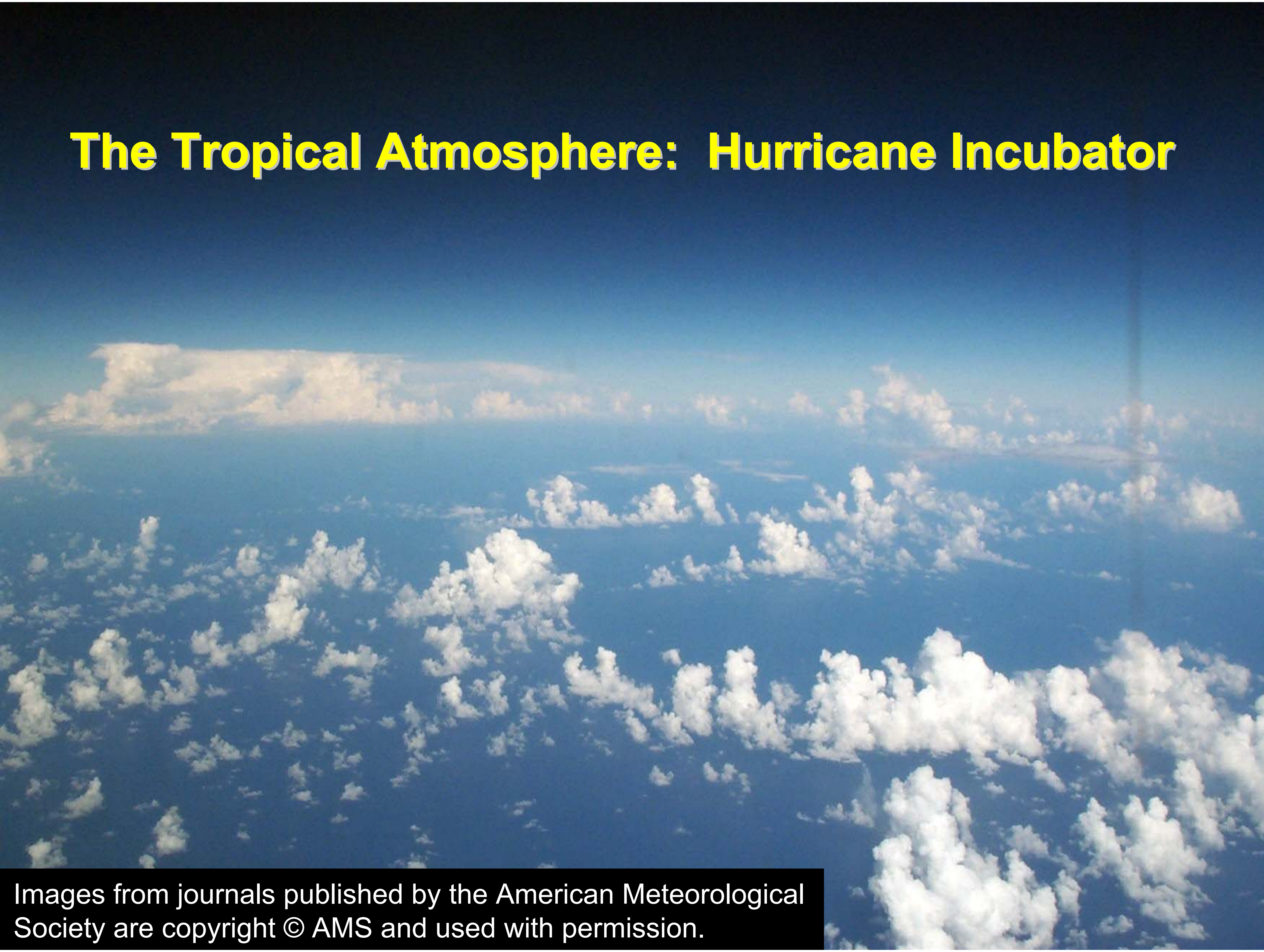
The Tropical Atmosphere: Hurricane Incubator



A One-Dimensional Description of the Tropical Atmosphere

Elements of Thermal Balance: Solar Radiation

- Luminosity: 3.9×10^{26} J s^{-1} = 6.4×10^{7} Wm^{-2} at top of photosphere

- Mean distance from earth: 1.5×10^{11} m
- Flux density at mean radius of earth

$$S_0 \equiv \frac{L_0}{4\pi d^2} = 1370\, Wm^{-2}$$

Stefan-Boltzmann Equation: $F = \sigma T^4$

$\sigma = 5.67 \times 10^{-8}\ W m^{-2} K^{-4}$

Sun: $\sigma T^4 = 6.4 \times 10^7\ W m^{-2}$

$\rightarrow \quad T \simeq 6{,}000\, K$

Disposition of Solar Radiation:

$$\text{Total absorbed solar radiation} = S_0\left(1-a_p\right)\pi r_p^{\,2}$$

$$a_p \equiv \text{planetary albedo}\left(\simeq 30\%\right)$$

$$\text{Total surface area} = 4\pi r_p^{\,2}$$

$$\text{Absorption per unit area} = \frac{S_0}{4}\left(1-a_p\right)$$

Absorption by clouds, atmosphere, and surface

Terrestrial Radiation:

Effective emission temperature:

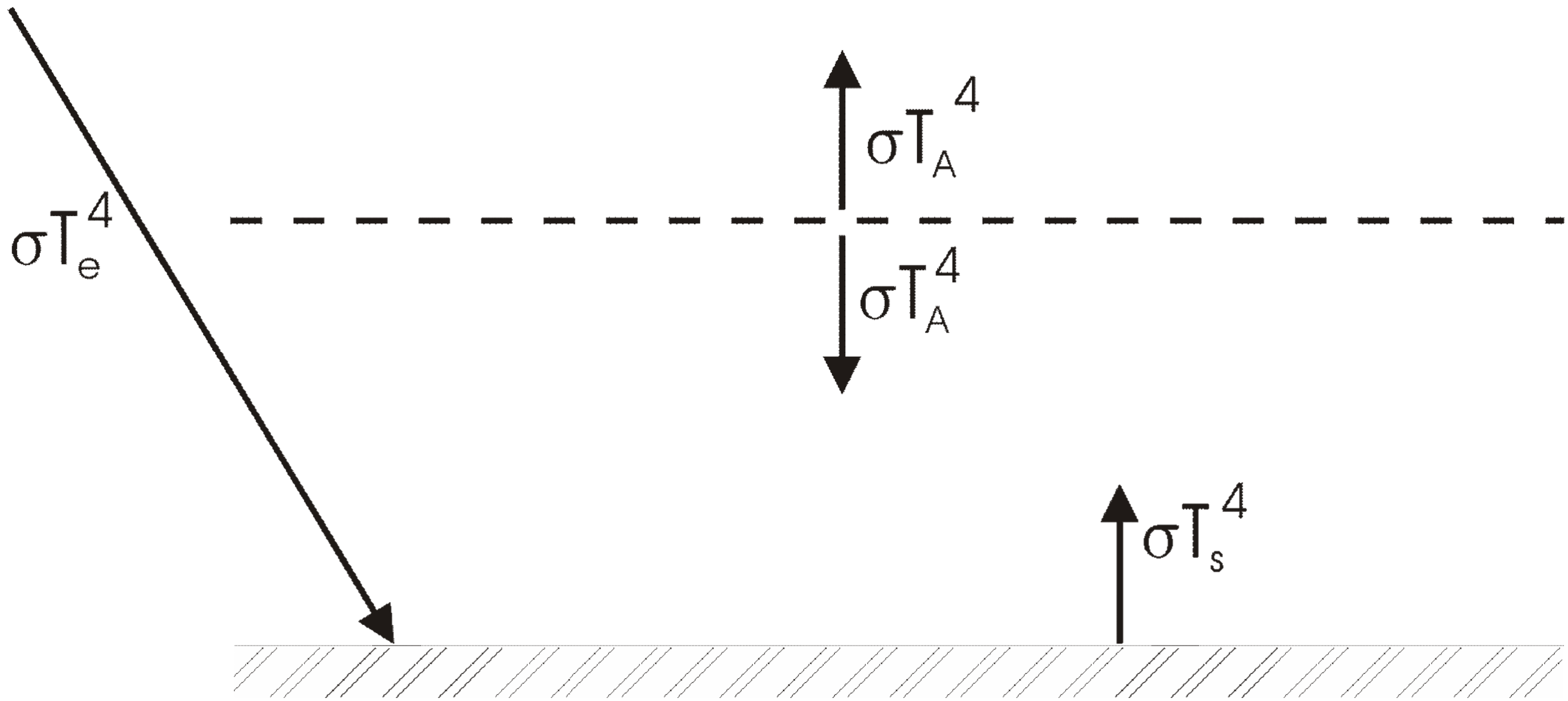
$$\sigma T_e^4 \equiv \frac{S_0}{4}\left(1-a_p\right)$$

Earth: $T_e = 255K = -18°C$

Observed average surface temperature $= 288K = 15°C$

Highly Reduced Model

- Transparent to solar radiation
- Opaque to infrared radiation
- Blackbody emission from surface and each layer



Radiative Equilibrium:

Top of Atmosphere:

$$\sigma T_A^{\,4} = \frac{S_0}{4}\left(1 - a_p\right) = \sigma T_e^{\,4}$$

$$\rightarrow \quad \boxed{T_A = T_e}$$

Surface:

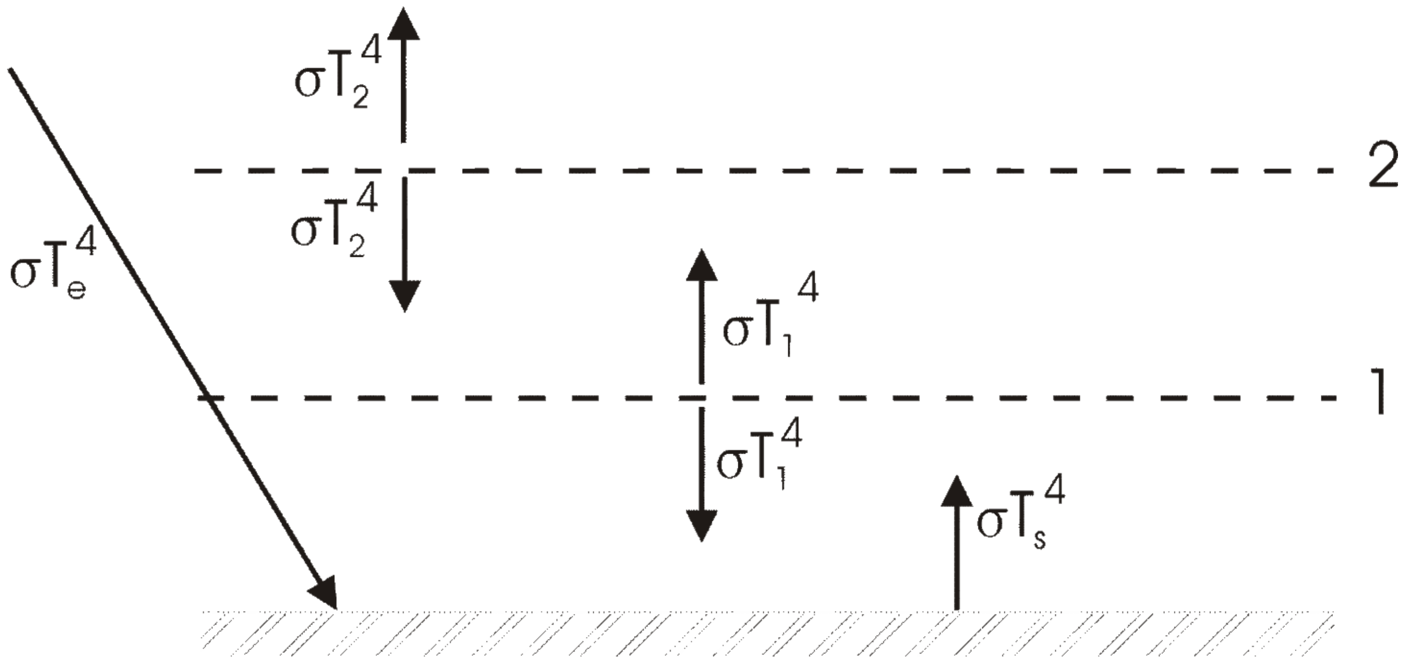
$$\sigma T_s^{\,4} = \sigma T_A^{\,4} + \frac{S_0}{4}\left(1 - a_p\right) = 2\sigma T_e^{\,4}$$

$$\rightarrow \boxed{T_s = 2^{1/4} T_e} = 303\ K$$

Surface temperature too large because:

