12.005 PROBLEM SET 1
DUE 2/22/06

Each problem set (~ weekly) will be weighted equally. The % for each problem is given.

1) Turcotte & Schubert (T&S) problem 2-8 (10%). Consider a rectangular block of rock with a height of 1 m and horizontal dimensions of 2 m. The density of the rock is 2.75 Mg/m^3. If the coefficient of friction f is 0.8, what force is required to slide the rock over a horizontal surface?

2) (30%) Consider a rock mass of density ρ and thickness h resting on an inclined plane, with the dip angle of the plane θ shown in the figure. The plane is just steep enough that frictional sliding continues after it begins.
a) Calculate the relation between the coefficient of friction f and θ.
b) Give the components of the normal vector to the plane, $\hat{n}$, in terms of θ.

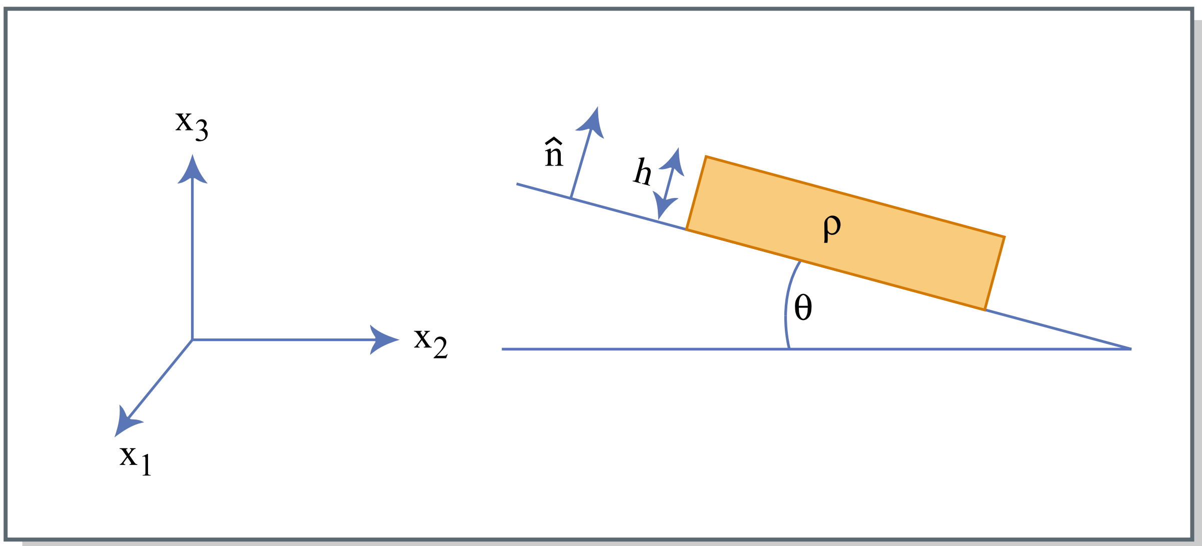

Figure by MIT OCW.

3) (60%) It is a good approximation in many geodynamical situations that variations in topography are compensated isostatically. That is, above the depth of compensation, the weight of the material in any column is a constant. The purpose of this problem is to determine whether isostatic compensation and a state of lithostatic stress are compatible. As a specific example, we will consider the simplified model of a mid-oceanic ridge shown in the figure. Assume that the lithosphere has a uniform density, $\rho_l = 3{,}300\,\text{kg}/\text{m}^3$, which is slightly greater than that of the underlying asthenosphere, which has a density $\rho_a = 3{,}250\,\text{kg}/\text{m}^3$. Assume that water has a density $1{,}000\,\text{kg}/\text{m}^3$. The lithosphere has zero thickness under the ridge crest, and thickens as it cools to a constant thickness (say 135 km) far from the ridge. As a result of isostasy, the ridge is at an elevation which is higher than the ocean basin.

