

12.005: Problem set 3 - Due 3/8/06

1) (20%) In the x_1, x_2, x_3 coordinate system, the stress tensor σ_{ij} is given by:

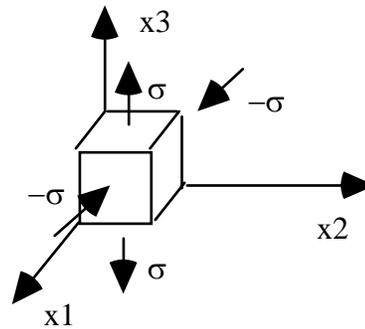
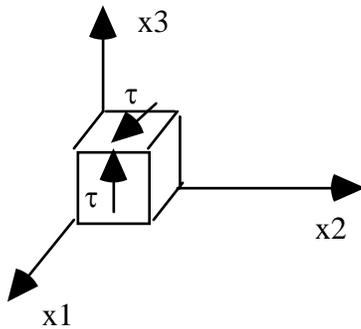
$$\sigma_{ij} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & -\sqrt{3} \\ 0 & -\sqrt{3} & 9 \end{bmatrix}$$

Consider the new ("primed") coordinate system obtained by rotating by 60° about the x_1 axis.

a) Determine σ'_{ij} .

b) Determine, compare and contrast the principal stresses and principal directions of σ_{ij} and σ'_{ij} .

2) (20%) Give the stress tensors, Mohr's circles and maximum shear stresses for the following cases of tractions on a unit cube:



3) (10%) If there are no body forces, demonstrate whether equilibrium exists, in general, for stresses:

$$\sigma_{xx} = 3x^2 + 4xy - 8y^2$$

$$\sigma_{yy} = 2x^2 + xy + 3y^2$$

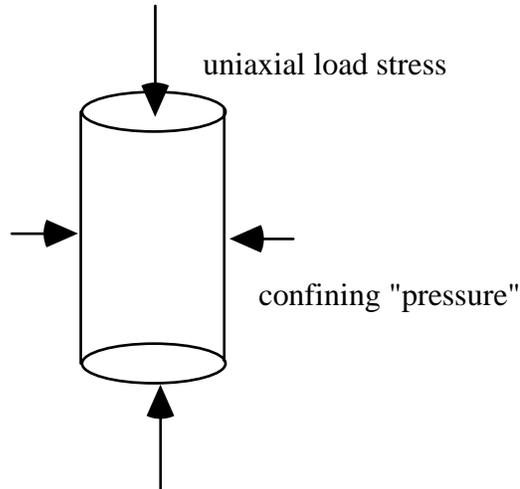
$$\sigma_{xy} = 1/2 x^2 - 6xy - 2y^2$$

$$\sigma_{zz} = \sigma_{xz} = \sigma_{yz} = 0$$

4) (30%) In a laboratory test (see picture on next page), samples of sandstone failed at the conditions:

a) confining "pressure" = 200 bars, uniaxial load stress = 1100 bars

b) confining "pressure" = 400 bars, uniaxial load stress = 1700 bars



If the strength of rock can be described by the Navier-Coulomb criterion, what are:

- (i) the intrinsic strength σ_0 ,
- (ii) the coefficient of internal friction μ ,
- (iii) the orientation of the fractures in each test,
- (iv) the maximum shear stress in each test,
- (v) the shear traction on the failure plane in each test?

Give the answers for dimensional quantities both in these units and in SI units.

5) (20%) One way of measuring the state of stress in a rock unit is to use 3 flat jacks arranged in a "delta" pattern, with each cut separated by 60° . (The name comes from the resemblance to the Greek character Δ .) The normal traction σ_n across each of the cuts can be measured easily using a "flat jack," as discussed in class. Write the expression for the normal traction across each of the three cuts, σ_n^i , $i = 1, 2, 3$, in terms of the components of the (two-dimensional) stress tensor, σ_{11} , σ_{12} , and σ_{22} .

