The Bending Strength of Pasta

1.105   Lab #1

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Lab Partners:

[Name1]

[Name2]

Data File: Tgroup3.txt

On the cover page, include your name, the names of your lab partners, the name of the data file which you will email (or have emailed) to me - [See Important Note on page 7] - the date the lab was performed, any other information you think belongs on a cover page.

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1.0 Summary

The objective of this experiment is to determine the *ultimate strength of spaghetti in bending* as defined by the *maximum strain* a strand can endure. We also (were meant to) learn about the "propagation of uncertainty", i.e., how uncertainty in the measurements we make propagate through and give us estimates of uncertainty in the value of the maximum strain.

The experimental determination of the maximum strain a strand can endure draws upon the theory describing the large deflections of an *elastic lamina*; a strand of spaghetti can be considered as such. ...

The procedure was (is) simple; the loading of the strand was (is) done "by hand". The mid-span deflection of the strand at failure was (is) measured by the naked eye. This measure and our measure of the original length was(is) all that was(is) needed, together with the theory, to predict the minimum radius of curvature of the strand at failure. To compute the strain, we also needed (need) to .....

Our data shows that the maximum strain a strand of pasta can endure is ..... +/- ......

In this summary of your report you state your objectives, present an overview of procedure, and describe the main results you obtained, using words alone. You can include numerical results but only for the most significant variables. This is best kept to a paragraph or two.

2.0 Introduction

Spaghetti is not usually considered a structural material. People eat spaghetti; they don't use it to build bridges or space robots. But pasta to a person responsible for its production, packaging and distribution might very well be concerned with when a single strand might fail. No one wants their spaghetti but two inches long. Optimum twirling requires at least 20 centimeters of uniform length.

For a long, slender, uniform structural member, failure is generally due to bending. Knowing what strain — as measured by the ratio of the radius of the member to the deformed curvature of its centerline — can be tolerated is essential to controlling and avoiding failure.

Euler's 18th century analysis of the large deflections of an elastic lamina - the elastica - will provide the basis for a determination of the maximum extensional strain at fracture. It is not clear whether Euler himself addressed this particular phenomenon, that is, whether he put spaghetti to the test, but certainly he must have consumed a pound or two.

In the Introduction, describe the motivations for the experiment. If the experiment, has a theoretical basis, give the reader enough of a description of the theory so that they can locate the experiment within that framework. (You can refer to the Appendix about the "Elastica"- the pdf file which you can append).

It is very important that you have in mind your audience. The general instructions for 1.105 describe for whom you are writing the report - a fellow student who has not yet done the experiment but will do so in the near future. In the "real" world, some audiences will be more knowledgeable than others. Hence your report may not need to be as detailed nor as lengthy as it would be were you writing for someone less experienced in the study of the engineering mechanics of structures. Of course the "real" audience is the professor, but you get the idea.

3.0 Experimental Method

Here you describe your experimental method. While you can not possibly describe everything you did, you should provide sufficient information so that a knowlegeable reader, e.g., a fellow student in Course 1C, could make an attempt at repeating what you did, and do so with some reasonable chance of success.

To determine the ultimate strain in bending, we use the following procedure: A single strand is placed on a sheet of linear graph paper with scale of appropriate resolution (1mm). The noodle is aligned with one of the heavy, longer lines. (See Fig. 3.1)

Include a figure, hand drawn is fine, which shows the original and deflected state of the speciman, atop the graph paper. This figure should be something like the one in the Appendix.

Equal and opposite loads are applied to the ends of the specimen, holding one end fixed against a stop cemented to the paper and applying a force, directed perpendicular to the support, at the other end. The strand, if of sufficient length, will deflect transversely.

Before loading, the diameter and length of straight strand are measured. Then measurements of the horizontal distance, s, between the ends and the corresponding vertical displacement, h, are made just prior to fracture.

