

12.005: Problem set 2
Due 3/21/06

1) (20%) Write out symbolically the relationship between the traction vector acting on a surface, T_i , the normal vector describing the orientation of that surface, n_j , and the local stress tensor, σ_{ji} , using a) vector-matrix notation, b) the summation symbol Σ , c) Einstein summation notation, and d) full explicit evaluation, writing out each term.

2) (20%) [Modified from Turcotte & Schubert, 2-12 (old edition), 2-14 (new edition)]. The state of stress at a point on a fault plane is $\sigma_{yy} = 150$ MPa, $\sigma_{xx} = 200$ MPa, and $\sigma_{xy} = 0$ (y is depth and the x axis points westward). What are the normal traction and shear traction on the fault plane if the fault strikes N-S and dips 35° to the west? Do you expect the fault to slip, given your expectation of the value of the coefficient of friction?

3) (20%) For practice in Einstein summation, expand the following expressions using Kronecker deltas, simplify and evaluate the numerical values, where possible:

a) δ_{ii} b) $\delta_{ij} \delta_{ij}$ c) $\delta_{ij} \delta_{jk}$ d) $\delta_{ij} \delta_{jk} \delta_{kl}$ e) $\delta_{ij} A_{ik}$

Briefly explain what you are doing.

4) (40%) For the stress tensor

$$\sigma_{ij} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

a) Find the principal stresses and directions. (Use a right-handed coordinate system.)

b) Find the deviatoric stress tensor σ_{ij}^{dev}

c) Find the principal stresses and principal directions of the deviatoric stress tensor.

d) What are the relations of the principal stresses, directions, shears, etc. for the two tensors? Can you say anything in general about these relations for an arbitrary stress tensor and its deviator?