Real-Time Control Strategies for Rail Transit

Outline:

- Problem Description and Motivation
- Model Formulation
- Model Application and Results
- Implementation Issues
- Conclusions

Problem Context

- High frequency urban rail service (e.g. headways of 2-10 minutes)
  - passengers arrive randomly
  - service regularity is a key goal
  - $E(WT) = \frac{E(H)}{2} [1 + cov(H)^2]$
- Branching route structure
- Central real-time train location information and dispatch capability

Three Levels of Control Problems

Routine disturbances - several minutes' deviation from schedule

Control Strategies:

- speed adjustment
- dwell time adjustment (selective holding) terminal recovery

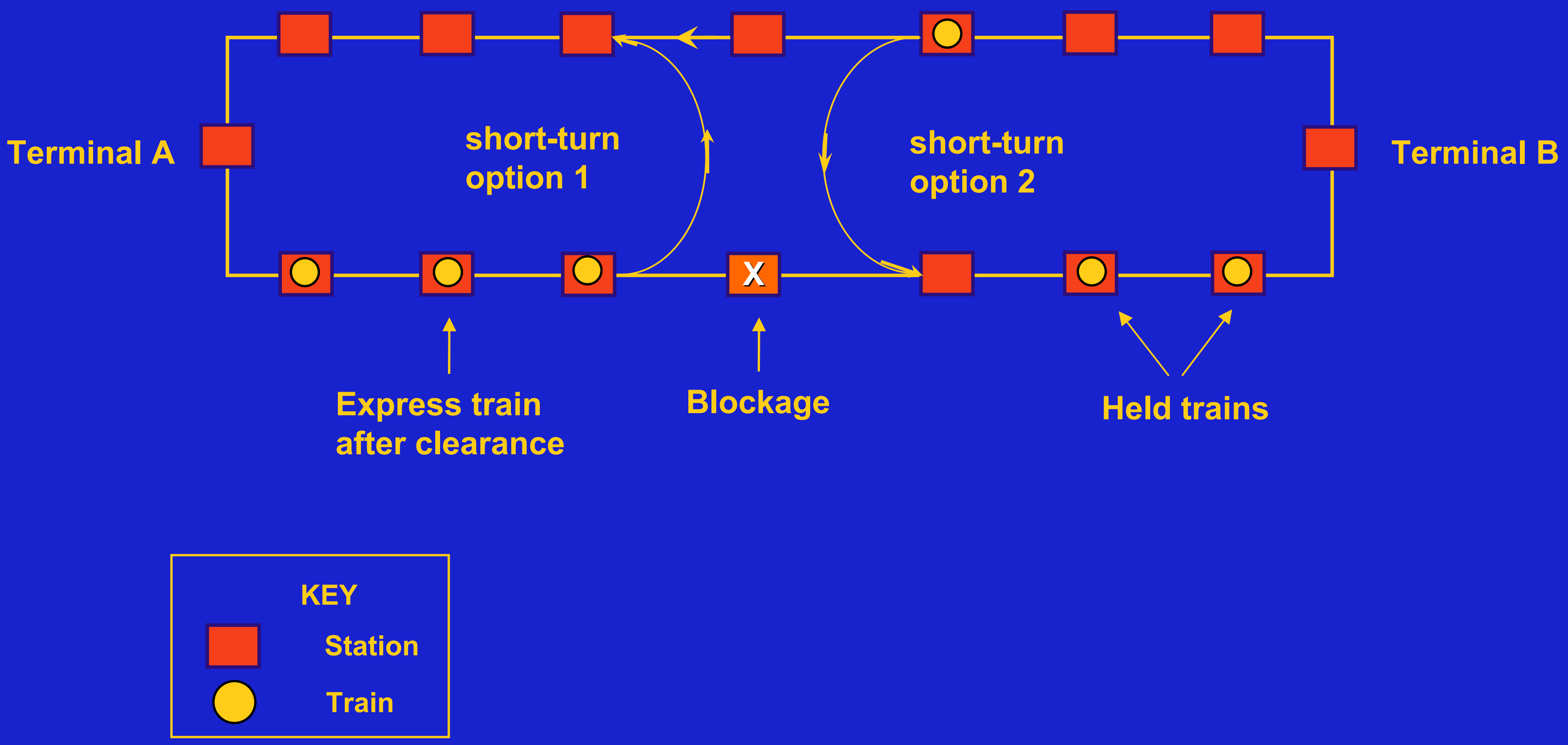
Short-term disruptions: 5-30 minute blockages on the line