- Real atmosphere is not opaque
- Heat transported by convection as well as by radiation

Extended Layer Models



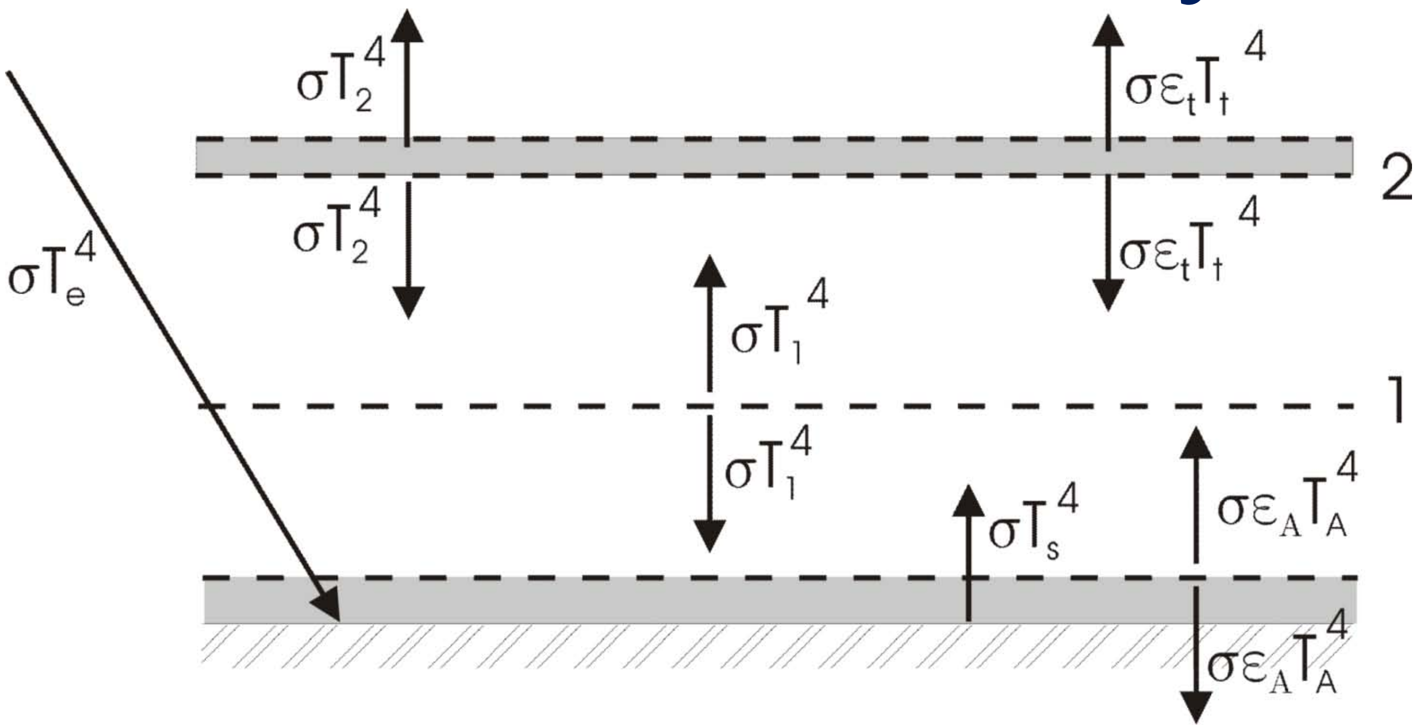
$$TOA: \quad \sigma T_2^{\,4} = \sigma T_e^{\,4} \rightarrow T_2 = T_e$$

$$Middle\ Layer: \quad 2\sigma T_1^{\,4} = \sigma T_2^{\,4} + \sigma T_s^{\,4} = \sigma T_e^{\,4} + \sigma T_s^{\,4}$$

$$Surface: \quad \sigma T_s^{\,4} = \sigma T_e^{\,4} + \sigma T_1^{\,4}$$

$$\rightarrow \quad T_s = 3^{1/4} T_e \qquad\qquad T_1 = 2^{1/4} T_e$$

Effects of emissivity<1



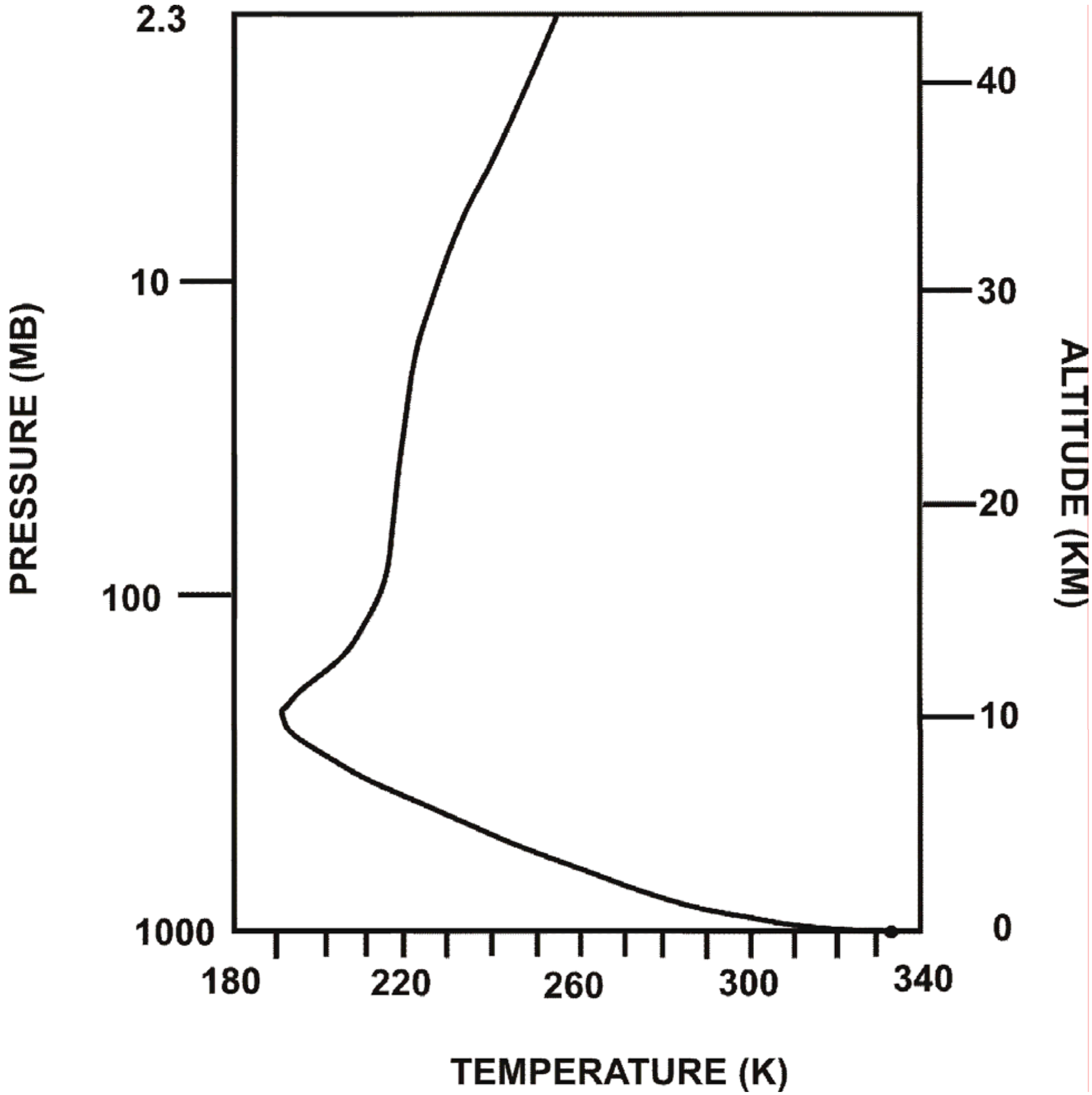
$$Surface: \quad 2\varepsilon_A \sigma T_A{}^4 = \varepsilon_A \sigma T_1{}^4 + \varepsilon_A \sigma T_s{}^4$$

$$\rightarrow \quad T_A = \left(5/2\right)^{1/4} T_e \simeq 321K \quad < T_s$$

$$Stratosphere: \quad 2\varepsilon_t \sigma T_t{}^4 = \varepsilon_A \sigma T_2{}^4$$

$$\rightarrow \quad T_t = \left(1/2\right)^{1/4} T_e \simeq 214K \quad < T_e$$

Full calculation of radiative equilibrium:



Problems with radiative equilibrium solution:

- Too hot at and near surface
- Too cold at a near tropopause
- Lapse rate of temperature too large in the troposphere
- (But stratosphere temperature close to observed)

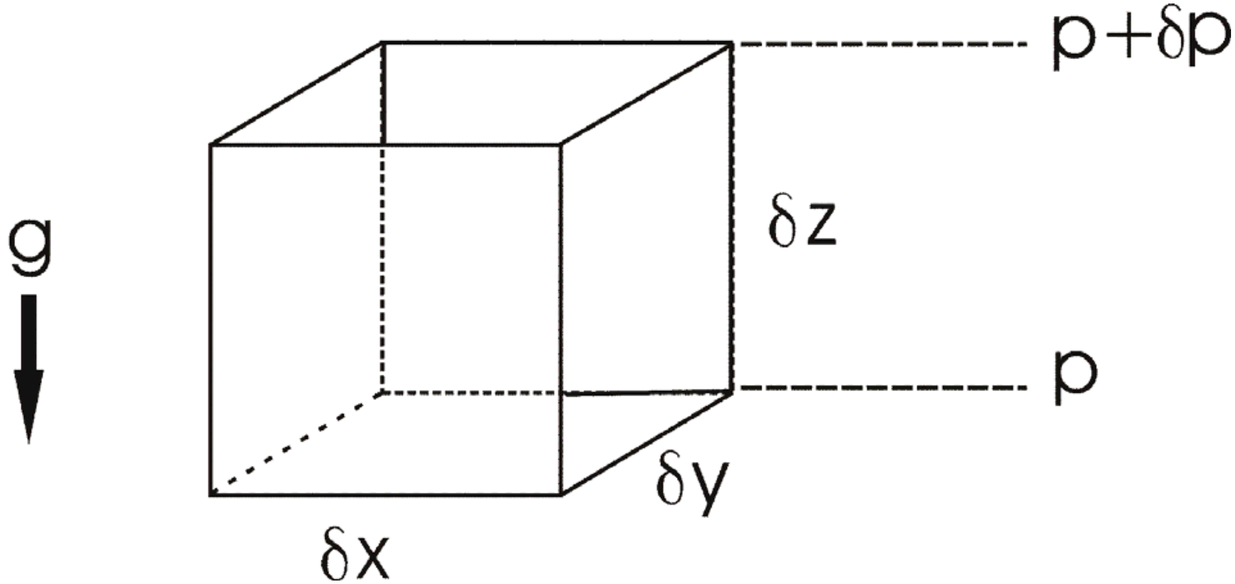
Missing ingredient: Convection

- As important as radiation in transporting enthalpy in the vertical
- Also controls distribution of water vapor and clouds, the two most important constituents in radiative transfer

When is a fluid unstable to convection?

