12.103

Strange Bedfellows: The Science and Policy of Natural Hazards

Seismic waves

Spring 2008

Seismic wave equation

$$mass \times acceleration = \sum forces$$

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + f_i$$

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Please see:
http://epscx.wustl.edu/seismology/book/chapter2/chap2_sr/2_3_05_s.JPG

Stein and Wysession, 2003

Seismic wave equation

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Stein and Wysession, 2003

strain tensor

$$e_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right) = \begin{pmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right) & \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}\right) & \frac{\partial u_3}{\partial x_3} \end{pmatrix}$$

Seismic wave equation

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Stein and Wysession, 2003

strain tensor

$$e_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right) = \begin{pmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right) & \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}\right) & \frac{\partial u_3}{\partial x_3} \end{pmatrix}$$

Seismic wave equation

Constitutive equation (Hooke's Law)

$$\sigma_{ij} = c_{ijkl} e_{kl} = \sum_{k=1,3} \sum_{l=1,3} c_{ijkl} e_{kl}$$

$$\sigma_{ij} = c_{ijkl} \sum_{k=1,3} \sum_{l=1,3} \left[\frac{1}{2} \left(\partial_k u_l + \partial_l u_k \right) \right]$$

$$= c_{ijkl} \sum_{k=1,3} \sum_{l=1,3} \left[\partial_l u_k \right]$$

$$= c_{ijkl} u_{k,l}$$

Back to the equation of motion:

$$mass \times acceleration = \sum forces$$

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + f_i$$

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \left[c_{ijkl} u_{(k,l)} \right]_{,j} + f_i$$

$$\rho \frac{\partial^2 u_i}{\partial t^2} \cong c_{ijkl} u_{(k,l),j} + f_i$$

Seismic wave equation

$$\rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} u_{(k,l),j} + f_i$$

isotropic medium:

$$c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{il} \delta_{jk} + \delta_{ik} \delta_{jl})$$

Helmholtz decomposition:

$$\underline{u} = \underline{\nabla} \phi + \underline{\nabla} \times \underline{\psi}$$

$$\underline{\nabla} \times \underline{\nabla} \phi = 0$$

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{\psi} = 0$$

Seismic wave equation

P-wave

wave eq: $\alpha^2 \underline{\nabla}^2 \phi - \frac{\partial^2 \phi}{\partial t^2} = -\frac{1}{\rho} F_P$

velocity: $\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}$

S-wave

$\beta^2 \underline{\nabla}^2 \underline{\psi} - \frac{\partial^2 \underline{\psi}}{\partial t^2} = -\frac{1}{\rho} \underline{F_S}$

$\beta = \sqrt{\frac{\mu}{\rho}}$

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Please see:
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Stein and Wysession, 2003

Seismic velocities in the Earth

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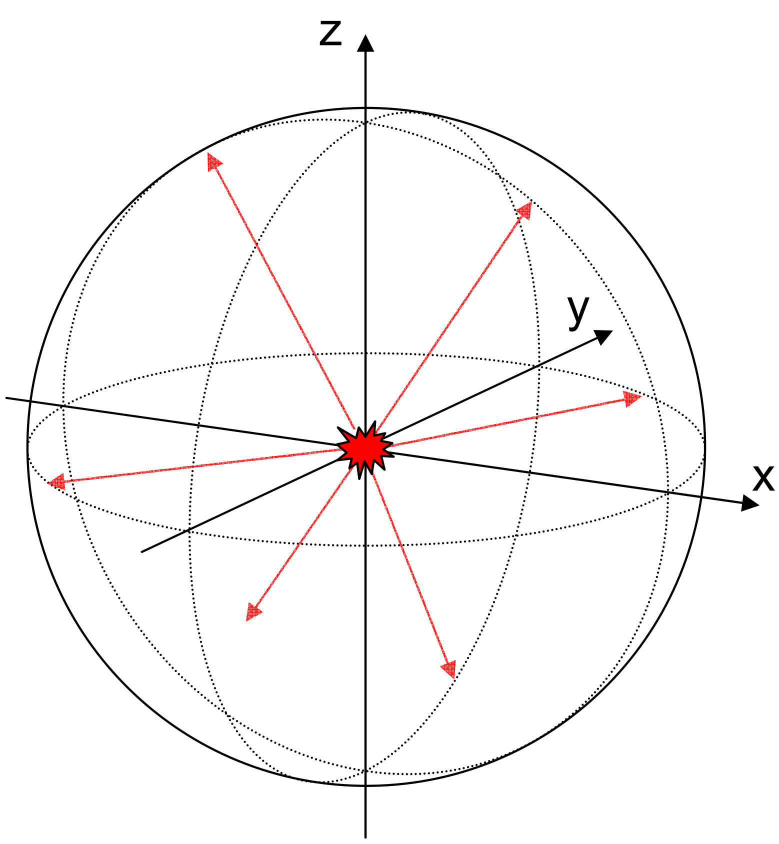
Stein and Wysession, 2003

