12.005 Lecture Notes 22

Plates (continued)

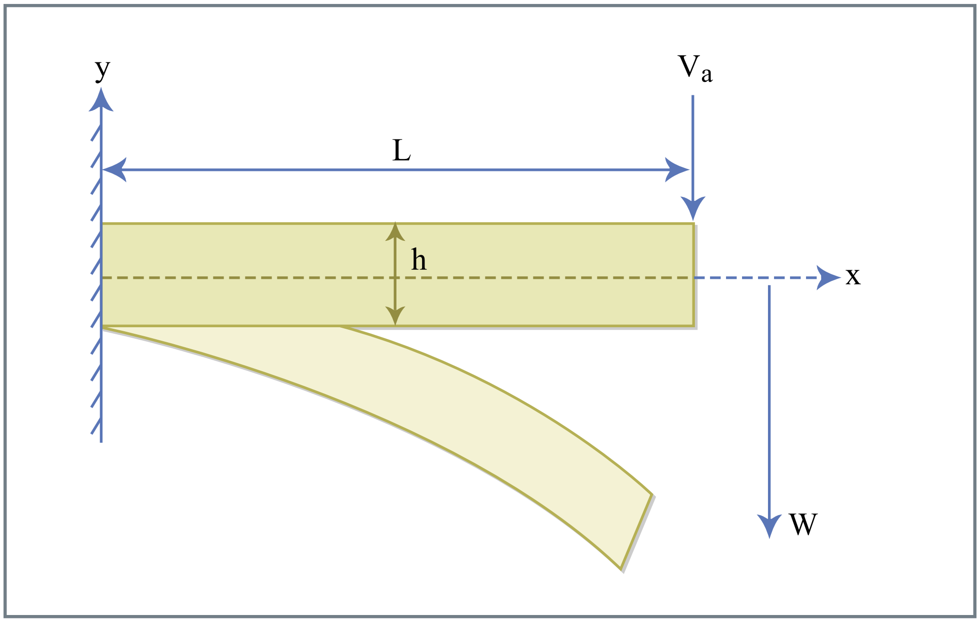

Figure 22.1

Figure by MIT OCW.

$$w = \frac{V_a}{2D} x^2 (L - \frac{x}{3})$$

Assumption $|\sigma_{xy}| = |\sigma_{xx}|$

$$\sigma_{xx} = \frac{E}{1-\nu^2} \varepsilon_{xx}$$

$$\varepsilon_{xx} = -y \frac{d^2 w}{dx^2}$$

$$M = -D \frac{d^2 w}{dx^2}$$

$$\varepsilon_{xx} = \frac{y}{D} M$$

$$\sigma_{xx}^{\max} = \frac{E}{1-\nu^2} \frac{h}{2} \frac{1}{D} V_a L = \frac{6 V_a L}{h^2} = \frac{6 V_a}{h} \left(\frac{L}{h} \right)$$

$$\langle \sigma_{xy} \rangle = \frac{V_a}{h} = \frac{1}{6}\frac{h}{L}\sigma_{xx}^{\max}$$

Response to a line load (e.g. volcanic chain under water)

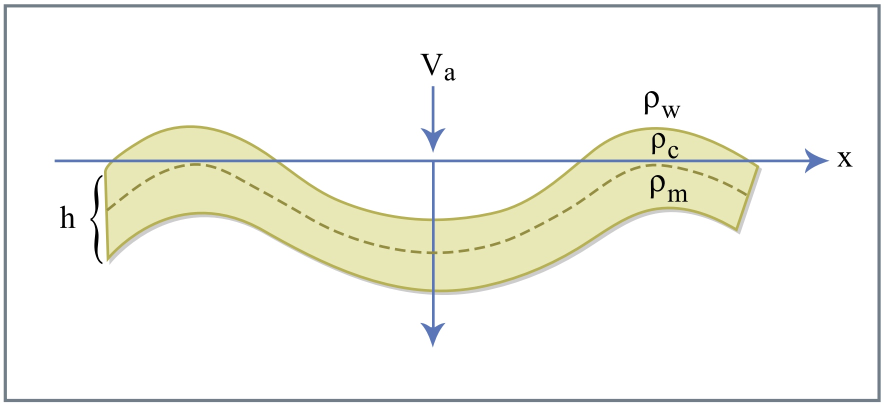

Figure 22.2

Figure by MIT OCW.

$$D\frac{d^4 w}{dx^4} + (\rho_m - \rho_w)gw = \begin{cases} 0 & \text{at } x \neq 0 \\ V_0 & \text{at } x = 0 \end{cases}$$

Solution to homogeneous equation

$$w = e^{x/\alpha}(C_1 \cos\frac{x}{\alpha} + C_2 \sin\frac{x}{\alpha}) + e^{-x/\alpha}(C_3 \cos\frac{x}{\alpha} + C_4 \sin\frac{x}{\alpha})$$

with $\alpha = \left[\frac{4D}{(\rho_m - \rho_w)g}\right]^{1/4}$ α is flexural parameter.