Longer-term disruptions - greater than 30 minute blockages

Control Strategies:

- single-track reverse direction operations
- replacement bus service around blockage

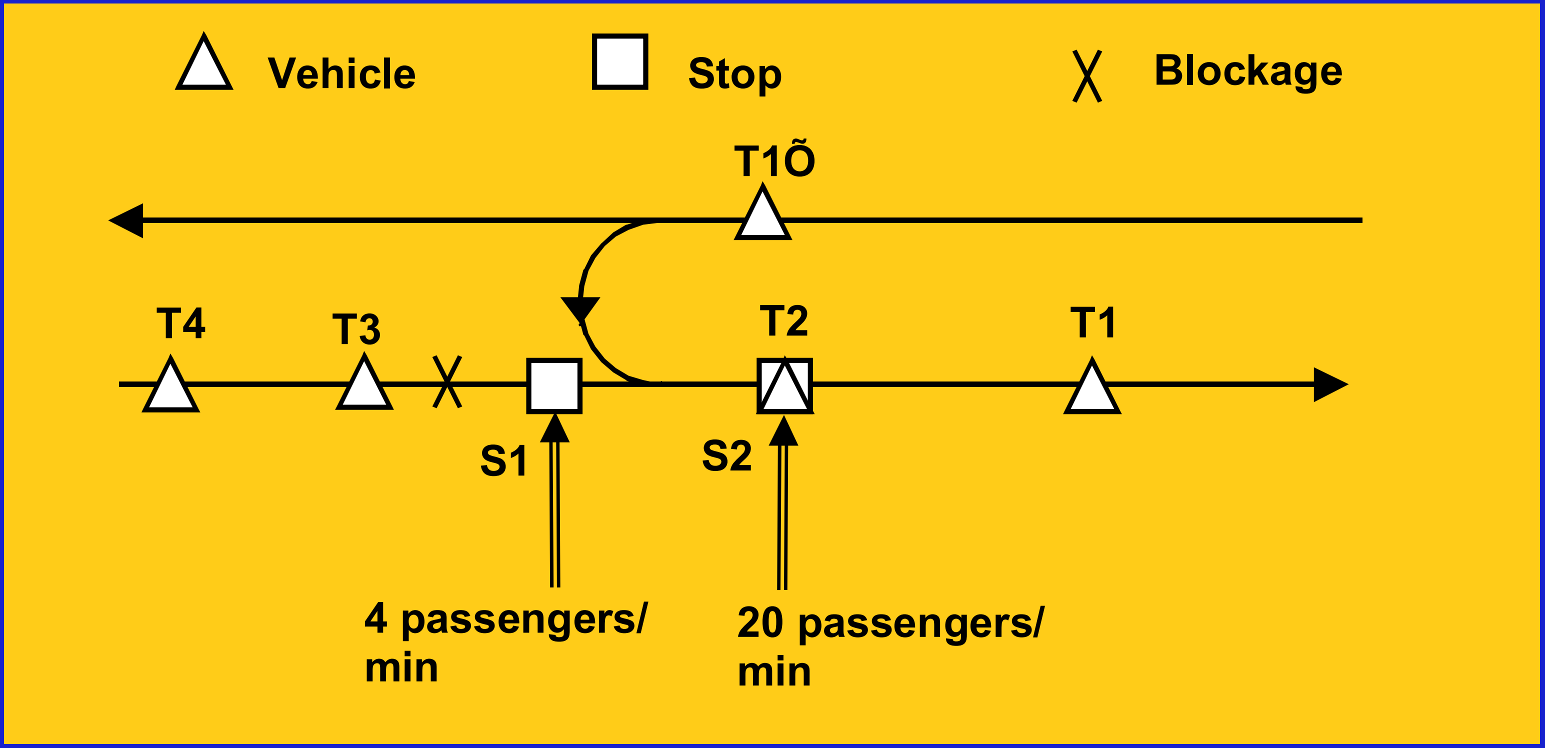
Disruption Response Strategies



Problem Description

- **Overall Objective:**
  -- Develop a real-time decision support system to determine control strategies to recover from disruptions
- **Specific Objective:**
  -- Minimize passenger waiting times (implies maintaining even headways)
- **Key Characteristics:**
  -- Instability of even headways
  -- Passenger sensitivity to long waiting time and crowding
  -- Cost insensitivity to different strategies
- **Possible Strategies:**
  -- Holding
  -- Short-turning
  -- Expressing

Example of Transit Control Strategies



- 6-minute scheduled headways
- 3-minute minimum safe headway
- 10-minute disruption
- impact set includes trains T2, T3, and T4 and stations S1 and S2

Example Results

1. **Do nothing:** h_{T_2} = 6 mins.; h_{T_3} = 16 mins.; h_{T_4} = 3 mins.

$$\textit{Total Passenger Waiting Time} = \frac{1}{2}\left[4(16^2 + 3^2)\right] + \frac{1}{2}\left[20(6^2 + 16^2 + 3^2)\right] = 3540 \text{ pass} - \text{mins.}$$

2. **Holding:** Hold T2 at S2 for 4 mins.
   Then at S2: h_{T_2} = 10 mins.; h_{T_3} = 12 mins.; h_{T_4} = 3 mins.

$$TPWT = \frac{1}{2}\left[\left(4(16^2 + 3^2) + 20(10^2 + 12^2 + 3^2)\right)\right] = 3060 \text{ pass} - \text{mins.}$$

