Homework problems on Fluid Dynamics

(1.63J/2.21J)

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hotspring.tex

1. **Hot spring at Yellowstone** In a heated rock there is a fracture in the shape of a tube of circular cross section of radius a. The axis of the tube is a semicircular arc of large radius $R \gg a$. Referring to figure 1 both ends of the fracture pierce through the ground surface. At one end A water enters the fracture from a reservoir at depth h and shoots out at the other end B.

Let the geothermal temperature in the rock be varying linearly with depth:

$$T_R = T_o(1 - bz) = T_o(1 + bR\sin\theta) \tag{1}$$

where b is a positive constant, Derive an approximate theory for the cross-sectional averaged temperature of water at the exit. Identify the the central dimensional parameter in the problem, and plot the exit temperature as a function of this parameter. This problem may

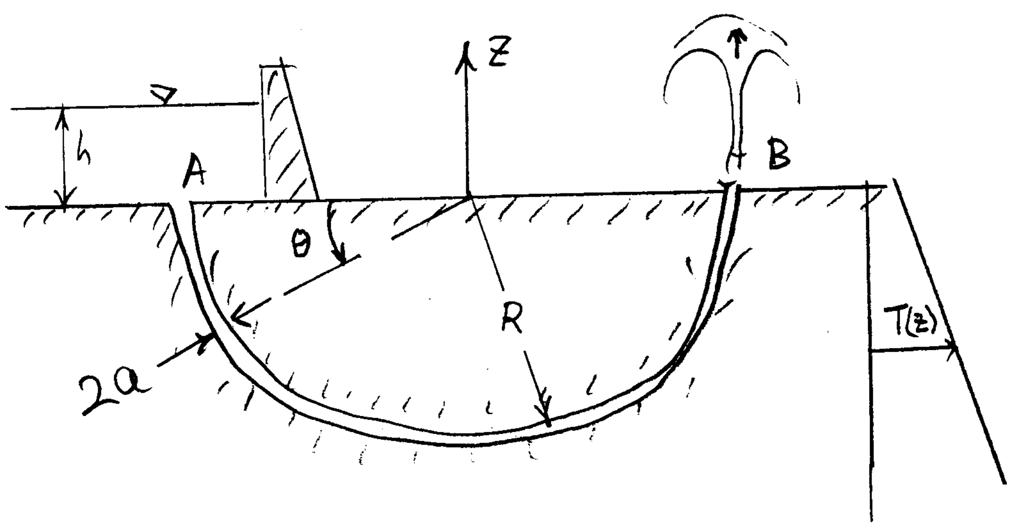

Figure 1: Hotspring at Yellowstone Park

suggest ways to extract geothermal energy from hydraulic fractures.

Remarks:

To go through the formal procedure of normalization, etc., would take too much time. Try to get to an approximation quickly by, ignoring the effects of curvature whenever possible (since $a/R \ll 1$), assuming that the flow is driven by pressure gradient alone, but the fluid temperature in the crack depends on the flow.

Suggested steps

- Ignore the curvature of the tube, find the velocity distribution $u_\theta(r)$ in the tube due to the pressure head difference $\rho g h$.
- Let the fluid temperature be

$$T_f(r, \theta) = T_R(\theta) + T'(r, \theta) \tag{2}$$

  Invoke linearity if necessary and solve first for the radial dependence of T', then take the cross-sectional average, e.g.,

$$\bar{T}_f \equiv \frac{1}{\pi a^2} \int_0^a 2\pi r T_f dr \tag{3}$$

- Use integrated energy balance to derive a differential equation for $\bar{T}_f$.
- Recall that the ODE

$$\frac{dy}{dx} + hy = g(x) \tag{4}$$

  can always be solved by rewriting it as

$$e^{-hx} \frac{d}{dx} \left(y \, e^{hx} \right) = g(x) \tag{5}$$