- Pressure and hydrostatic equilibrium
- Buoyancy
- Stability

Hydrostatic equilibrium:



$$Weight: \quad -g\rho\delta x\delta y\delta z$$

$$Pressure: \quad p\delta x\delta y-(p+\delta p)\delta x\delta y$$

$$F=MA: \quad \rho\delta x\delta y\delta z\frac{dw}{dt}=-g\rho\delta x\delta y\delta z-\delta p\delta x\delta y$$

$$\frac{dw}{dt}=-g-\alpha\frac{\partial p}{\partial z}, \qquad \alpha=1/\rho=specific\ volume$$

Pressure distribution in atmosphere at rest:

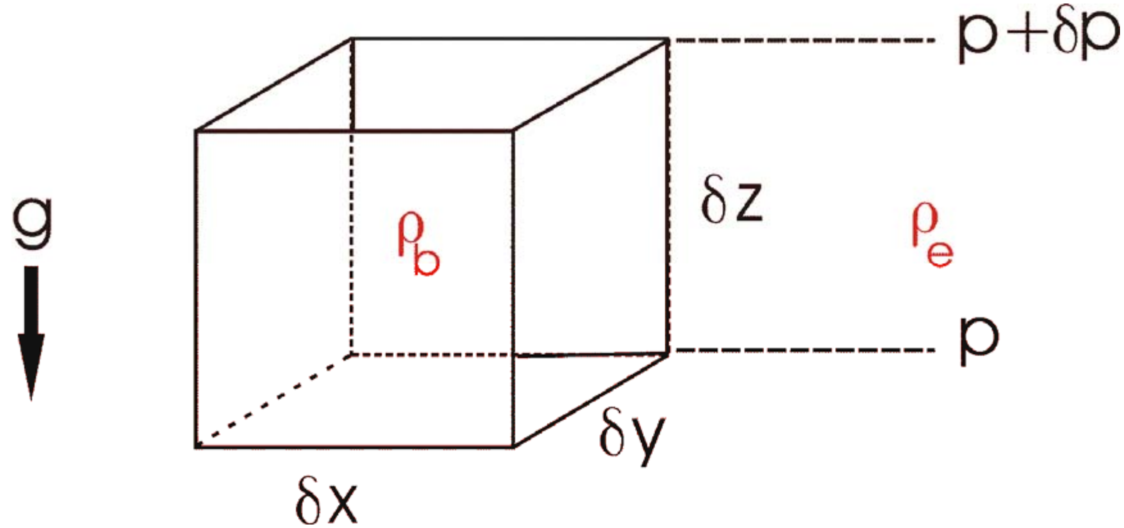
$$Ideal\ gas: \quad \alpha = \frac{RT}{p}, \qquad R \equiv \frac{R^*}{\bar{m}}$$

$$Hydrostatic: \quad \frac{1}{p}\frac{\partial p}{\partial z} = \frac{\partial \ln(p)}{\partial z} = -\frac{g}{RT}$$

$$Isothermal\ case: \quad p = p_0 e^{-z/H}, \qquad H \equiv \frac{RT}{g} = \text{"}scale\ height\text{"}$$

Earth: H~ 8 Km

Buoyancy:



$$Weight: \quad -g\rho_b \delta x \delta y \delta z$$

$$Pressure: \quad p\delta x\delta y - (p+\delta p)\delta x \delta y$$

$$F = MA: \quad \rho_b \delta x \delta y \delta z \frac{dw}{dt} = -g\rho_b \delta x \delta y \delta z - \delta p \delta x \delta y$$

$$\frac{dw}{dt} = -g - \alpha_b \frac{\partial p}{\partial z} \qquad but \quad \frac{\partial p}{\partial z} = -{g}/{\alpha_e}$$

$$\rightarrow \quad \frac{dw}{dt} = g\frac{\alpha_b - \alpha_e}{\alpha_e} \equiv B$$

Buoyancy and Entropy

Specific Volume: $\alpha = {}^{1}\!/_{\rho}$

Specific Entropy: s

$$Maxwell: \left(\frac{\partial \alpha}{\partial s}\right)_p = \left(\frac{\partial T}{\partial p}\right)_s$$

$$\alpha = \alpha(p, s$$

$$\left(\delta\alpha\right)_p = \left(\frac{\partial \alpha}{\partial s}\right)_p \delta s = \left(\frac{\partial T}{\partial p}\right)_s \delta s$$

$$B = g\frac{\left(\delta\alpha\right)_p}{\alpha} = \frac{g}{\alpha}\left(\frac{\partial T}{\partial p}\right)_s \delta s = -\left(\frac{\partial T}{\partial z}\right)_s \delta s \equiv \Gamma \delta s$$

The adiabatic lapse rate:

$$\textit{First Law of Thermodynamics}:$$

$$\dot{Q} = T\frac{ds_{rev}}{dt} = c_v\frac{dT}{dt} + p\frac{d\alpha}{dt}$$

$$= c_v\frac{dT}{dt} + \frac{d\left(\alpha p\right)}{dt} - \alpha\frac{dp}{dt}$$

$$= \left(c_v + R\right)\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

$$= c_p\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

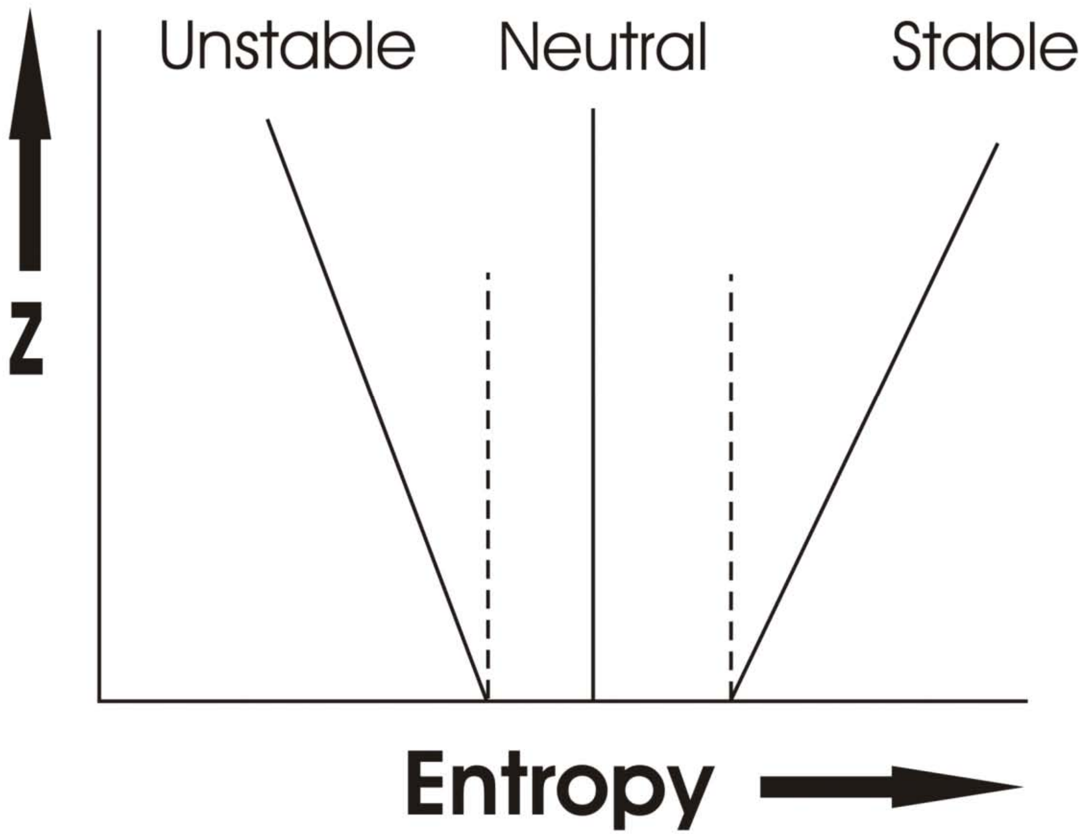
$$\textit{Adiabatic}: \quad c_p dT - \alpha dp = 0$$

$$\textit{Hydrostatic}: \quad c_p dT + g dz = 0$$

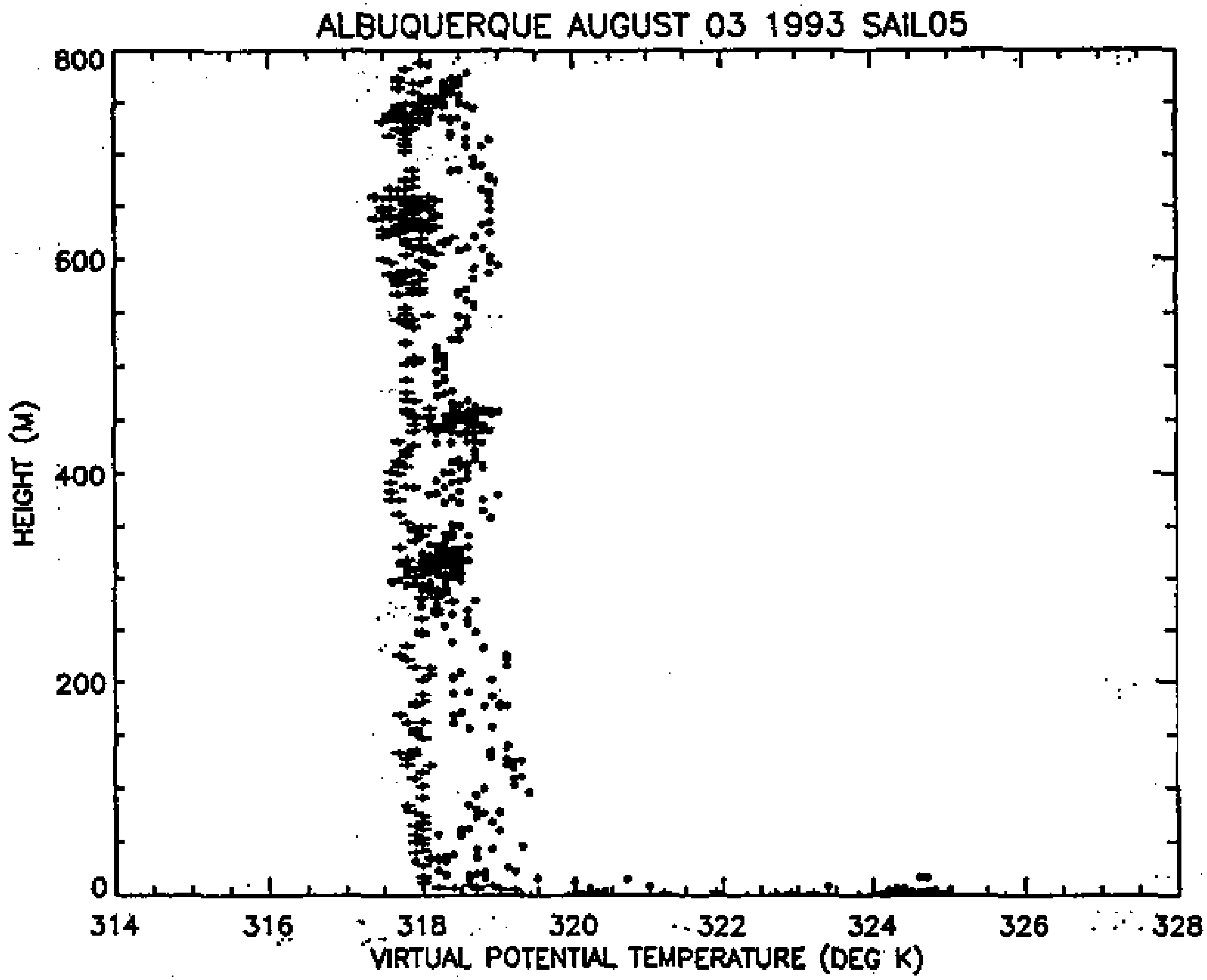
$$\rightarrow \quad \left({}^{dT}\!/_{dz}\right)_s = -\,{}^{g}\!/_{c_p} \equiv -\Gamma_d$$