3. **Expressing:** Express T3 past S1 to save 1 minute in travel time.
   Then at S2: h_{T_2} = 6 mins.; h_{T_3} = 15 mins.; h_{T_4} = 4 mins.

$$TPWT = \frac{1}{2}\left[4 * 19^2 + 20(6^2 + 15^2 + 4^2)\right] = 3492 \text{ pass} - \text{mins.}$$

Model Formulation

Key Features:

- station specific parameters: passenger arrival rates, alighting fractions, minimum safe headways
- station dwell time a linear function of passengers boarding, alighting and crowding
- train order is variable
- train capacity constraint

Simplifications:

- predictable disruption length
- passenger flows estimated from historical data
- system is modelled as deterministic
- strategies selected to produce minimum inter-station travel times.

Model Formulation

Decision Variables: departure time of train *i* from station *k*

Objective function: minimization of passenger waiting time

- *quadratic function approximated by a piecewise linear function*

Impact Set: consider a finite set of trains and stations and approximate the effects beyond this set

Constraints: train running time and minimum safe headways

- *other relationships govern passenger loads, train dwell times*

Model Structure: mixed integer program except if passenger capacity is not binding when it is a linear program

Specific Models

Holding Strategy Models:

- Hold all
- Hold once
- Hold at first station

Combined Short-turning and Holding Models:

