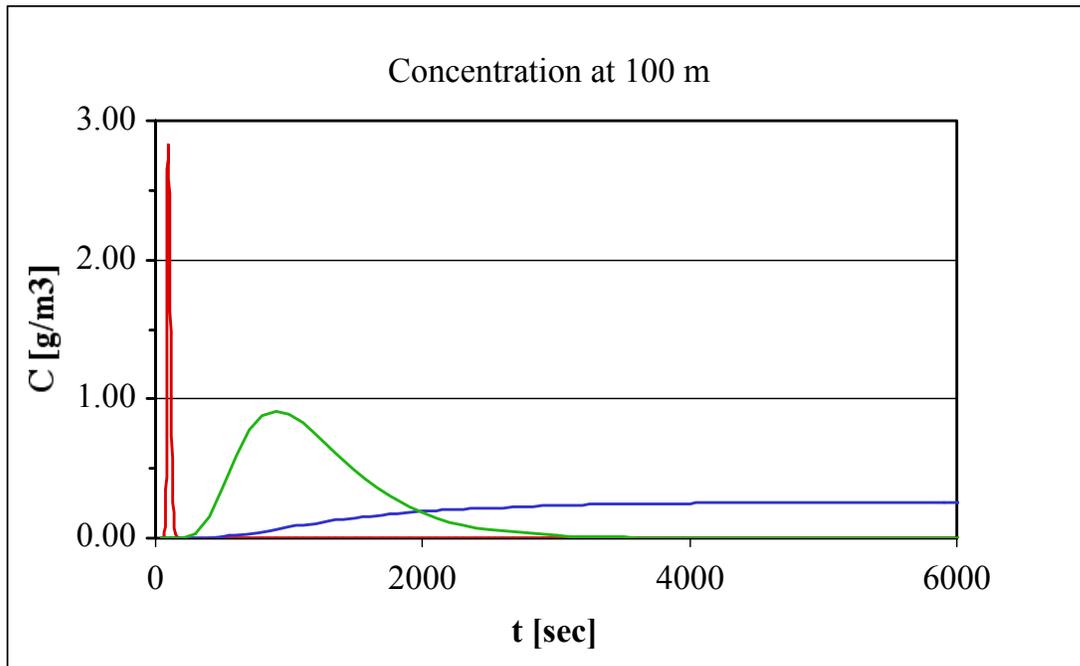
1.061 / 1.61 Transport Processes in the Environment  
Fall 2008

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**Problem 5.1.** One gram of dye is injected into a long, thin tube and instantly mixes across the cross-section,  $A = 1 \text{ cm}^2$ . Water is pumped through the tube at  $Q = 0.1 \text{ cm}^3\text{s}^{-1}$ . Assume that the flow is uniform across  $A$ . If the flow is laminar, assume a molecular diffusion coefficient,  $D_M = 10^{-5} \text{ cm}^2\text{s}^{-1}$ . Otherwise, assume  $D = 100 D_M$ . The concentration is measured at  $x = L = 100 \text{ cm}$  downstream from the injection point. Estimate the length of the dye patch as it passes  $x = L$  and the maximum concentration experienced at this point.

**Problem 5.2.** A 1-kg mass of dye is dumped into a  $10\text{-m}^2$  channel at a location designated as  $x = 0$ . Downstream, at  $x = L = 100 \text{ m}$ , the concentration of the dye is measured. The test is repeated for three different flow rates,  $u = 0.001, 0.1, 1 \text{ ms}^{-1}$ . For simplicity, the diffusion coefficient is  $D = 1 \text{ m}^2\text{s}^{-1}$  for each condition.

- For each case estimate the advection and diffusion time-scales, and the Peclet Number.
- The curves below represent the three cases. Label each curve with the appropriate Peclet number.
- For which curves will the peak concentration observed at  $L$  occur at the advection time scale? Calculate the peak concentration for these cases.
- The injection of mass required 10 seconds. Considering the other relevant time scales, can this injection be considered instantaneous?



