

1.105 Solid Mechanics Laboratory

Fall 2003

Experiment 6 The linear, elastic behavior of a Beam

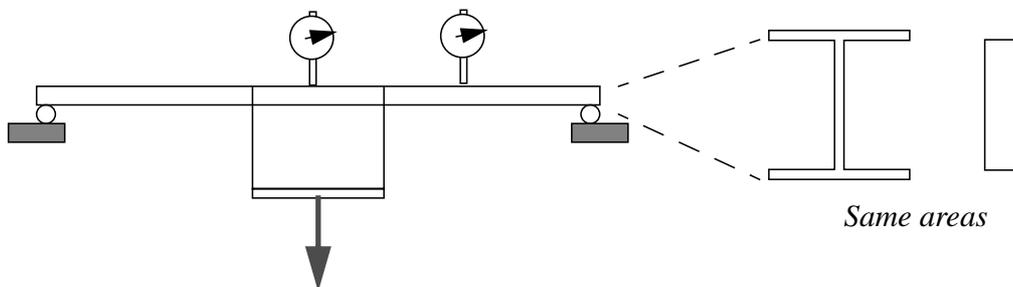
The objectives of this experiment are

- To experimentally study the linear elastic behavior of beams under four point bending.
- To compare the stiffness of beams of the same length and cross-sectional area but having different profiles.
- To measure the extensional strain in the top and bottom fibers of the beam specimens and compare with the prediction of engineering beam theory.

Each beam specimen, made of Aluminum and 24 inches long, will be simply supported at its ends and loaded at two points symmetrically disposed with respect to midspan, each 3 inches off center. This “four point bending” produces “pure bending” (no shear) over the midspan section. Two dial gages will be employed, one to measure midspan deflection, the other will be positioned to measure vertical displacement roughly half the distance between midspan and one of the ends. Two strain gages are mounted on the top and on the bottom at midspan. The beams will be loaded using dead weights as in the previous experiment. You will compare bending strain and beam stiffness to theory.

Beam Specimens

The figure below shows a beam specimen under four point bending. To the right we show the cross sections of the two Aluminum specimens. The material is 6061-T6 as is clearly indicated on the beams' sides.



The two have the same cross sectional area as you can verify.

Measure all the relevant dimensions, e.g., the length of the span between the supports, all cross-section dimensions, the location of the two load points relative to the supports.

You will test the I beam in one orientation, as indicated in the figure. You will test the beam with rectangular cross section in two orientations - in the orientation indicated in the figure and with the beam lying flat on the supports. You will make a total of three runs minimum. Multiple runs are suggested, especially with the first specimen, to acclimate yourself to the beam's behavior and instrumentation performance.

Procedure - Instrumentation Setup

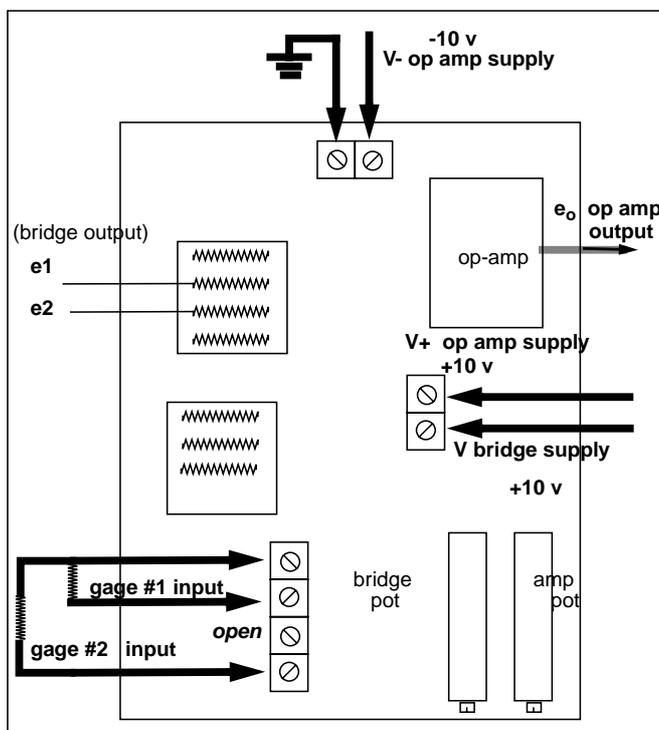
Before placing a specimen on the end supports, mark off two points, three inches to the left and right of the end of one of the two mid span strain gages. This end of the gage, away from the leads, will become the midspan point. Mark both the top and bottom flanges at these load points.

Position the specimen (we will first test the I beam) so that a dial gage can be positioned at mid-span without resting on the strain gage but as close as possible to the end of the gage away from the leads and adjust the end supports so that this point is truly midspan.

Loop the thin cables with eye hook ends over the beam at the two symmetrically disposed loading points, three inches off center.