Two different kinds of pasta are tested in the order below:

- Prince Spaghetti
- Prince Thin Spaghetti[1]

The sequence of test for each kind of pasta is as follows:

- First, measurement of the diameter of the kind is made. Take several measurements i) on a single strand, ii) of several strands of the kind. An estimate of uncertainty, as well as the diameter, is recorded.
- A full-length strand is loaded to failure, h and s recorded at the instant just prior to failure.
- The strand ordinarily breaks into two pieces of roughly the same length. The biggest of the two is then loaded to failure and measurements of h and s at fracture recorded. (Do not forget to measure the original length of the half strand).
- A second, full-length strand is next... and so on through 10 to 20 specimens.
- Estimates of uncertainty in measuring the original length L, and h and s at failure are made and recorded.

1. Note: Your types of spaghetti may differ from these.

All data is recorded in the quadrille composition book, or equivalent lab notebook. Allow one page for each kind of spaghetti, making a table of 9 columns. The first four columns are for

1) the Specimen # ,
2) its original Length L,
3) the deflected height, h, and
4) deflected span, s; the latter two are the values measure at fracture.

The next two columns - namely
5) h/L,
6) R/L, are for computations required in order to compute
7) the corresponding strain values at failure.

Include also columns for
8) the measured ratio s/L at failure and
9) the "theoretical value" obtained from the graph.

TABLE 1. *Data Sheet Format - included in an Appendix*

Spc #	**Length, L**	**Height, h**	**Span, s**	**h/L**	**R/L**	**$e\|_{max}$**	**s/L**	
	mm	**mm**	**mm**				**measd.**	**graph**
	+/- **mm**	+/- **mm**	+/- **mm**	+/-	+/-	+/-	+/-	

You should carry a few of these computations through in the lab period, to see that you are on the right track, but completion of the table can be left for later.

Photocopy your data from your lab notebook and include in an appendix. Show also in the appendix, sample calculations of the failure strain, s/L ratio, etc.

Note: You may have the resources to enter the raw data directly into a laptop. In which case, make sure your lab partners obtain the data file as well as soon after lab as possible. Include a print out of the data in the above format in your lab notebook and in an appendix to your report.

4.0 Results

Using the theoretical relationships for the shape of the elastica, provided in the Appendix, we determined a set of values for the ultimate strain in bending for different length specimens. The original data and calculations leading to values of strain at failure are included in Appendix yy..

Here you present your results. Most times this will include graphs and tables. Do not include your raw data here. Detailed data reduction and uncertainty calculations, with accompanying explanation, belong in the appendix with your raw data.. But here you may briefly summarize in words the relationships you used to determine the strain at failure.

Include histograms showing the frequency distributions for the different kinds of spaghetti. If you hand draw these, photocopy the original, reducing them if you like, cut out and paste or tape to the page in this section. Otherwise use a spread sheet and plot from there (or your TI calculator, or whatever technology you have available to make and print out a graph)..

Make sure you say something about the these results - e.g., the differences in failure strain for different types of spaghetti. Do the experiments verify the theory? How did the measured ratio of span to length compare with the theoretical? You want to explain likely sources of uncertainty and scatter. Is the spread in results explained by the uncertainties in making measurements?

5.0 Conclusions & Recommendations

Spaghetti fails at a strain of xxx +/- yy in bending. ...

We recommend further experiments to

- Determine the effect, if any, of variations in composition from one type of noodle to another, one manufacturer to another.
- ...

A final summary of the main result goes here along with recommendations for further work.

All Important Note*:*

Each group is to produce a text file -"Tgroup[n].txt" or "Rgroup[n].txt" where T is for Tuesday and R is for Thursday and n is your group number - of your raw data which includes ***two header lines:***

- The first gives the **kind of sphagetti** and **your group id**
- the second gives the diameter in millimeters and the number of samples

Then follows the raw data of the **first four columns of your data sheet: i.e., Speciman #, Original Length, L, height h at failure and span s at failure.**

All dimensions to be in **mm**.

Thin Sphagetti Group 3

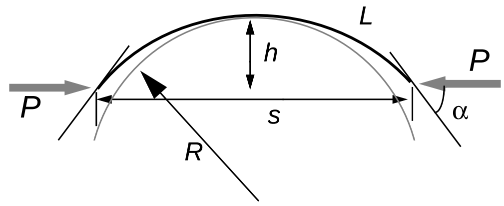
Diameter = x.x mm N=30

1	*241*	*80*	*161*
2	*105*	*19*	*95*
3 etc			

Each group should email me their text file.