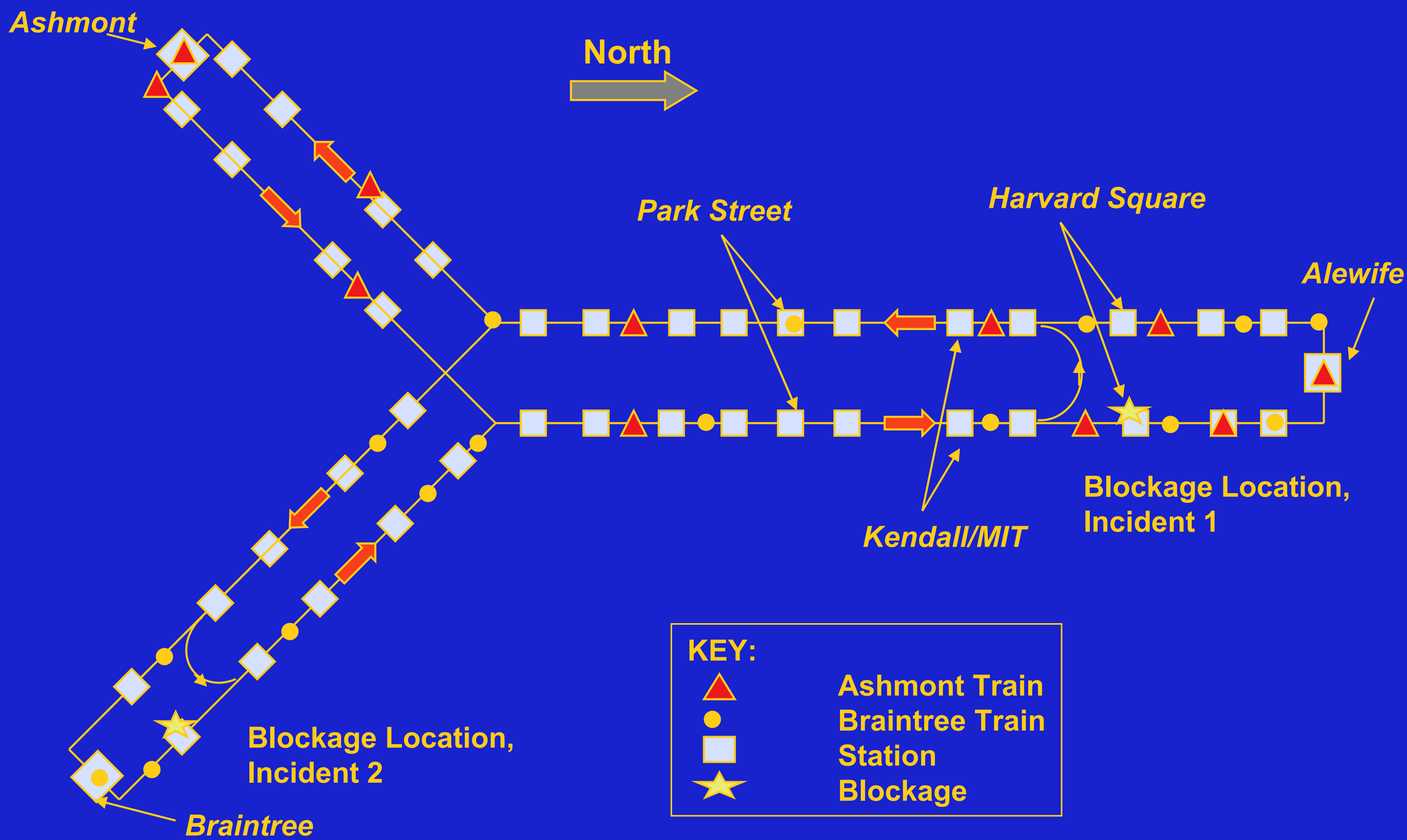
- Predetermined train order
- Undetermined train order

Model Application

MBTA Red Line Characteristics:

- 23 stations (including 3 terminals)
- 27 six-car trains in A.M. peak
- 3.4 minute trunk headways (6 and 8 minutes on branches)
- 30,000 passengers in peak hour