Invoke symmetry, boundedness. Determine solution only for $x \geq 0$.

$$\frac{dw}{dx} = 0 \quad \text{at } x = 0 \qquad C_3 = C_4$$

$w \to 0 \quad x \to \infty \qquad C_1 = C_2 = 0$

$$w = C_3 e^{-x/\alpha}(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha})$$

Now to evaluate C_3, go back to the end load problem (or original definition)

$$\frac{dM}{dx} = V + P\frac{dw}{dx}$$

C_3 depends on V_0.

$$\frac{d^3w}{dx^3} = -\frac{1}{2}\frac{V}{D}$$

$$\frac{1}{2}V_0 = D\frac{d^3w}{dx^3}\bigg|_{x=0} = \frac{4DC_3}{\alpha^3}$$

V_0 is negative load, half supported by each side.

$$w = \frac{V_0\alpha^3}{8D}e^{-x/\alpha}(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha})$$

$$w_0 = \frac{V_0\alpha^3}{8D}$$

$$w = w_0 e^{-x/\alpha}(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha})$$

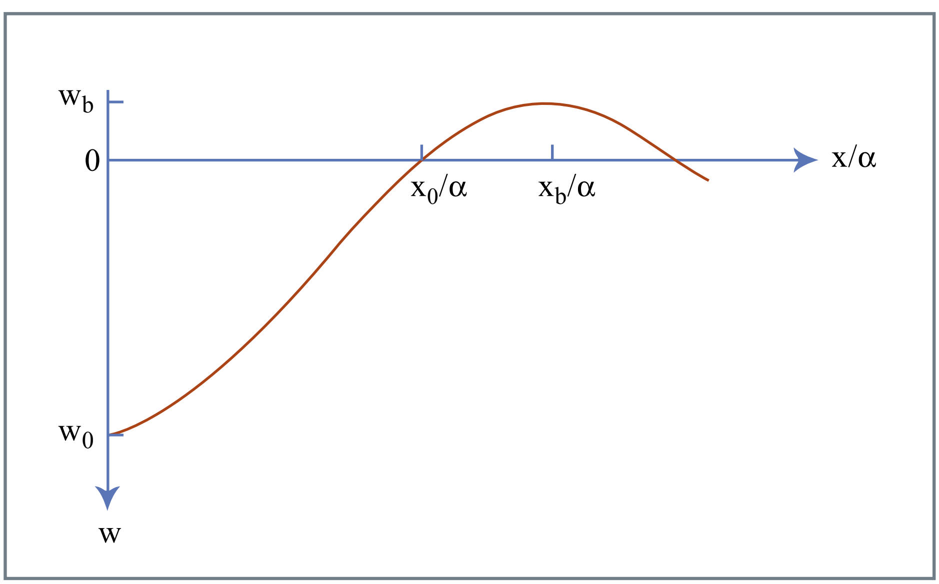

Figure 22.3

Figure by MIT OCW.

$$\frac{x_0}{\alpha} = \tan^{-1}(-1) = \frac{3\pi}{4}$$

$$\frac{x_b}{\alpha} = \sin^{-1}(0) = \pi$$

$$w_b = -w_0 e^{-\pi} = -0.0432 w_0$$

$$x\ ;\ 3^{\text{o}}\ ;\ 300\text{km}$$

$$\alpha = \frac{314}{\pi}\text{km}\ ;\ 100\text{km}$$

Suppose plate is fractured.

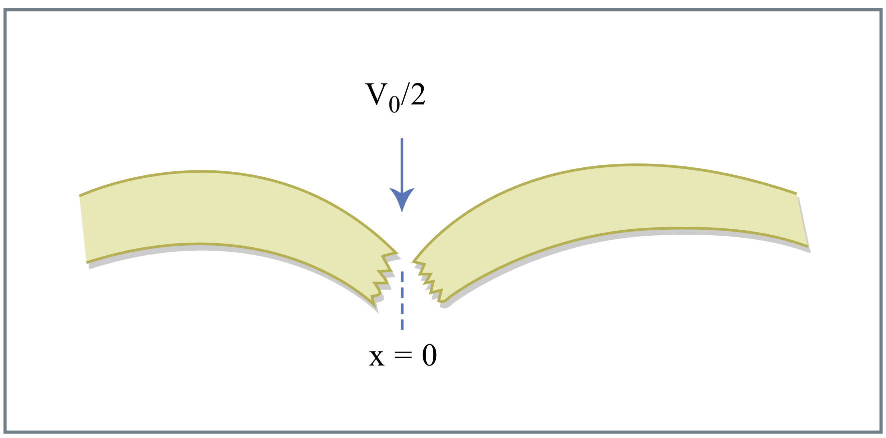

Figure 22.4

Figure by MIT OCW.