Seismic velocities in the Earth

Depth (km)	Radius (km)	Density (g/cc)	P (kbar)	Vp (km/s)	Vs (km/s)	Φ (km^2/s^2)	K (kbar)	μ (kbar)	σ
0	6371	1.02	0	1.45	0	2.10	21	0	0.500
3.0	6368	1.02	0.2	1.45	0	2.10	21	0	0.500
3.0	6368	2.60	0.3	5.80	3.20	19.99	520	266	0.281
15.0	6356	2.60	3.3	5.80	3.20	19.99	520	266	0.281
15.0	6356	2.90	3.3	6.80	3.90	25.96	753	441	0.254
24.4	6346.6	2.90	6.0	6.80	3.90	25.96	753	441	0.254
24.4	6346.6	3.38	6.0	8.11	4.49	38.89	1315	682	0.278
40.0	6331	3.37	11.2	8.10	4.48	38.81	1311	680	0.279
60.0	6311	3.37	17.8	8.08	4.47	38.71	1307	677	0.279
80.0	6291	3.37	24.5	8.07	4.46	38.60	1303	674	0.279

Seismic waves

Body Waves: P-waves and S-waves are body waves, as they can travel in all directions through an elastic volume



Seismic waves

Ray theory: seismic wavefield can be described by discrete, linear ray paths linking sources and receivers (infinite frequency approximation)

Please see:
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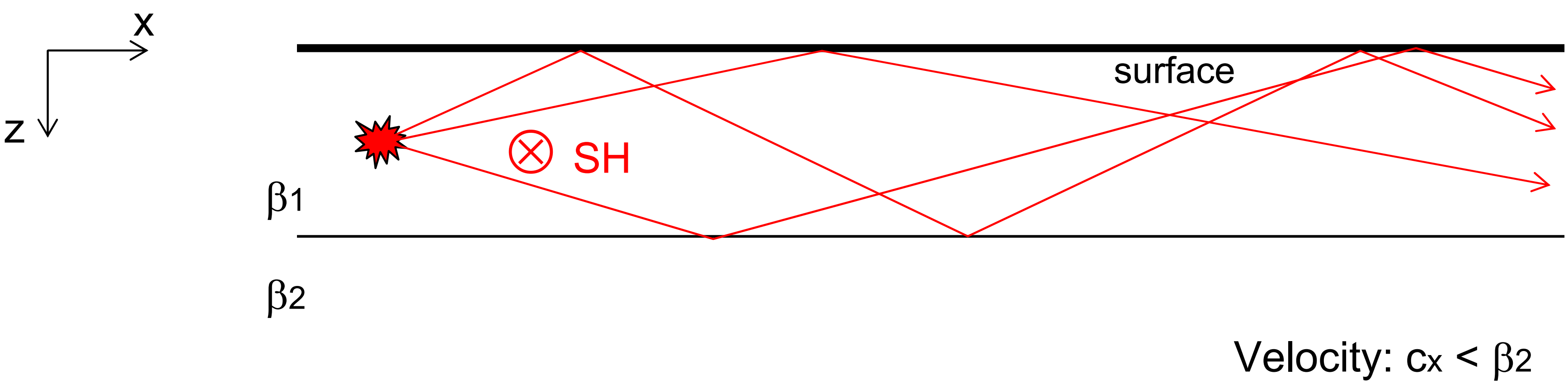
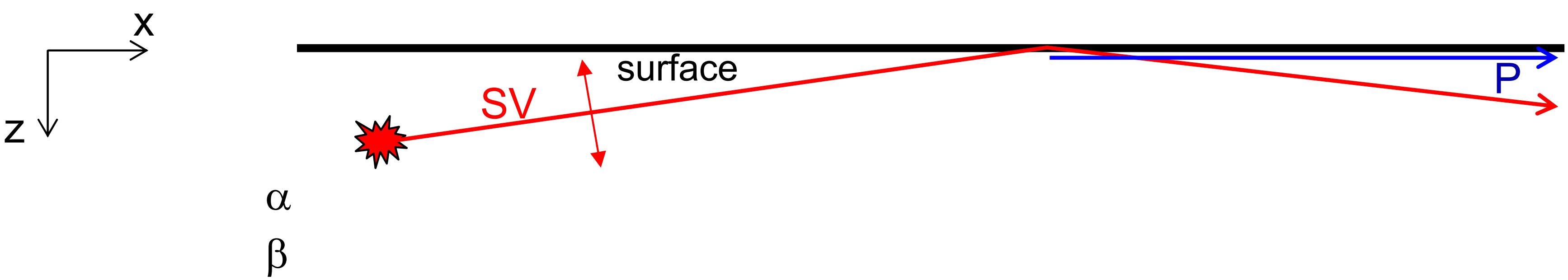
Surface waves