Connect the cables to the horizontal bar below the test bed with s hooks. Let this be your "no load" condition. (Before hooking up the bucket).

Position the dial gages as indicated in the figure on the previous page.



Connect the strain gages to the terminals on the circuit board as per the figure below. You will note that one of the leads of one of the gages is soldered to one of the leads of the other gage. Only three terminals on the board are used in this experiment and they are to be connected as shown in the figure. (See appendix for the bridge circuit layout).

With the power supply **disconnected** from the circuit board, set the +20v supply to +10v and verify that the -20v supply is set to -10v (with the full tracking nob turned all the way clockwise). **Turn off the power supply.**

Connect the +10v supply leads to the two terminals as indicated in the figure. Connect the common and the -10v supply to the two terminals also as indicated in the figure.

Turn on the power supply and, with the Digital Multimeter, check the voltages at the strain gage input terminals. One should read +10v, another approximately +5v and the third should read 0v.

Balance the bridge.

Null the amplifier.

Calibrate the amplifier by resetting the bridge balancing pot so that the output of the bridge (which is the input to the amplifier) is on the order of 10 mv, then measure the output of the amplifier to obtain the gain.

Repeat this for a bridge output of approximately 20mv.

Balance the bridge and null the amplifier.

Press down lightly on the beam to verify setup. Op amp output should be on the order of 10 mv.

Procedure - Loading Sequence

Weigh the bucket and other supporting hardware still to be attached.

Now load the I beam specimen but **do not exceed 50 lbs. total.**

Take data while unloading too.

Be sure too note any drift in the op-amp output and estimate uncertainties in all measured quantities.

Now position the beam of rectangular cross-section so that the long dimension of the cross-section is vertical.

Load the specimen but again, **do not exceed 50 lbs. total.**

Repeat if you deem worthwhile.

Now position the beam of rectangular cross-section flat on the supports, so that the short dimension of the cross-section is vertical.

You need not record the strain-gage output. (But note any interesting output).

Load the specimen.

but **do not exceed 30 lbs. total.**

When done, turn off the power supply but leave all connections as is.

Report

Start with a one paragraph executive summary of the purpose, the method, the results of the lab tests.

Include a short section on experiment procedure which, rather than reproduce the steps set out above, makes note of any particular difficulties you encountered and how you managed to overcome them. *Included in this section your op-amp calibration data and your calculation of amplifier gain.*

In your section on "results", you will compare experiment with engineering beam theory. The relationships summarized in the Appendix will allow you to deduce and evaluate:

- The bending stiffness, EI
- The bending moment, hence the normal stress, hence the extensional strain at the top and bottom of the beams.
- The deflection as a function of the geometry and bending stiffness.

Include then in your results:

- A plot of displacement versus load for each of the three beam specimens.¹ Plot also the off center displacement versus midspan load. Show on each plot the results of engineering beam theory.
- Plot the strain at the top (or bottom) versus load for each of the three specimens. Show on each plot the results of engineering beam theory.
- A table showing the bending stiffnesses obtained from the strain measures. (From the strain you can compute the curvature. From the load values and lengths measured you can compute the bending moment in the mid section. The ratio is the bending stiffness). Include in the table the "theoretical values".

In a "conclusions" section, include recommendations for improvement of the experience.

1. We consider the two orientations of the beam of rectangular cross-section as two different beam specimens.

Appendix

Engineering Beam Theory

Engineering beam theory shows that the most significant stress is the normal stress component on an “x face”; σ_x in the example at the right. It is related to the applied loads by

$$\sigma_x = -\frac{M_b \cdot y}{I}$$

where y is the distance from the “neutral axis” which, for a doubly symmetric beam, is at the center of the cross-section and I is the moment of inertia of the cross section.

$$I = \int_A y^2 dA$$

For a rectangular cross-section of width b and height h , this is $I = bh^3/12$

The applied loads come in through the bending moment M_b . The convention for positive shear and bending moment is shown in the figure.

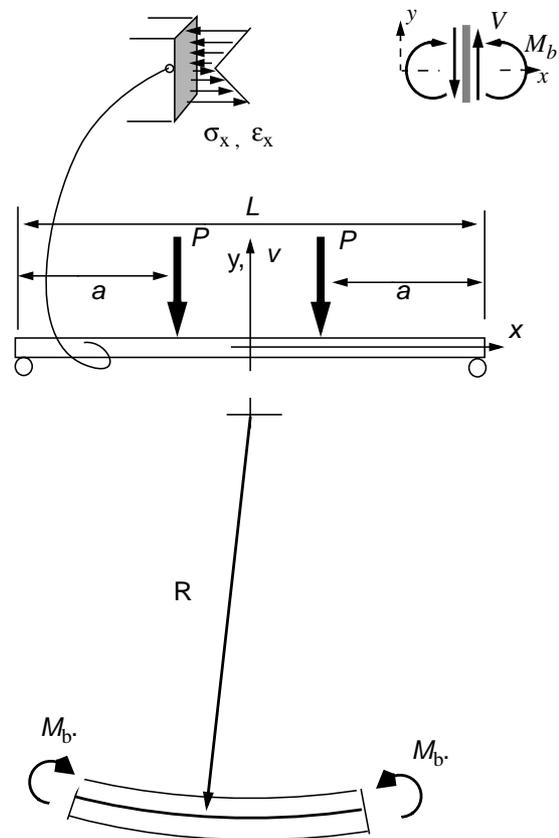
The extensional strain, from the stress/strain relations is just:

$\epsilon_x = \sigma_x/E$ where E is the Elastic Modulus. In terms of the geometry of deformation, the extensional strain is given by $\epsilon_x = -y/R$ where R is the radius of curvature of the neutral axis. ($1/R$) is the curvature.

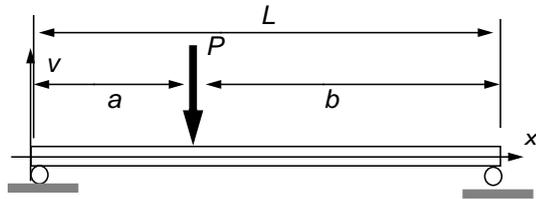
The “bending stiffness” is defined as the product EI as it appears in the “moment-curvature” relationship $M_b = (EI) \cdot \left(\frac{1}{R}\right)$ where the curvature, for small deflections, is related to the vertical displacement of the neutral axis by, $(1/R) = d^2v/dx^2$.

An integration of the differential equation obtained from the moment curvature relation gives, for the case where the beam is loaded as shown, the mid-span deflection

$$v|_{midspan} = -\left(\frac{Pa}{24EI}\right) \cdot (3L^2 - 4a^2)$$



While this is not required of you, to determine the displacement at some point other than the mid-span, you can use the relationships below and the superpositioning of two symmetrically placed, loads P , to obtain an expression for $v(x)$.



Note that the origin of the x axis is located at the left end of the beam.

For $x < a$:

$$v(x) = \left(\frac{Pb}{6LEI} \right) \cdot [-x^3 + (L^2 - b^2) \cdot x]$$

For $x > a$:

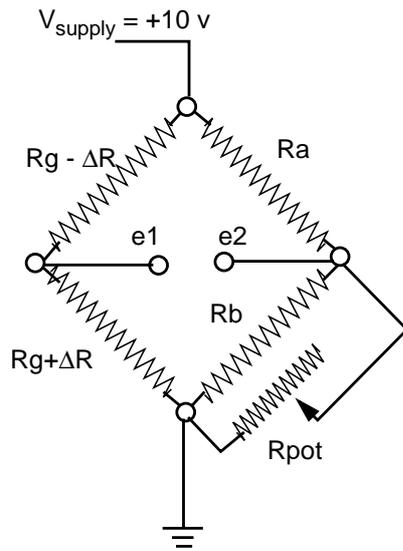
$$v(x) = \left(\frac{Pb}{6LEI} \right) \cdot \left[\left(\frac{L}{b} \right) \cdot (x - a)^3 - x^3 + (L^2 - b^2) \cdot x \right]$$

Later in the semester we will make use of the program, Fameworks, to model these three beams, then compare the computer results with experiment.

Strain-gage Circuit.

Two strain gages are fixed to top and bottom flanges. The top will experience contraction, the bottom extension. The bridge circuit is used to avoid attempting to measure the small difference of two large numbers and their placement in the bridge effectively doubles the sensitivity. The assumption is that the beam shows a doubly symmetric section so the magnitudes of the two strains are equal.

The circuit you used in Experiment 3 has been altered to give us the measures we seek.



The values of the resistances are:

$$R_a = 348 \pm 5\% \text{ ohms}$$

$$R_b = 561 \pm 5\% \text{ ohms}$$

$$R_g = 350 \pm 0.2\% \text{ ohms}$$

$$R_{pot} = 1 \text{ kohm (max)}$$

The output of the bridge as a function of the change in resistance, ΔR ,

$$e_1 - e_2 = (V_{supply}/2)(\Delta R/R_g)$$

The strain as a function of change in resistance is given by

$$\epsilon = (1/F_{gage})(\Delta R/R_g)$$

where F_{gage} is the "gage factor" stated by the manufacturer¹ to be

$$F_{gage} = 2.07 \pm 0.5\%$$

With these, you can compute the strain in the member, given the voltage difference $e_1 - e_2$.

The voltage difference $e_1 - e_2$ is obtained from your measured values at the op-amp output by dividing by the amplifier gain.

1. BLH Electronics, Inc.