$$\Gamma = g / c_p$$

Earth's atmosphere: $$\Gamma = 1\,K / 100\,m$$



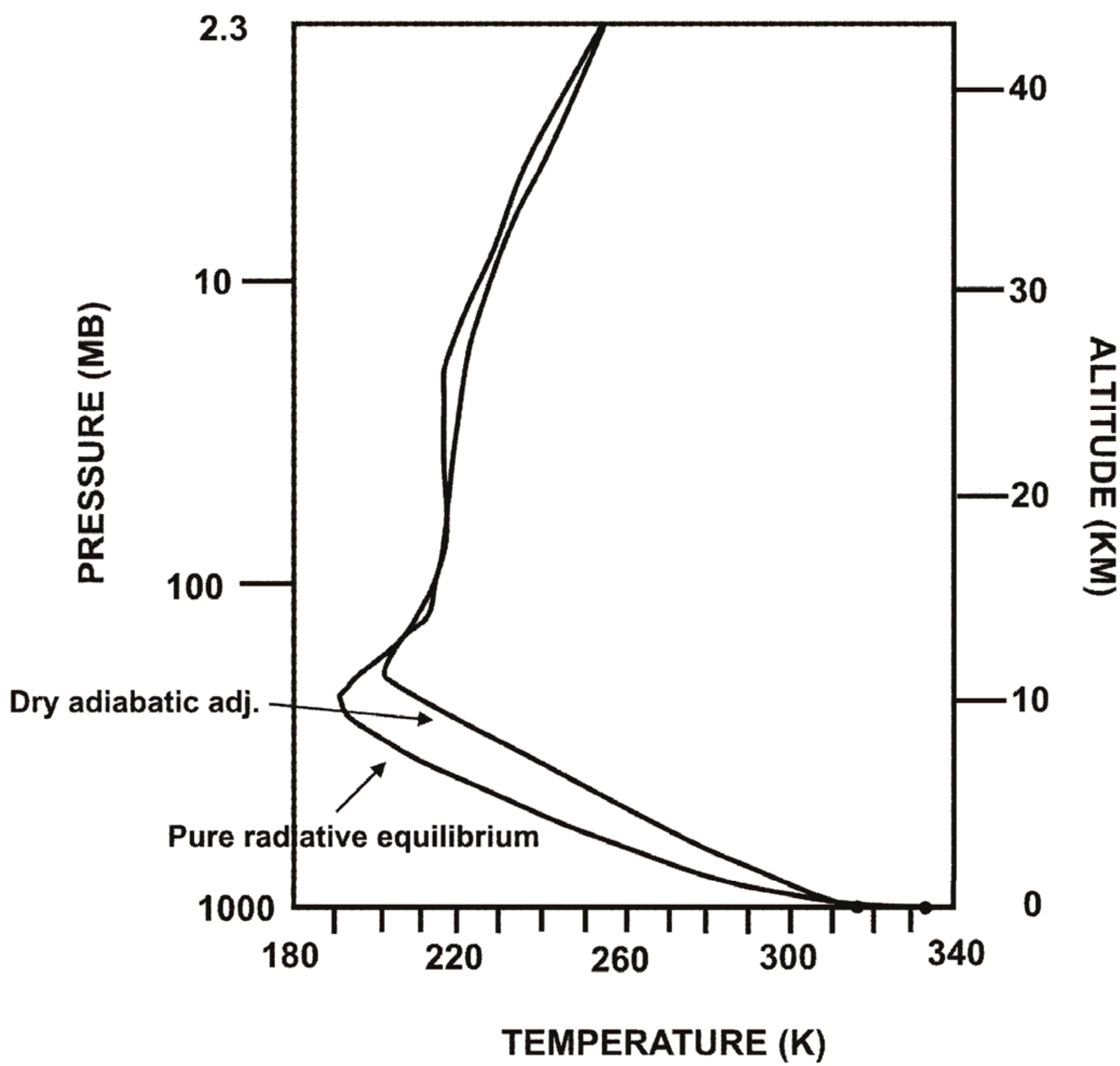
Model Aircraft Measurements (Renno and Williams, 1995)



Radiative equilibrium is unstable in the troposphere

Re-calculate equilibrium assuming that tropospheric stability is rendered neutral by convection:

Radiative-Convective Equilibrium



Better, but still too hot at surface, too cold at tropopause

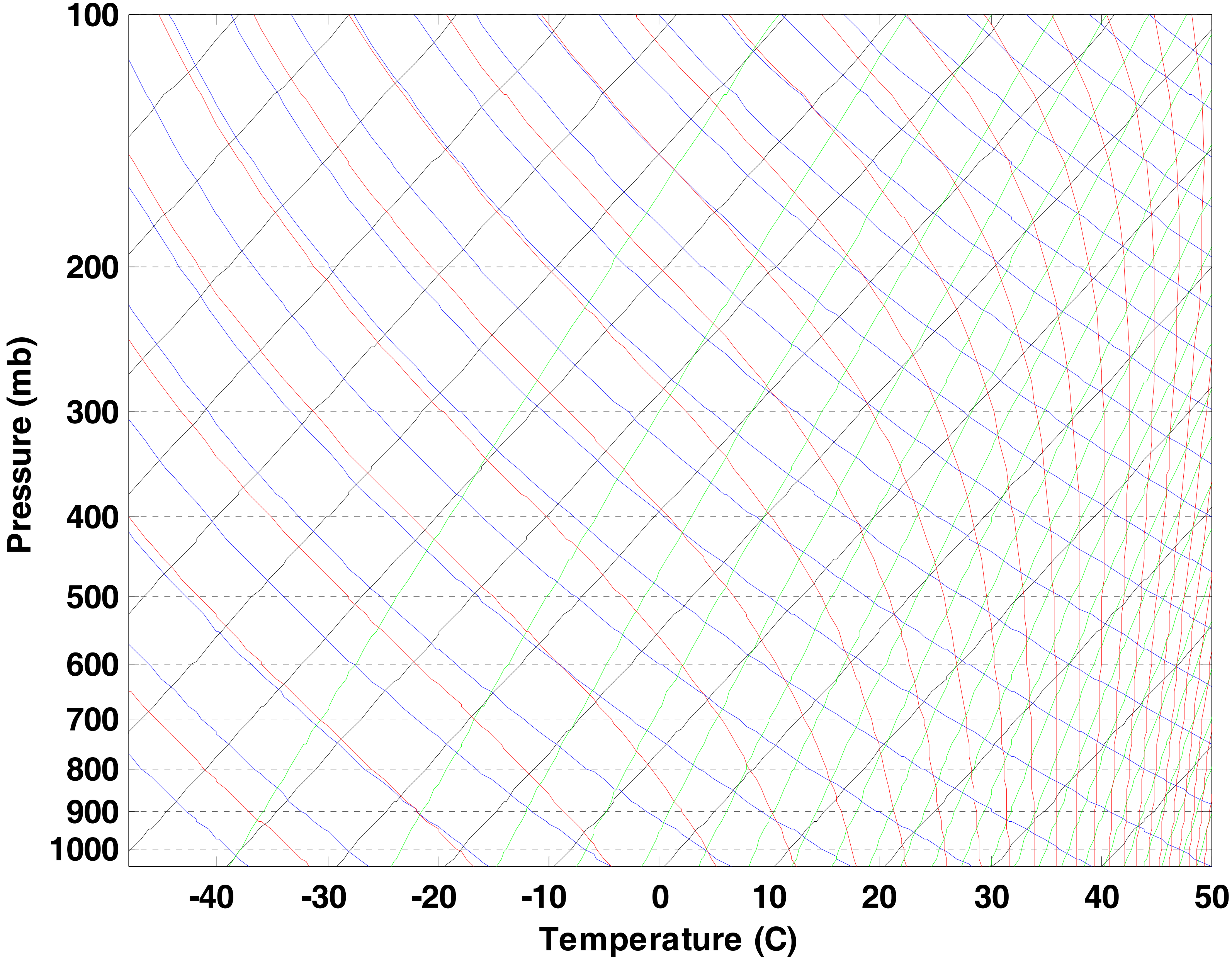
Above a thin boundary layer, most atmospheric convection involved phase change of water:

Moist Convection

Moist Convection

- Significant heating owing to phase changes of water
- Redistribution of water vapor – most important greenhouse gas
- Significant contributor to stratiform cloudiness – albedo and longwave trapping

Stability Assessment using Tephigrams:

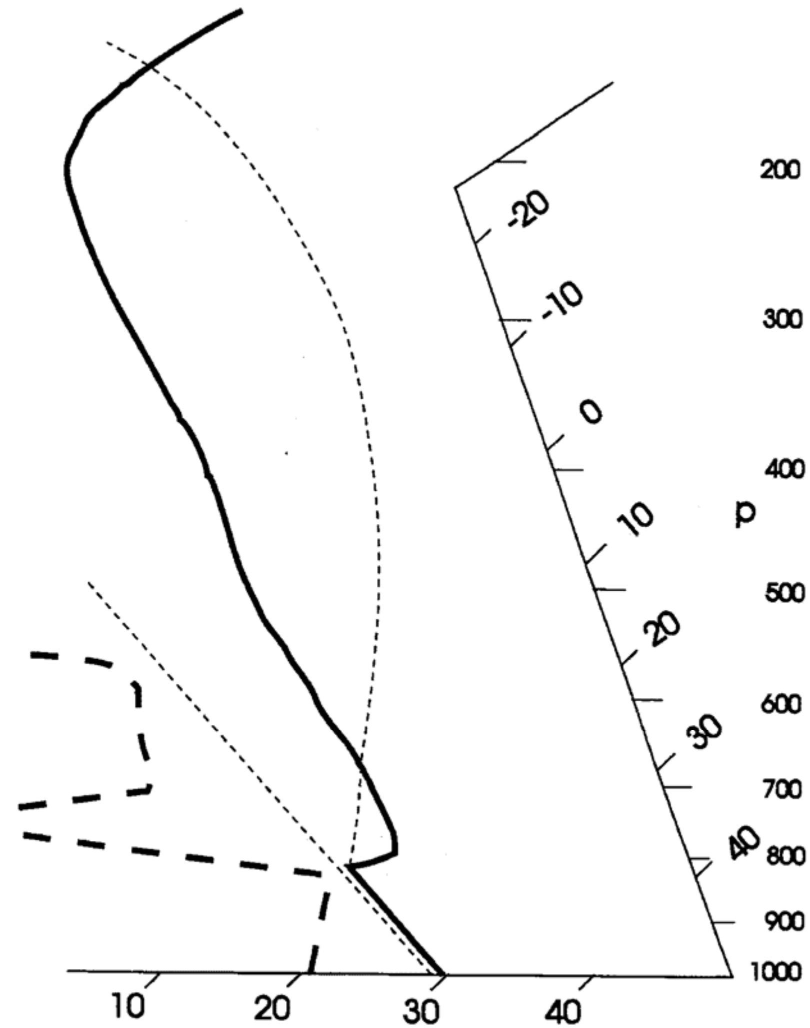


Stability Assessment using Tephigrams:

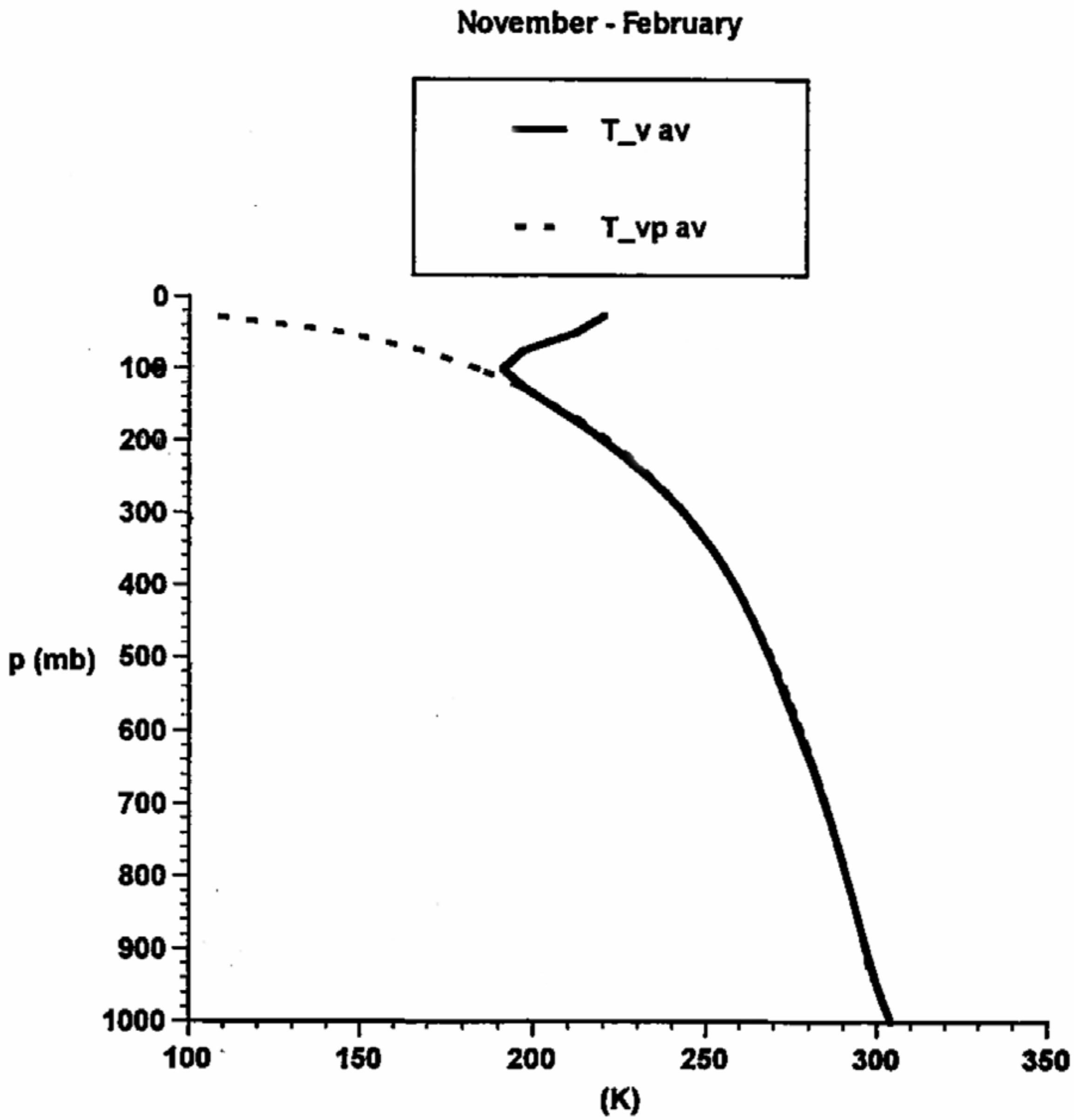
Convective Available Potential Energy (CAPE):

$$CAPE_i \equiv \int_{p_n}^{p_i} \left(\alpha_p - \alpha_e\right) dp$$

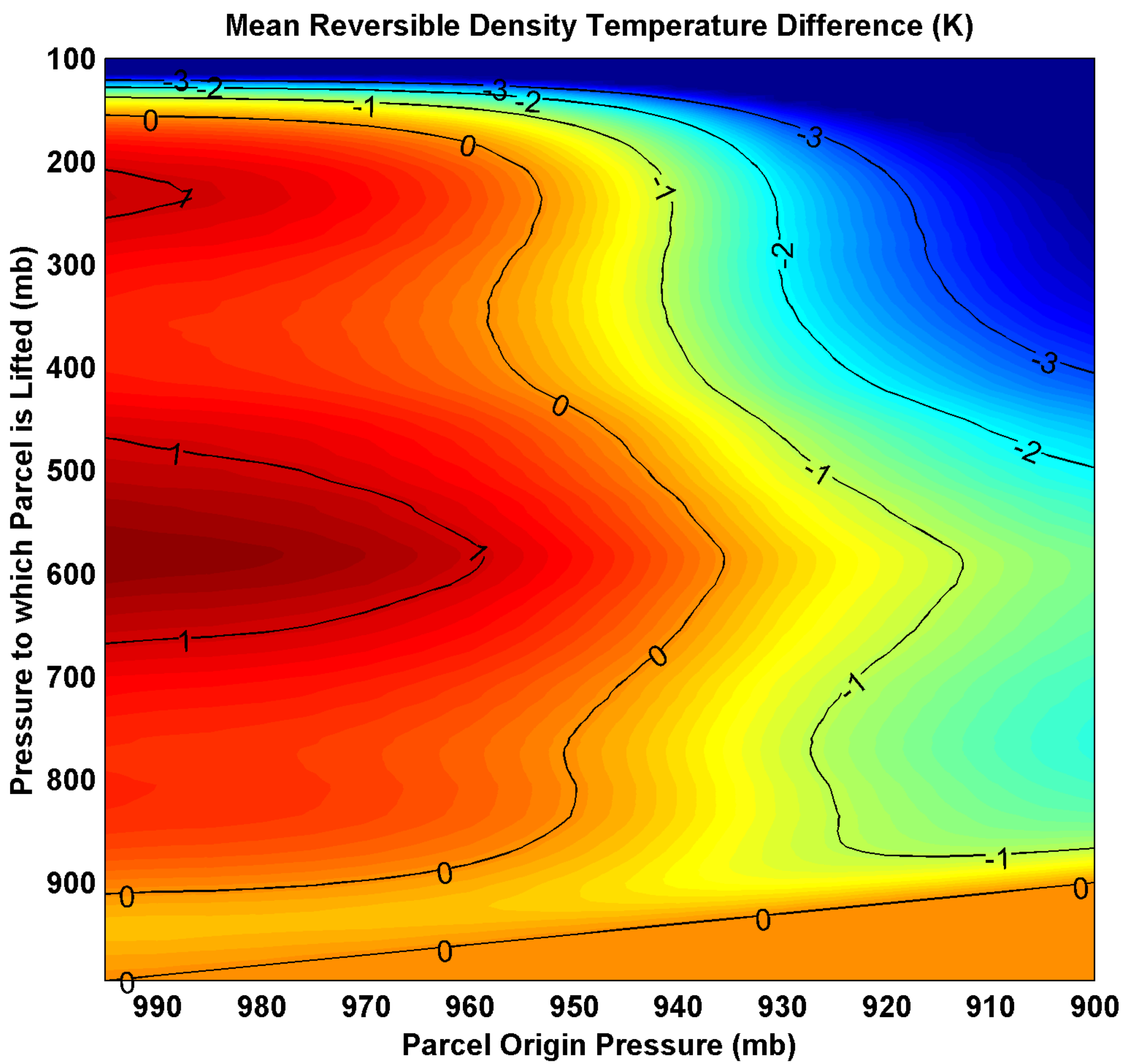
$$= \int_{p}^{p_i} R_d \left(T_{\rho_p} - T_{\rho_e}\right) d\ln\left(p\right)$$