Same equation. Different boundary conditions $\Rightarrow$ different profile.

$$M = 0 \ \text{ at } x = 0 \qquad \frac{d^2 w}{dx^2} = 0$$

$$w = \frac{V_0 \alpha^3}{4D} e^{-x/\alpha} \cos\frac{x}{\alpha}$$

$$w_0 = \frac{V_0 \alpha^3}{4D}$$

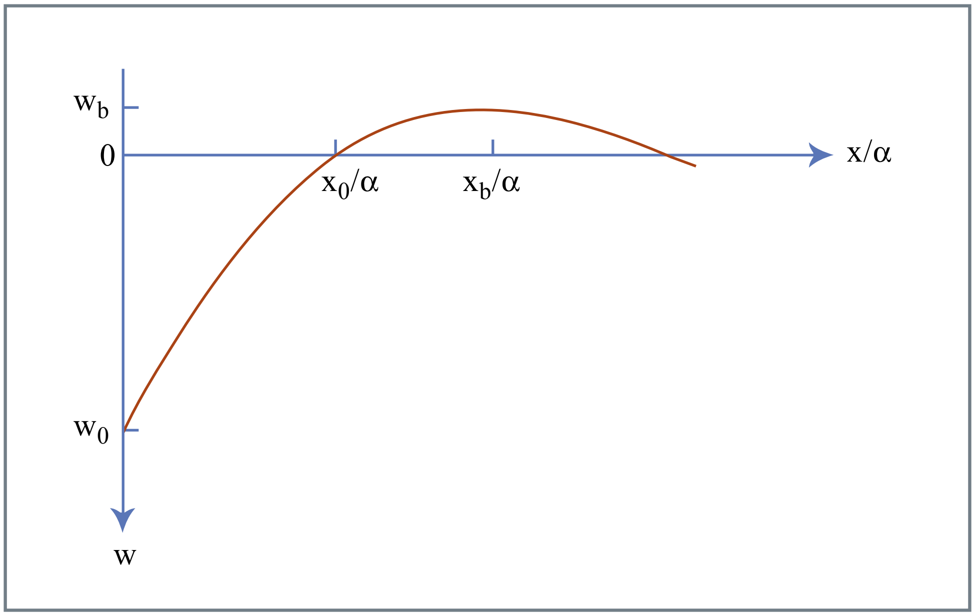

Figure 22.5

Figure by MIT OCW.

$$x_0 = \frac{\pi}{2}\alpha, \qquad x_b = \frac{3\pi}{4}\alpha, \qquad w_b = -0.067 w_0$$

h ; 35km unbroken

h ; 50km broken

Thickness of broken lithosphere is about 1.5 times thickness of the unbroken lithosphere.

Bending at a trench.

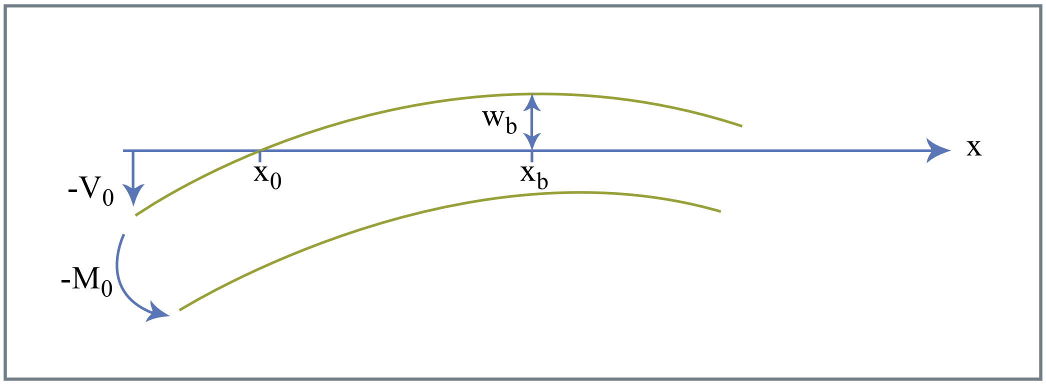

Figure 22.6

Figure by MIT OCW.

$$w = e^{-x/\alpha}(C_3 \cos\frac{x}{\alpha} + C_4 \sin\frac{x}{\alpha})$$

$$C_4 = -\frac{M_0 \alpha^2}{2D}$$

$$C_3 = (V_0 \alpha + M_0)\frac{\alpha^2}{2D} \quad \text{(entire } V_0 \text{ supported)}$$

$$w = \frac{\alpha^2 e^{-x/\alpha}}{2D}\left\{-M_0 \sin\frac{x}{\alpha} + (V_0 \alpha + M_0)\cos\frac{x}{\alpha}\right\}$$

Observables -- x_0, x_b, w_b

$$\tan\frac{x_0}{\alpha} = 1 + \alpha V_0 / M_0$$

$$\tan\frac{x_b}{\alpha} = -1 - 2M_0 / \alpha V_0$$

$$x_b - x_0 = \frac{\pi}{4}\alpha$$

$$w_b = \frac{\alpha^2 e^{-x_b/\alpha}}{2D}\left\{-M_0 \sin\frac{x_b}{\alpha} + (V_0\alpha + M_0)\cos\frac{x_b}{\alpha}\right\}$$

$$w_b = -\frac{\alpha^2 M_0}{2D} e^{-[(x_b - x_0)/\alpha]} e^{-x_0/\alpha} \frac{\sin\left((x_b - x_0)/\alpha\right)}{\cos\left(x_0/\alpha\right)}$$