Homework problems on Fluid Dynamics
(1.63J/2.21J)

Chiang C. Mei, 2002

13-Huppert.tex

Spreading of lava on a horizontal plane. Huppert, (1986).

Let a finite mass of lava is initially released on a horizontal plane and spread slowly in all radial directions. Invoke the lubrication approximation and assume the local radial veloity to be

$$u(r,z,t) = -\frac{g}{2\nu}\frac{\partial h}{\partial r}z(2h-z) \tag{1}$$

Then use the law of mass conservation

$$\frac{\partial h}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\int_0^h u dz\right) = 0 \tag{2}$$

to show that

$$\frac{\partial h}{\partial t} = \frac{g}{3\nu}\frac{1}{r}\frac{\partial}{\partial r}\left(rh^3\frac{\partial h}{\partial r}\right) \tag{3}$$

Show also that

$$2\pi\int_0^{R(t)} rh(r,t)dr = \text{constant} = V \tag{4}$$

where $R(t)$ is the front of the spreading lava. The boundary conditions are

$$h(R(t),t) = 0, \quad \text{and} \quad \frac{\partial h(0,t)}{\partial r} = 0 \tag{5}$$

Show that the similarity solution exists and is of the form

$$h(r,t) = \frac{A}{t^{1/4}}f(\eta), \quad \text{with} \quad \eta = \frac{Cr}{t^{1/8}} \tag{6}$$

subject to the integral constraint (4).

Derive the governing equation and boundary conditions for $f(\eta)$ and adjust the constants A and C so that the governing equations look the simplest. What condition determines η_R?

Try to solve the problem analytically.