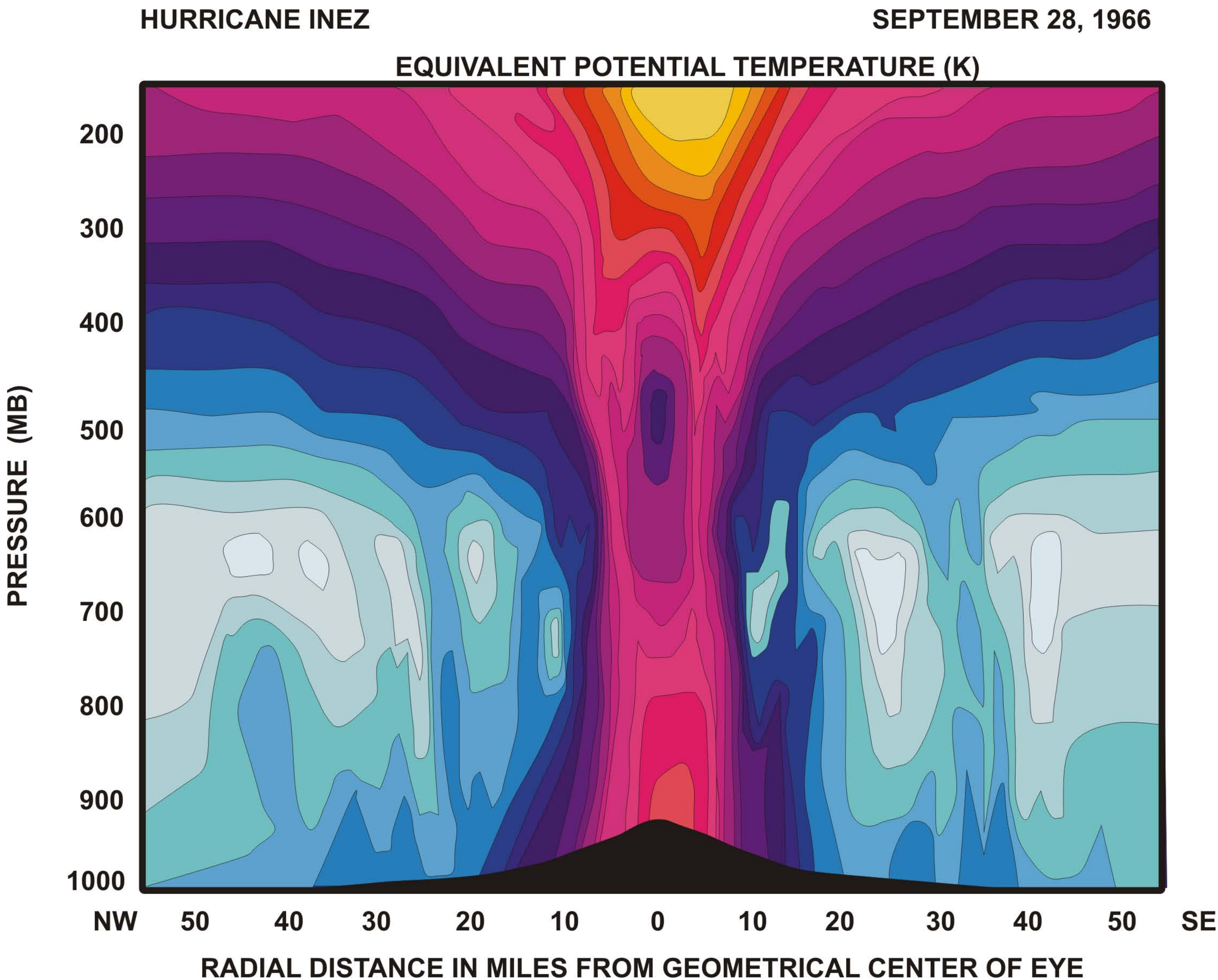
Tropical Soundings: Virtually no CAPE

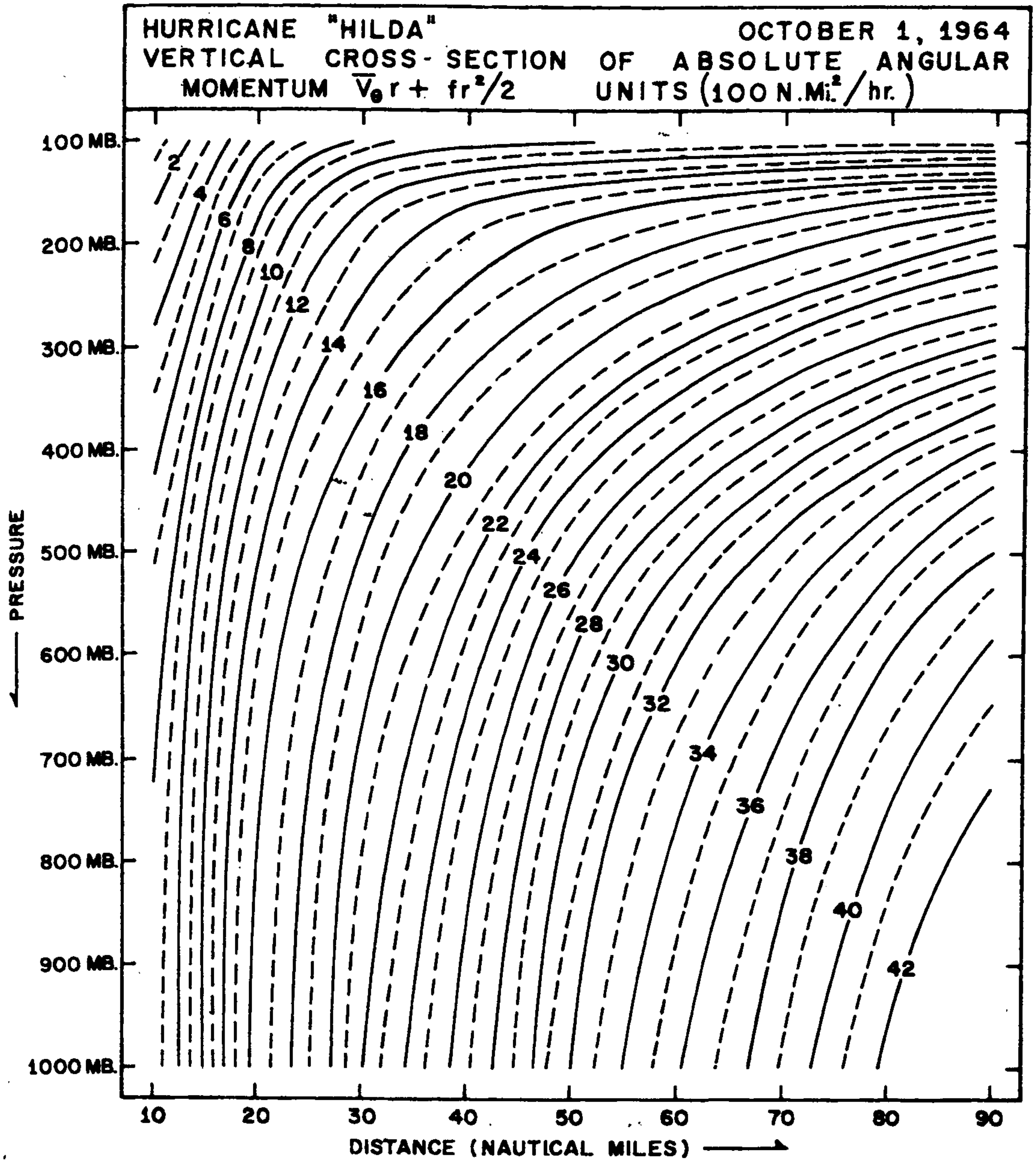


Annual Mean Kapingamoronga



Tropical Cyclones: Steady State Physics. Where does the Energy Come From?

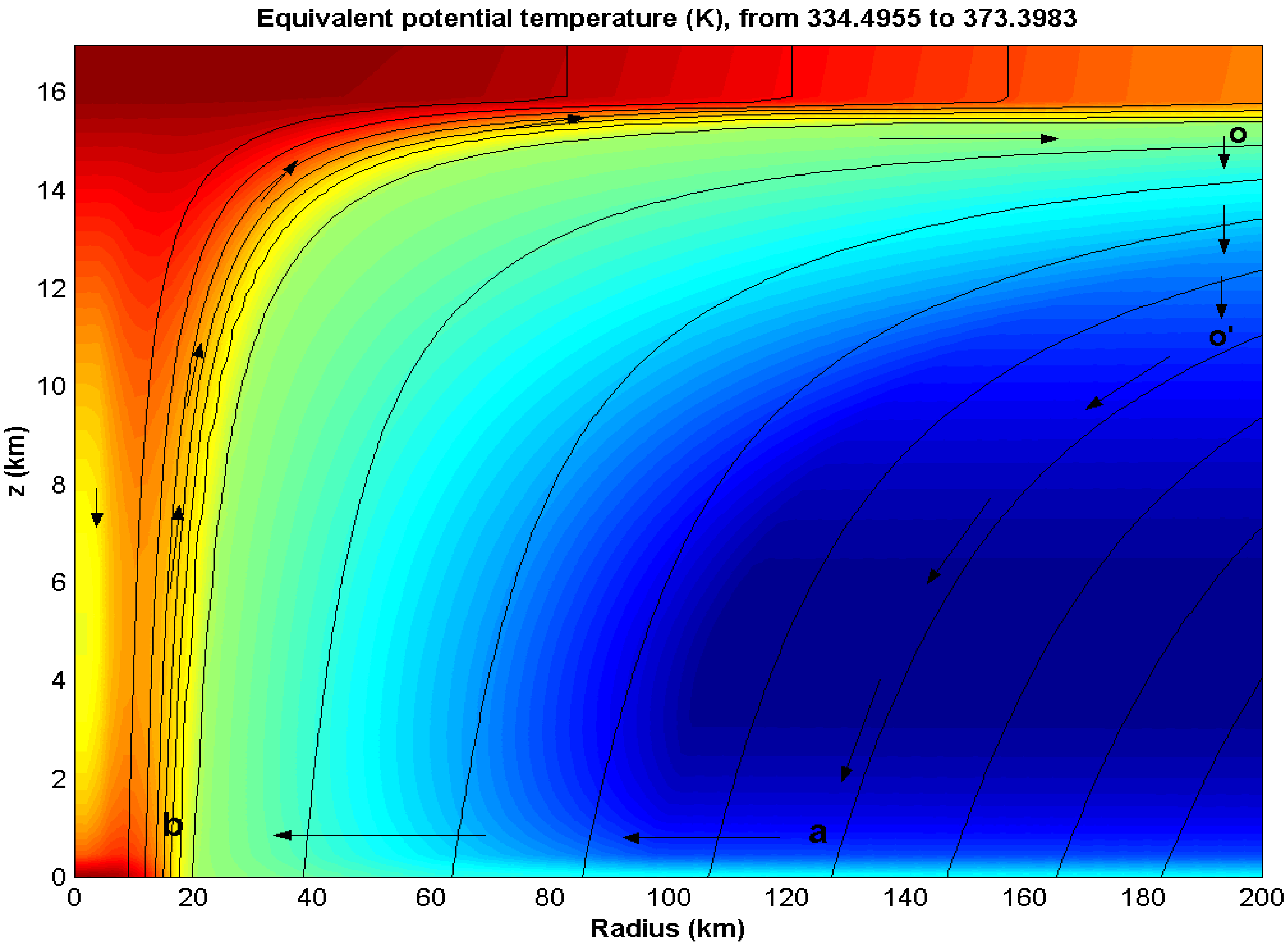

HURRICANE INEZ
SEPTEMBER 28, 1966
EQUIVALENT POTENTIAL TEMPERATURE (K)
PRESSURE (MB)
200
300
400
500
600
700
800
900
1000
NW
50
40
30
20
10
0
10
20
30
40
50
SE
RADIAL DISTANCE IN MILES FROM GEOMETRICAL CENTER OF EYE



Angular momentum per unit mass

$$M = rV + \Omega \sin(\theta) r^2$$

Energy Production



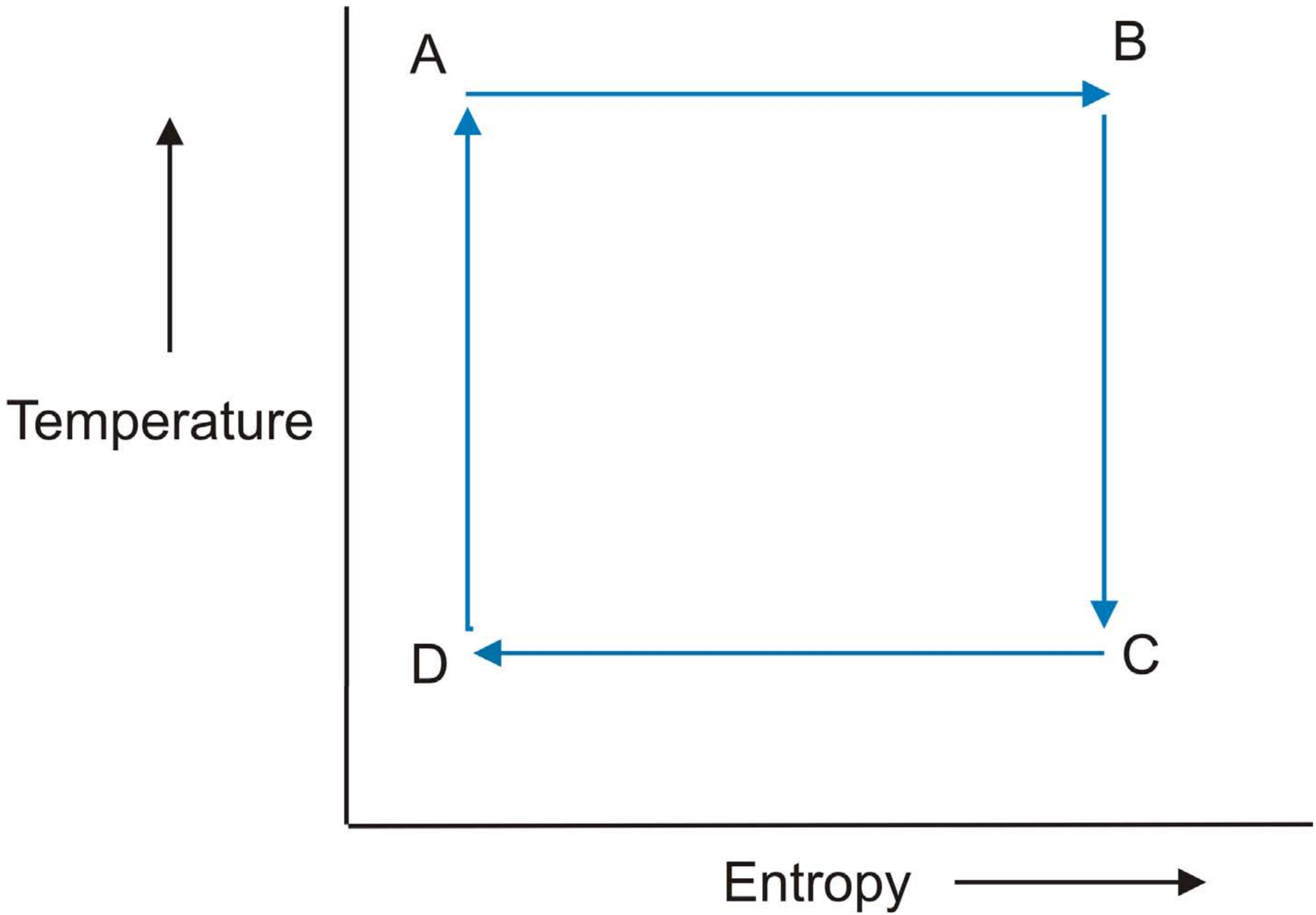
Carnot Theorem: Maximum efficiency results from a particular energy cycle:

- Isothermal expansion
- Adiabatic expansion
- Isothermal compression
- Adiabatic compression

Note: Last leg is not adiabatic in hurricane: Air cools radiatively. But since environmental temperature profile is moist adiabatic, the amount of radiative cooling is the same as if air were saturated and descending moist adiabatically.

Maximum rate of working:

$$W = \frac{T_s - T_o}{T_s} \dot{Q}$$


A
B
C
D
Temperature
Entropy

Total rate of heat input to hurricane:

$$\dot{Q} = 2\pi \int_0^{r_0} \rho \left[C_k \,|\mathbf{V}|\left(k_0^* - k\right) + C_D \,|\mathbf{V}|^3 \right] r\,dr$$

Surface enthalpy flux

Dissipative heating

In steady state, Work is used to balance frictional dissipation:

$$W = 2\pi \int_0^{r_0} \rho \left[C_D \,|\mathbf{V}|^3 \right] r\,dr$$

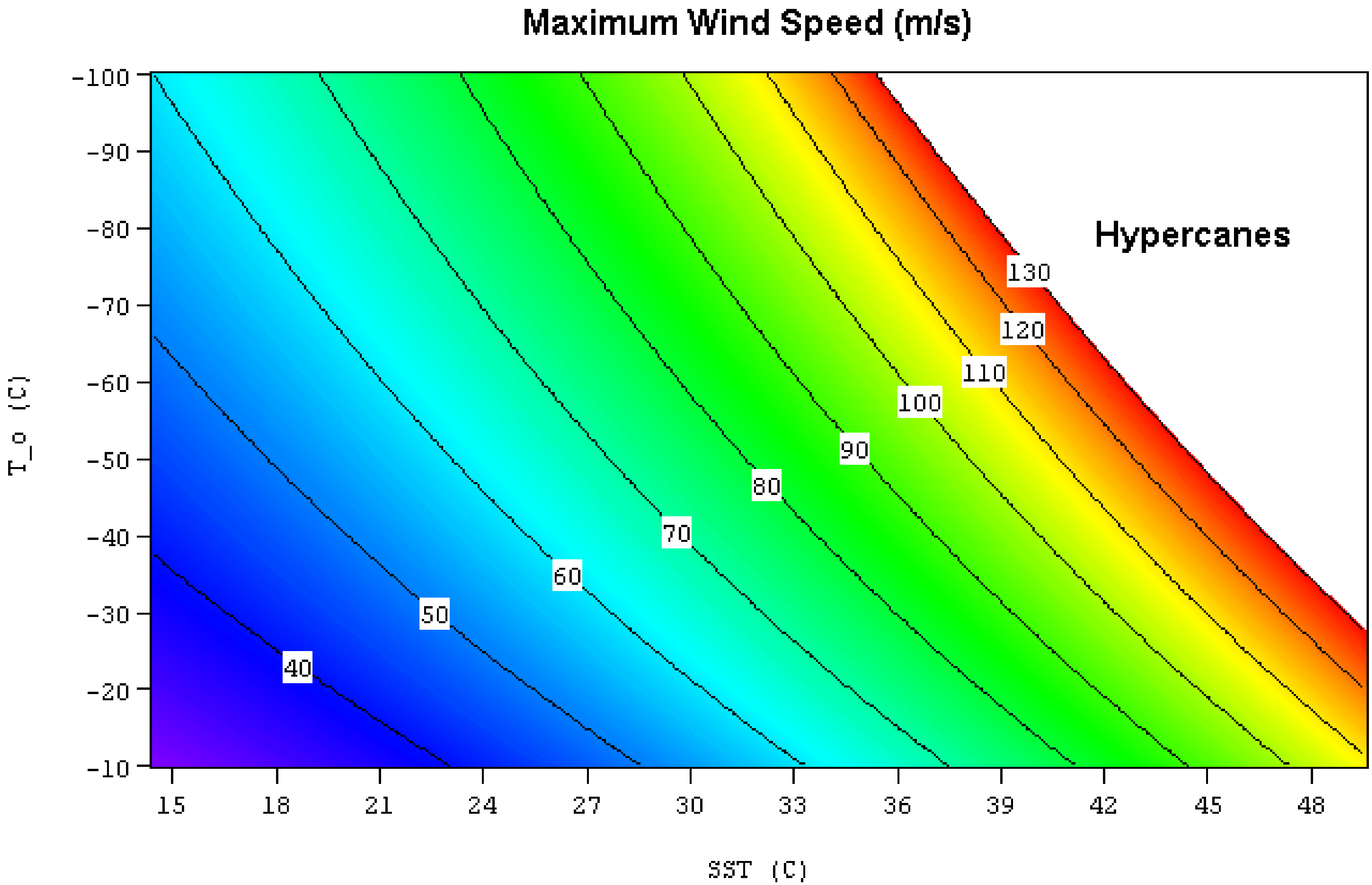
Plug into Carnot equation:

$$\int_0^{r_0} \rho\left[C_D |\mathbf{V}|^3\right] r\,dr = \frac{T_s - T_o}{T_o} \int_0^{r_0} \rho\left[C_k |\mathbf{V}| \left(k_0^* - k\right)\right] r\,dr$$

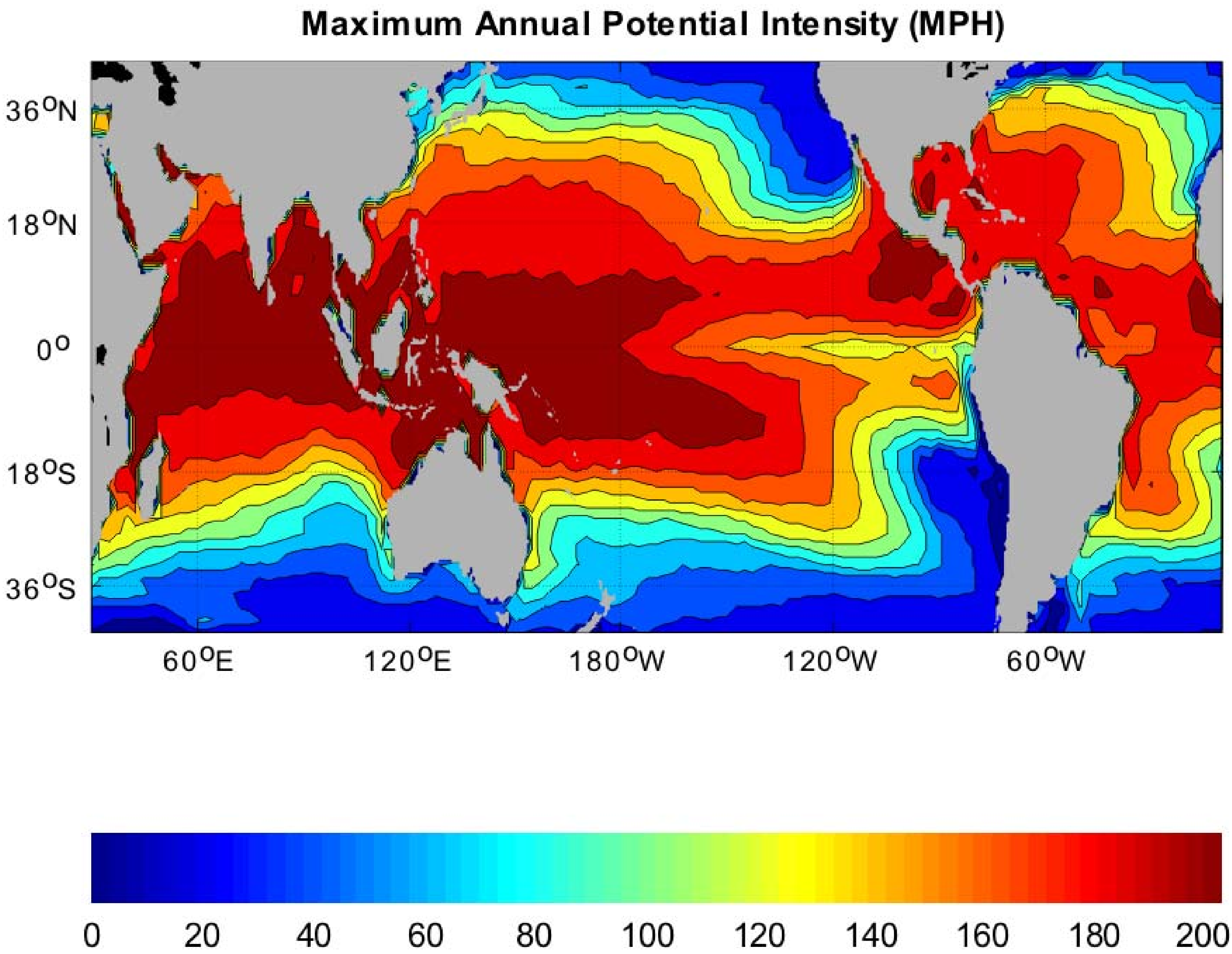
If integrals dominated by values of integrands near radius of maximum winds,