Red Line


Ashmont
North
Park Street
Harvard Square
Alewife
Kendall/MIT
Blockage Location,
Incident 1
Blockage Location,
Incident 2
Braintree
KEY:
Ashmont Train
Braintree Train
Station
Blockage

Incident 1, Ten Minute Delay

		Control Strategies			
		FOHPC			STPP
Passenger Waiting Time (Passenger-Minutes)	Do Nothing	Hold All	Hold Once	Hold at First	Hold All
Ahead of Blockage	11202	8863	8931	8961	9997
Savings (percent)		15%	14%	14%	8%
Behind Blockage	4791	4763			4753
Savings (percent)		0%			0%
Maximum Train Load	988	603	614	666	603
Problem Size		95	95	95	88
CPU Time (seconds)		22	37	21	16

Incident 1, Twenty Minute Delay

		Control Strategies			
		FOHPC			STPP
Passenger Waiting Time (Passenger-Minutes)	Do Nothing	Hold All	Hold Once	Hold at First	Hold All
Ahead of Blockage	36868	16934	17306	17385	16836
Savings (percent)		43%	42%	42%	43%
Behind Blockage	9218	7833			6842
Savings (percent)		3%			5%
Maximum Train Load	1646	666	759	805	651
Problem Size		95	95	95	88
CPU Time (seconds)		25	82	27	17