6.0 Appendix - The Elastica

6.1 General Remarks



The Elastica, deformed.

An *elastic lamina* is a slender element, uniform in its geometric and physical properties along its length, - e.g., a long rod, or beam- and capable of supporting a compressive or a tensile load. When subject to an axial compressive force, if the length is sufficiently long, the lamina will *bend* out of line as shown in the figure.

An analysis of equilibrium and deformation of the lamina, based upon the principles of Solid Mechanics, produces an ordinary, but nonlinear differential equation whose solution describes the geometry of the deformed lamina. We present here only the essential results of the analysis; we defer a more complete presentation until later in the subject. The solution yields expressions for the height *h* and the span *s* in terms of two parameters, one related to the slope at the ends

(1) $k \equiv \sin(\alpha/2)$ which is dimensionless, and a second related to the applied force *P* and the bending stiffness, *EI*[1] .

(2) $c \equiv \sqrt{EI/P}$ which has the dimensions of length. The solution gives:

(3a) $$s = 2c \cdot [2 \cdot F_1(k) - F_2(k)]$$

(3b) $$h = 2ck$$

(3c) $$L = 2c \cdot F_2(k)$$

(3d) $$R_{min} = c^2/h = h/(4k^2)$$

R_{min} in the above is the minimum radius of curvature which occurs at mid-span. L is the original straight length of the lamina.

In these, $F_1(k)$ and $F_2(k)$ are functions of the parameter *k* alone and are defined by the following two, definite, so called "elliptic integrals". Values of F_1 and F_2 for a range of val-

$$F_1(k) \equiv \int_0^{\pi/2} [1 - k^2 \cdot \sin z^2]^{1/2} dz \quad \text{and} \quad F_2(k) \equiv \int_0^{\pi/2} (1/[1 - k^2 \cdot \sin z^2]^{1/2}) dz$$

1. of which we will learn more later on in the subject.

ues of *k* have been computed and are available from almost any book of mathematical tables.

In our experiment, we want the maximum strain the lamina can withstand before fracture. **The strain is a function of the radius of the lamina and the radius of curvature.** It is proportional to the former and inversely proportional to thelatter.[1] Hence the *maximum strain* occurs where the radius of curvature is a *minimum*, i.e.,

$$\varepsilon|_{max} = r/R_{min}$$

So, to determine the maximum strain in bending we need to measure the radius (diameter) of the lamina and determine the minimum radius of curvature from equation (3d). This, means we need to know

i) the value of *k* and
ii) the value of *h*.

The value of *h* is measured in the experiment. In the next section, we show how to determine the value of *k*.

6.2 The Elastica - Useful Results

In the lab experiment, we measure the span, *s*, the mid-span height, *h*, at fracture. We measure the original straight length, *L*, before applying the end loads. From equations (3b) and (3c) on the previous page we can eliminate *c*, which remains an unknown, and write

$$\text{(4a)} \qquad k = (h/L) \cdot F_2(k)$$

I have found the roots of this expression for a range of values of (*h/L*); think, of *k* as a function of *h/L*. (Note: I do not show what this relationship looks like). With these values of *k* for a given ratio (*h/L*), the above relationship for the radius of curvature may be rewritten, normalizing with respect to L:

$$\text{(4b)} \qquad R/L = \frac{h/L}{4 \cdot k^2}$$

So measuring and knowing *h/L* then fixes *k* and the ratio *R/L*, hence the radius of curvature. Having measure the radius of the strand we also can compute the maximum strain due to bending.

From equations (3a) and (3b), we can also write an expression for the span, again normalized with respect to *L*. (4c) $(s/L) = 2 \cdot \frac{F_1(k)}{F_2(k)} - 1$

1. Again, this is the subject of a later part of our course, that devoted to the analysis of the bending of beams.

Again, once having a value of *k*, for some *h/L*, we can compute, or obtain from the tables, the definite integrals *F1(k)* and *F2(k)* and so compute the span. This will serve as a check since we independently have measured and recorded *s* at fracture.

Plots of (s/L) and (R/L) as a function of (h/L), determined in accord with the above relationships are shown below. It is this plot you will use to directly obtain values for the two ratios s/L and R/L having measured *h* and *L*.