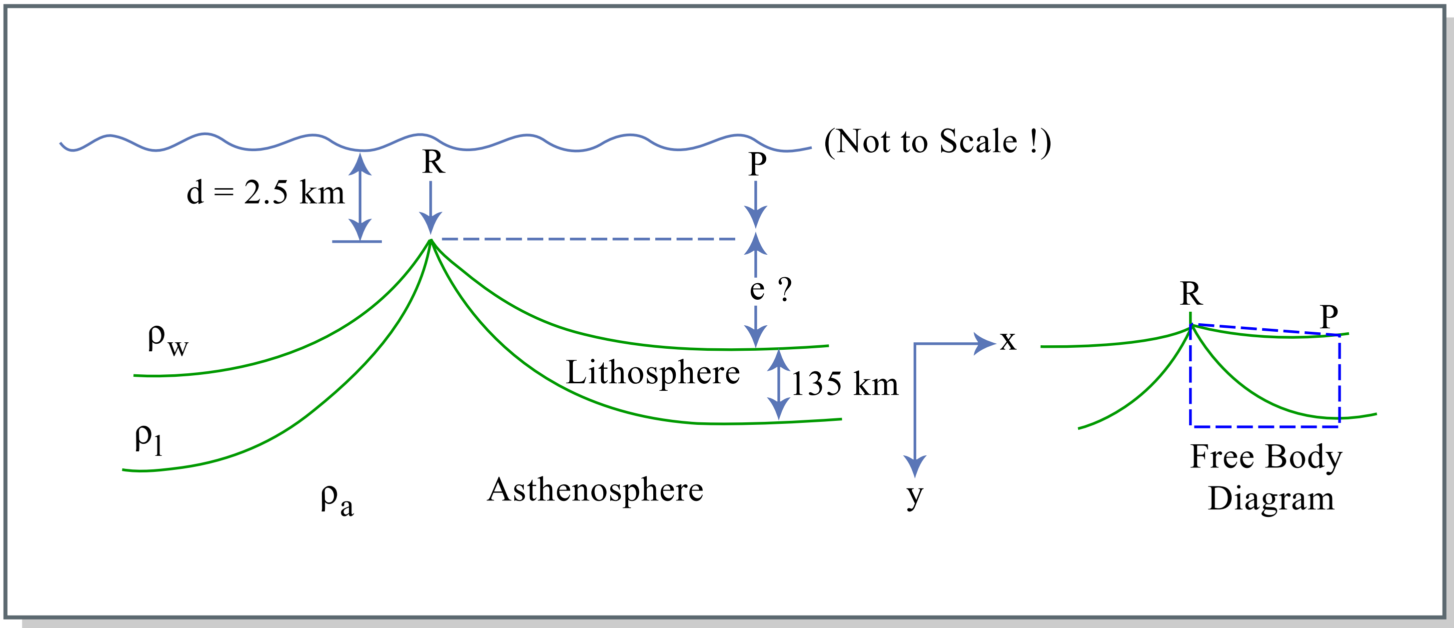

Figure by MIT OCW.

a) What is the elevation of the ridge, if it is in isostatic equilibrium?
b) Assuming that the state of stress is lithostatic at both places, make a graph of the horizontal normal stress, σ_{xx} as a function of depth beneath both the ridge crest (point R) and the abyssal plain (point P).
c) F_x, the horizontal force per unit length (into the page) acting on the lithosphere, can be determined by integrating σ_{xx} over the thickness of the lithosphere. For this problem, with constant densities, this integration is easy to do graphically. Consider a free body diagram of the lithosphere made by drawing a box with edges beneath points R and P. Determine the net horizontal force per unit length acting on the lithosphere if the assumption of lithostatic stress applies.
d) In order that there not be a net force acting on the lithosphere, the assumption of lithostatic stress must be modified. Calculate the magnitude of the average nonlithostatic stress $\Delta\sigma_{xx}$ acting over the 135 km thickness of the lithosphere required to balance the forces on the lithosphere.
e) How does the magnitude of $\Delta\sigma_{xx}$ compare to the average value of the lithostatic stress σ_{xx}?
f) If the departure from lithostatic stress, $\Delta\sigma_{xx}$, occurs in the old lithosphere, is it extensional or compressional? What if it occurs at the ridge?