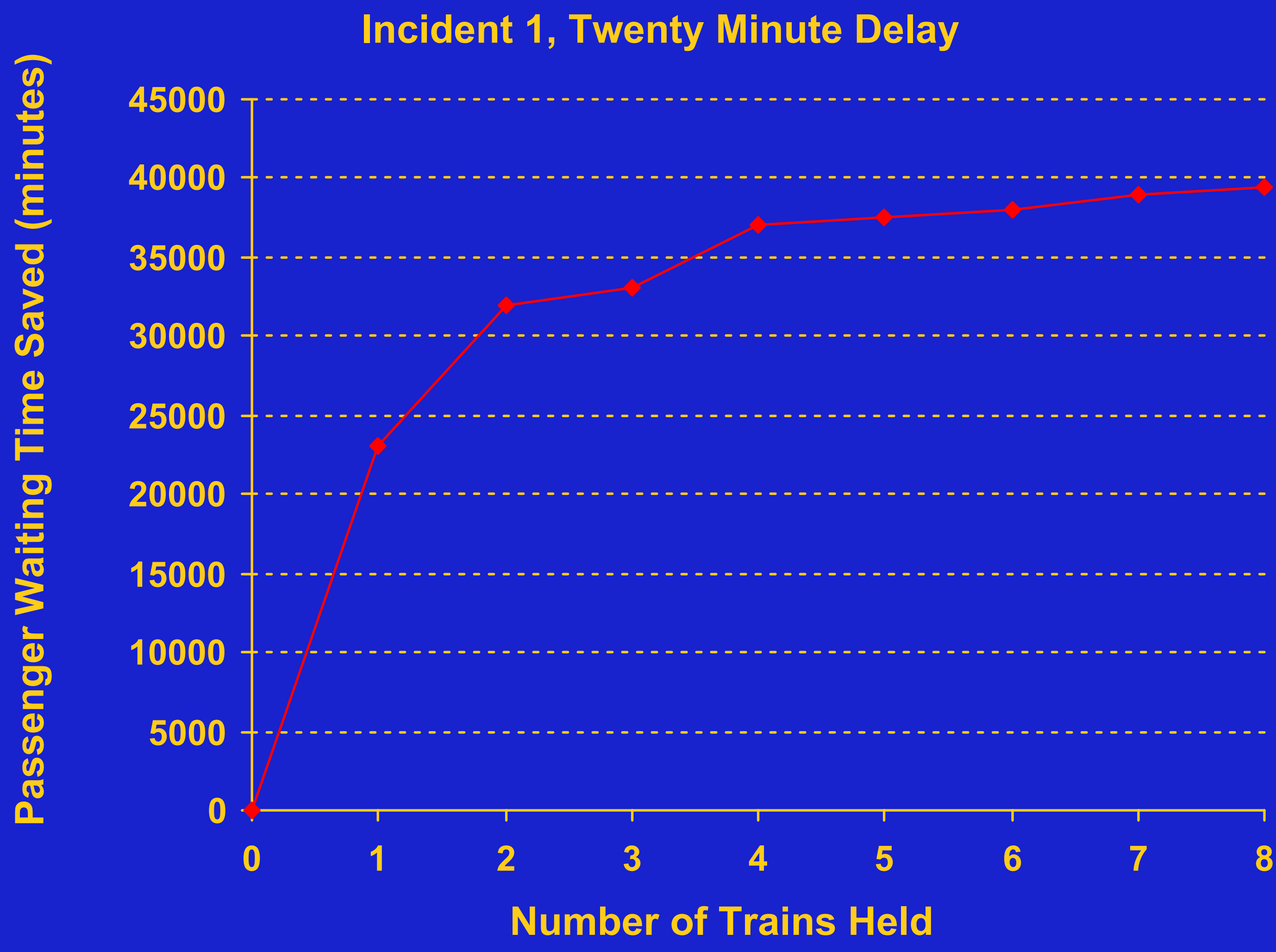
Incident 2, Ten Minute Delay

		Control Strategies			
		FOHPC			STPP
Passenger Waiting Time (Passenger-Minutes)	Do Nothing	Hold All	Hold Once	Hold at First	Hold All
Ahead of Blockage	32495	23101	24465	25327	23016
Savings (percent)		25%	21%	19%	25%
Behind Blockage	5593	5320			5404
Savings (percent)		<1%			<1%
Maximum Train Load	1336	1137	964	985	776
Problem Size		69	69	69	78
CPU Time (seconds)		17	274	23	12

Incident 2, Twenty Minute Delay

		Control Strategies			
		FOHPC			STPP
Passenger Waiting Time (Passenger-Minutes)	Do Nothing	Hold All	Hold Once	Hold at First	Hold All
Ahead of Blockage	88204	48978	52620	55487	38244
Savings (percent)		41%	37%	34%	52%
Behind Blockage	6773	6124			5964
Savings (percent)		<1%			<1%
Maximum Train Load	1653	1422	1343	1307	1200
Problem Size		69	69	69	78
CPU Time (seconds)		25	2458	763	62

Impact Set Size



Passenger On-Board Time

Incident	Delay	Objective Function	Passenger Time		
			Waiting	On-Board	Total (Weighted)
1	10 Min.	PWT	8961	1543	9578
		TPT	9074	271	9182
1	20 Min.	PWT	17385	2372	18334
		TPT	17659	806	17982
2	10 Min.	PWT	23411	8666	26877
		TPT	23702	5920	26070
2	20 Min.	PWT	50018	17617	57065
		TPT	51201	10488	55396

Execution Times

- Sun SPARC 20 workstation
- GAMS V. 2.25
- CPLEX V. 3.0
- Simple front-end heuristic to fix some binary variables

Large Problems: 11-13 trains, 69-95 train/station decision var.
Execution Time: 10 out of 16 <30 sec.

Realistic Size: 7-8 trains, 40-50 train/station decision var.
Execution Time: 16 out of 16 <34 sec.

Conclusions

- Holding and short-turning models formulated and solved to optimality
- Active control strategies result in significant passenger waiting time savings
- Train control set can be reduced to trains ahead of the blockage
- Train control set need not be large

Conclusions

- Hold at First or Hold Once strategies *can* be almost as effective as Hold All strategy
- Short-turning most effective where:
  - -- blockage is long relative to short-turn time
  - -- number of stations outside the short-turn loop is small
- Consideration of on-board time is desirable
- Execution time is 30 seconds or less but faster heuristics are probably achievable