12.005 Lecture Notes 16

Simple Example: Uniaxial Strain

Uniaxial strain is excellent approximation, but not exact.

Sediment loading erosion:

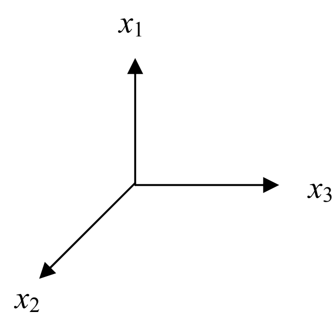

Figure 16.1

Assume $e_{22} = e_{33} = 0, \quad e_{11} \neq 0$

$$\tau_{11} = (\lambda + 2\mu) e_{11}$$

$$\tau_{22} = \tau_{33} = \lambda e_{11} = \frac{\lambda}{\lambda + 2\mu} \tau_{11} = \frac{\nu}{1-\nu} \tau_{11}$$

$$\tau_{11} = \frac{(1-\nu) E e_{11}}{(1+\nu)(1-2\nu)}$$

$$\tau = -\rho g d \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \underset{\approx}{\tau}^{\text{uniaxial}}$$

For $\gamma = 0.25$, $\tau_{11} = -\rho g h$; -0.27 kbar/km (crustal rock)

$$\tau_{22} = \tau_{33} = \frac{\tau_{11}}{3} \; ; \; -0.1 \text{ kbar/km} \Rightarrow \text{ deviatoric stress}$$

$$\begin{bmatrix} -0.27 & 0 & 0 \\ 0 & -0.09 & 0 \\ 0 & 0 & -0.09 \end{bmatrix} = \begin{bmatrix} -0.15 & 0 & 0 \\ 0 & -0.15 & 0 \\ 0 & 0 & -0.15 \end{bmatrix} + \begin{bmatrix} -0.12 & 0 & 0 \\ 0 & -0.06 & 0 \\ 0 & 0 & -0.06 \end{bmatrix}$$

Unloading:

Assume initially lithostatic

$$\tau_{11} = \tau_{22} = \tau_{33} = -\rho g d$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{2}{3}\rho g d & 0 \\ 0 & 0 & -\frac{2}{3}\rho g d \end{bmatrix} = \begin{bmatrix} \frac{4}{9} & 0 & 0 \\ 0 & -\frac{2}{9} & 0 \\ 0 & 0 & -\frac{2}{9} \end{bmatrix} \rho g d + \begin{bmatrix} -\frac{4}{9} & 0 & 0 \\ 0 & -\frac{4}{9} & 0 \\ 0 & 0 & -\frac{4}{9} \end{bmatrix} \rho g d$$

$\Rightarrow$ thrust faults

popups

give direction of compressional stress

Assume $\sigma_2 < \sigma_3$ tectonic stress
Type of faulting: (compression negative)

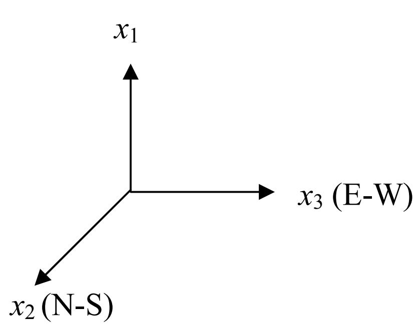

Figure 16.2

$\sigma_2 < \sigma_3 < \sigma_1 \quad \Rightarrow$ thrusting on E-W plane

$\sigma_2 < \sigma_1 < \sigma_3 \quad \Rightarrow$ strike-slip

$\sigma_1 < \sigma_2 < \sigma_3 \quad \Rightarrow$ normal faulting on N-S plane

Stress increment from burial

$$\underset{\sim}{\tau} = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \rho g d = -\begin{bmatrix} \frac{5}{9} & 0 & 0 \\ 0 & \frac{5}{9} & 0 \\ 0 & 0 & \frac{5}{9} \end{bmatrix} \rho g d + \begin{bmatrix} -\frac{4}{9} & 0 & 0 \\ 0 & \frac{2}{9} & 0 \\ 0 & 0 & \frac{2}{9} \end{bmatrix} \rho g d$$

Near surface in Ventura Basin, thrusting occurs. Stress tensor could look something like:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\tau & 0 \\ 0 & 0 & -\alpha\tau \end{bmatrix}$$

$\tau \equiv$ "tectonic" stress

$$0 \le \alpha \le 1$$
$$p = -\frac{1+\alpha}{3}\tau$$

Assume tectonic stress independent of depth.

For simplicity, assume

$$\alpha = \frac{1}{2} \quad \Rightarrow \quad p = -\frac{\tau}{2}$$

Total deviatoric stress is

$$\begin{bmatrix} -\frac{4}{9} & 0 & 0 \\ 0 & \frac{2}{9} & 0 \\ 0 & 0 & \frac{2}{9} \end{bmatrix} \rho g d - \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tau$$

Normal faulting occurs if

$$-\frac{4}{9}\rho g d+\frac{\tau}{2}<\frac{2}{9}\rho g d-\frac{\tau}{2}<\frac{2}{9}\rho g d$$

$$-\frac{2}{3}\rho g d<-\tau$$