**Problem 5.3.**

You are the safety officer for a new factory with a permit to discharge a treated waste stream into the center of a river at a flow rate of  $2 \text{ m}^3\text{s}^{-1}$ . The river has a square cross-section with an area of  $20\text{m}^2$ . Under standard operating procedures the discharge is fully treated before release. The discharge is monitored, and if the chemical concentration in the waste stream becomes elevated above a safety threshold of  $100 \text{ mg l}^{-1}$  the factory will shut-down. However, the response time for the shut down is 100 seconds. You are asked to prepare an environmental impact report for a fish hatchery located 1-km downstream of the plant. The hatchery draws water from the river to flush the hatchery tanks. Based on studies of chronic exposure to juvenile fish the safety standard at the hatchery is set at  $C = 0.5 \text{ mg l}^{-1}$

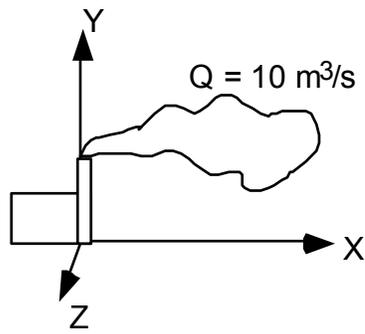
Write an expression describing the concentration experienced at the hatchery. Then, for each scenario below find the peak concentration and the duration of exposure at the intake to the fish hatchery. Then describe the exposure threat - considering duration and level - for each flow regime and state whether you would issue a water quality warning.

Case 1: High Discharge.  $U = 50 \text{ cm s}^{-1}$ ,  $D_x = 10 \text{ m}^2\text{s}^{-1}$ ,  $D_Y = D_Z = 0.1 \text{ m}^2\text{s}^{-1}$   
Case 2: Low Discharge.  $U = 2 \text{ cm s}^{-1}$ ,  $D_x = 1 \text{ m}^2\text{s}^{-1}$ ,  $D_Y = D_Z = 0.01 \text{ m}^2\text{s}^{-1}$

**Hint 1 - Start by defining the system dynamics using time scales**

**Problem 5.4**

A chemical plant releases exhaust gas at a flow rate of  $10 \text{ m}^3/\text{s}$  through a smokestack of height  $H = 20 \text{ m}$ . A failure in the filter system allows a pulse of sulfur dioxide,  $\text{SO}_2$ , to be released with the exhaust. The concentration of  $\text{SO}_2$  in the exhaust is normally zero, but during the failure increases to  $10 \text{ mg/m}^3$ . This release occurs for  $T_R = 5 \text{ minutes}$ . Assume an isotropic diffusivity,  $D = 1 \text{ m}^2/\text{s}$ , and uniform wind speed,  $U = 2 \text{ m/s}$ .



- Using the coordinate system shown in the sketch above, and assuming the release occurs as an instantaneous slug of mass,  $M$ , write an expression for concentration of chemical XX downstream. Assume that the ground is a no-flux boundary.
- Calculate the  $Pe$  for  $x = 500$  m and explain what the value indicates about transport.
- At what distance downstream will the release appear as an instantaneous release?
- Your home is 500 m downwind of the plant. What is the maximum concentration you would expect to observe at ground level at your house?

**Problem 5.5.**

Consider a long, narrow channel connected to a harbor that receives drainage water from the city at its landward end ( $x = 0$ ) at an average rate,  $Q = 0.005\text{m}^3/\text{s}$ . The channel is 75-m long, 2-m deep, and 10-m wide. One morning a boat spills a tank of soluble degreasing fluid at  $x = 25$  m. Use the following questions to assess the impact of this spill on the harbor. Assume an isotropic diffusivity,  $D = 0.1\text{ m}^2/\text{s}$ .

- Estimate the Peclet No.,  $Pe = \frac{UL}{D}$ , for the system and explain its meaning.
- What time do I have to execute preventive measures to keep the contaminant out of the harbor?
- Estimate the time scale for the contaminant to become well-mixed in the lateral and vertical.
- To describe the concentration of contaminant observed at the harbor ( $x = 75$  m), should I use the 1-D, 2-D, or 3-D evolution equation? Justify your answer.
- Using symbolic language [ $U$ ,  $D$ ,  $x$ ,  $t$ ,  $M$ , etc] write a mathematical expression that describes the evolution of the spill in time and space, after the concentration has become well-mixed across the width and depth of the channel. Assume that the contaminant cannot pass through the back wall of the inlet at  $x = 0$ .
- State two limitations of the model you have chosen.