Surface waves

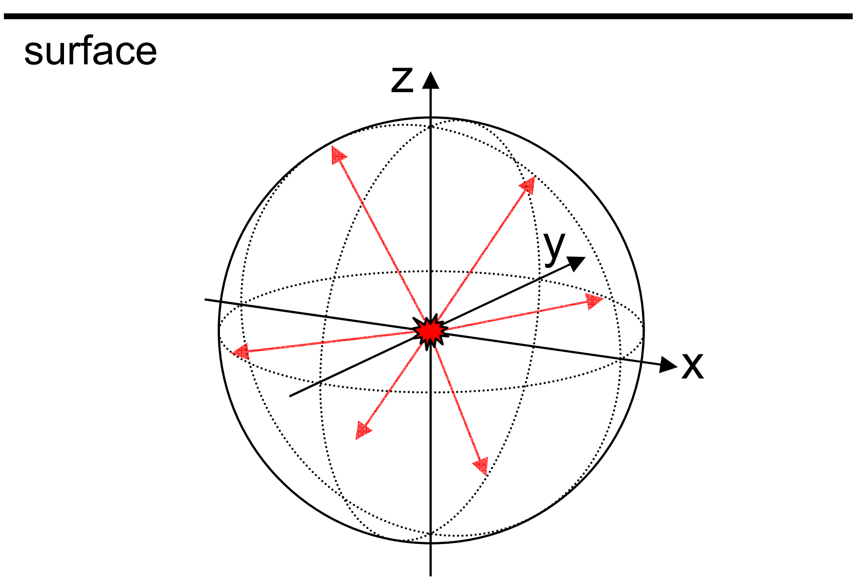


Velocity: $c_x = 0.92\ \beta$

Energy of seismic waves

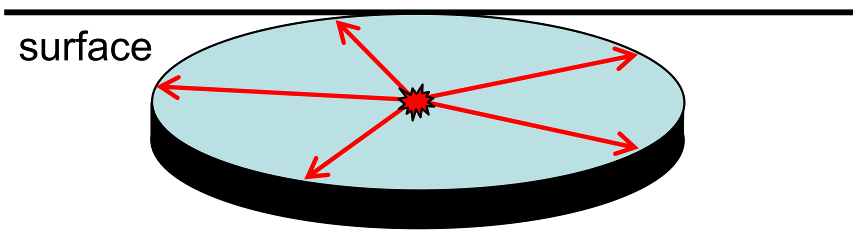
$$\overline{E} = \frac{1}{2}\rho A^2 \omega^2$$

