

**Homework problems on Fluid Dynamics**  
(1.63J/2.21J)

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Ex3-M-energy.tex

Ex. 3: *Mechanical energy.*

From the governing equation for an incompressible fluid

1. Show that

$$\rho \frac{D}{Dt} \left( \frac{q_i q_i}{2} \right) = \rho f_i q_i + \frac{\partial(\sigma_{ij} q_i)}{\partial x_j} - \sigma_{ij} \frac{\partial q_i}{\partial x_j} \quad (1)$$

where  $\sigma_{ij}$  is the viscous stress tensor.

2. What is the physical meaning of each term above?

3. Derive the explicit expression for the last term

$$\Phi = \sigma_{ij} \frac{\partial q_i}{\partial x_j} \quad (2)$$

for a two dimensionnal flow in term of  $u, v$  and  $x, y$  and comment on the sign of  $\Phi$ .

4. If there is no body force, show that within a fixed container of volume  $V$  filled with viscous fluid,

$$\frac{\partial}{\partial t} \iiint_V \frac{1}{2} q_i q_i dV = - \iiint_V \Phi dV \quad (3)$$

What is the physical meaning of  $\Phi$ .

5. For a uniform fluid layer with a free surface flowing steadily down an inclined plane (depth  $h$ , plane slope  $\theta$ ), work out the velocity field  $u(y)$  and calculate the function  $\Phi$ .