

Homework problems on Fluid Dynamics
(1.63J/2.21J)

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Ex 11. *Vorticity evolution in an inhomogeneous fluid.* Vorticity is generated in an inhomogeneous fluid if the gradients of density and pressure are not parallel.

1. Referring to Figure (1a), where the pressure contours (isobars) and density contours (isopycnics) are sketched. What is the direction of vorticity generated?
2. If the fluid can be regarded as incompressible, show that in steady two dimensional flows the density is constant along a streamline so that $\rho = \rho(\psi)$.
3. In many meteorological situations the horizontal scale is much greater than the vertical scale, so that the vertical acceleration is relatively small. The pressure is hydrostatic and the isobars are near horizontal. Referring to Figure (1b), consider a steady wind past a long mountain range. Far upstream the density decreases with height (stable atmosphere). As the wind blows towards the 2D mountain the streamlines are forced upwards on the upwind side. Explain why the streamlines may turn downward on the downwind side, and then upward again some distance down stream, hence lead to lee waves which can be seen through cloud streaks due to condensations of moist air.

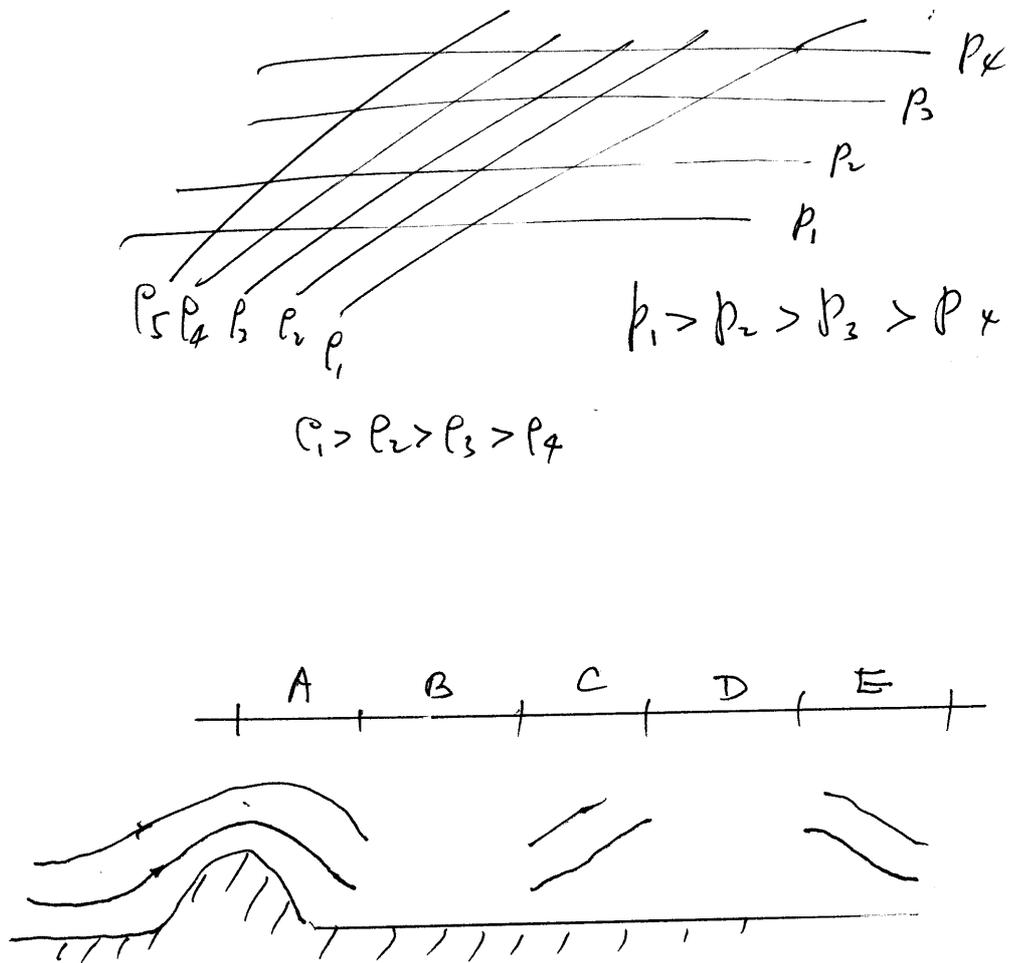


Figure 1: (a) Isobars and density contours, (b) Streamlines past a mountain.