Body waves



$$\overline{E}(r) \propto \frac{1}{r^2}$$

Surface waves



$$\overline{E}(r) \propto \frac{1}{r}$$

Kobe earthquake, Jan 17, 1995, M 7.2

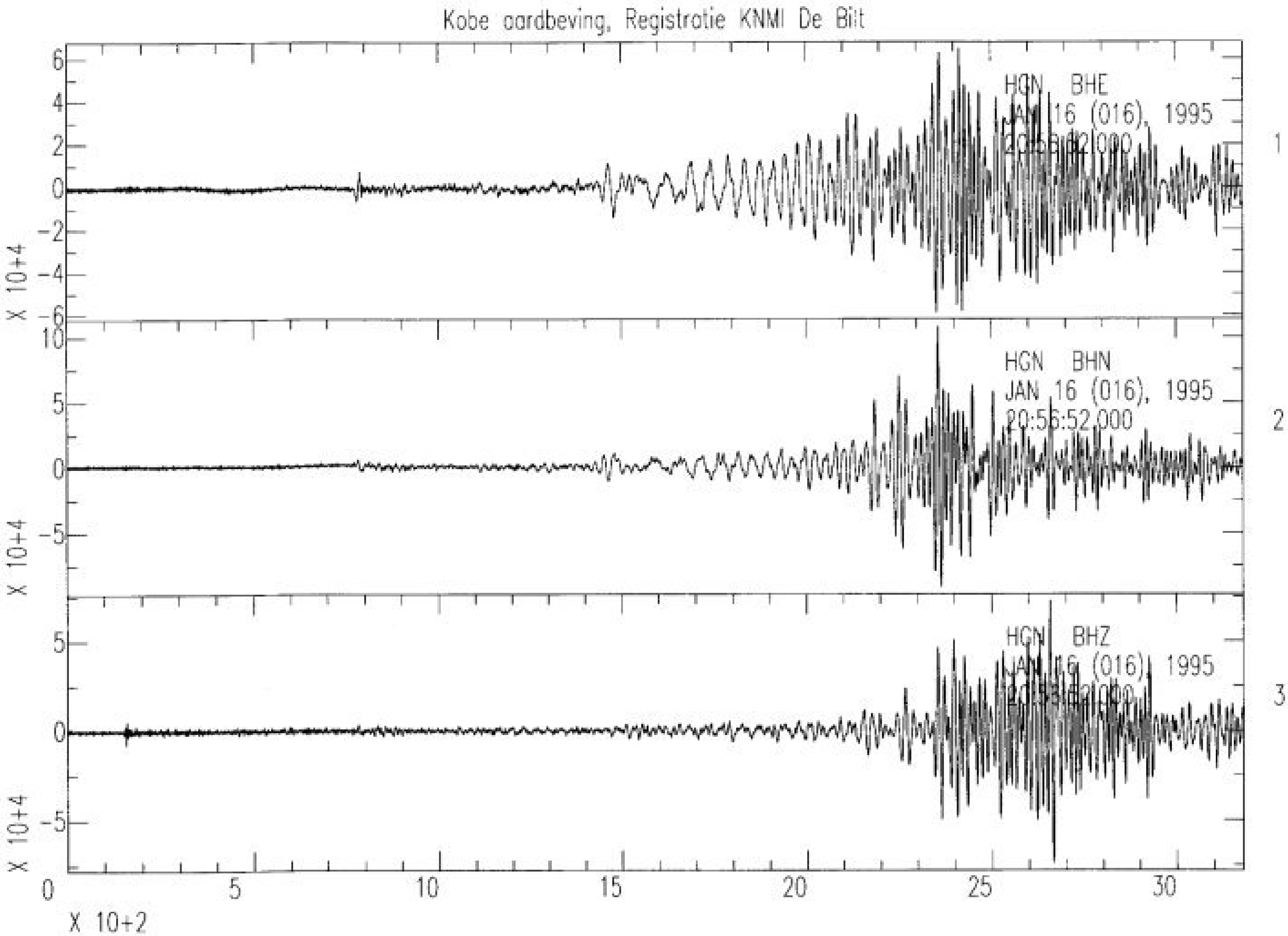


Image courtesy of Oklahoma Geological Survey.

Kobe earthquake, Jan 17, 1995, M 7.2

links to video footage of surface waves:

http://www.youtube.com/watch?v=pXATR6vOcfQ

http://www.youtube.com/watch?v=0plbf5w0sbA&NR=1

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