Homwork, Fluid Dynamics (1.63J/2.21J),
C. C. Mei, 1-cavity.tex, 1999

Cavity collapse

A spherical cavity of initial radius R_o suddenly collapses.Ignore surface tension and water coressibiiilty. Th ecavity has the pressure p_o which is smaller than th pressure at infinity p_∞.

Use continuity that

$$4\pi r^2 u = \text{constant for} \quad r \geq R(t) \tag{0.1}$$

where $R(t)$ is the radius of the cavity, and $u(r,t)$ is the radial velocity. Use also the law of mometum conservation

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial r} \tag{0.2}$$

Integrate the momentum equation from $r = R$ to $r = \infty$ to get a differential equation of the following form

$$R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right) = -\frac{p_\infty}{\rho} \tag{0.3}$$

and show that it can be rewritten as

$$\frac{1}{2}\frac{d}{dt}\left(R^3\left(\frac{dR}{dt}\right)^2\right) = -\frac{\Delta p}{3\rho}\frac{d(R^3)}{dt} \tag{0.4}$$

With the initial conditions

$$R(0) = R_o, \quad \frac{dR(0)}{dt} = 0 \tag{0.5}$$

Show that the time to complete collapse is

$$T = \sqrt{\frac{3\rho}{2\Delta p}}\int_0^{R_o}\frac{dR}{\sqrt{\frac{R_o^2}{R^2} - 1}} \tag{0.6}$$

Evaluate the integral in terms of Gamma functions.