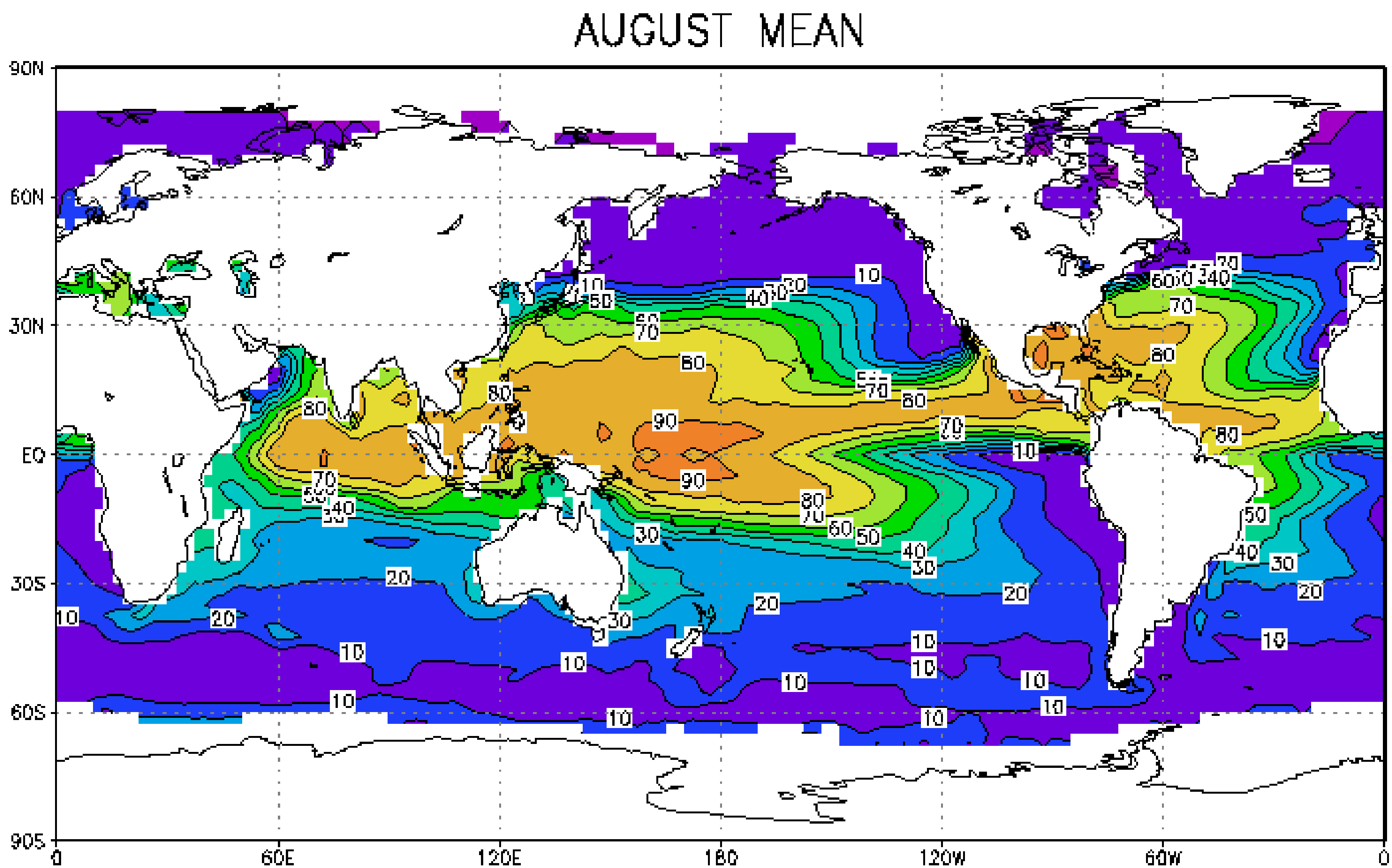
$$\rightarrow \quad |V_{\max}|^2 \cong \frac{C_k}{C_D} \frac{T_s - T_o}{T_o} \left(k_0{}^* - k\right)$$

Graph of Solution for particular values of coefficients:

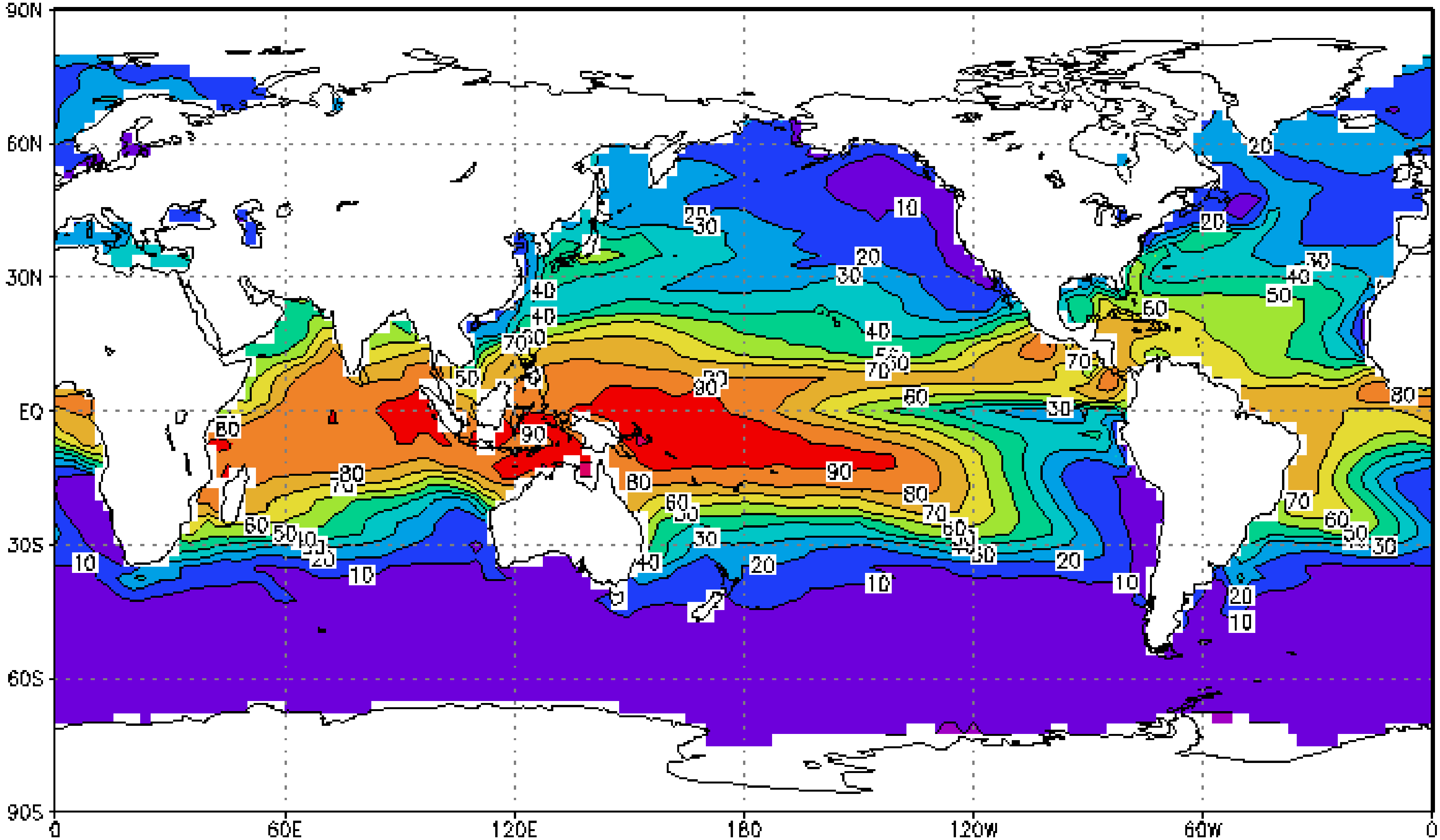


$\mathcal{H} = 0.75 \quad C_k/C_D = 1.2$

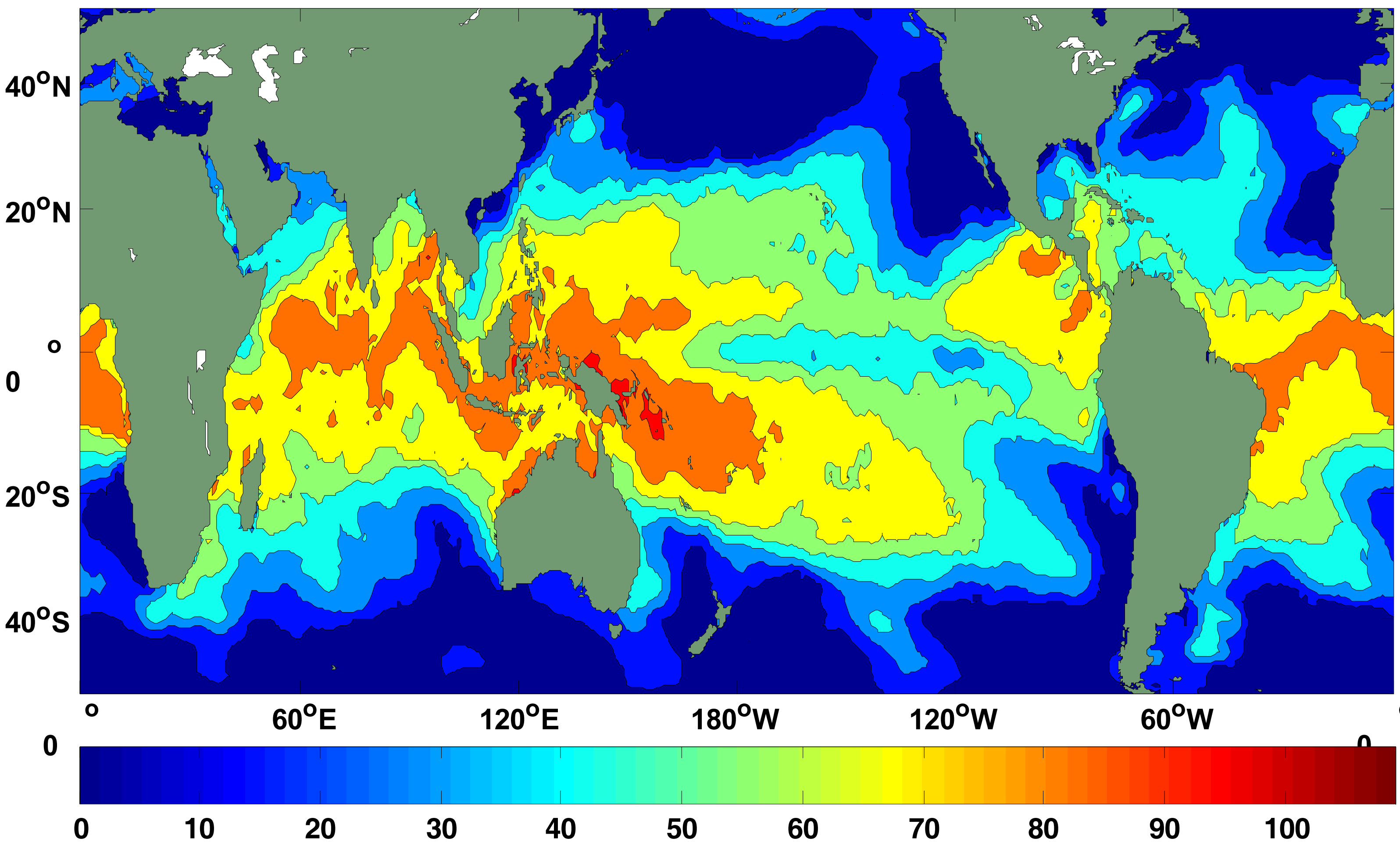

Maximum Annual Potential Intensity (MPH)
36°N
18°N
0°
18°S
36°S
60°E
120°E
180°W
120°W
60°W
0
20
40
60
80
100
120
140
160
180
200


AUGUST MEAN
90N
60N
30N
EQ
30S
60S
90S
0
60E
120E
180
120W
60W
0

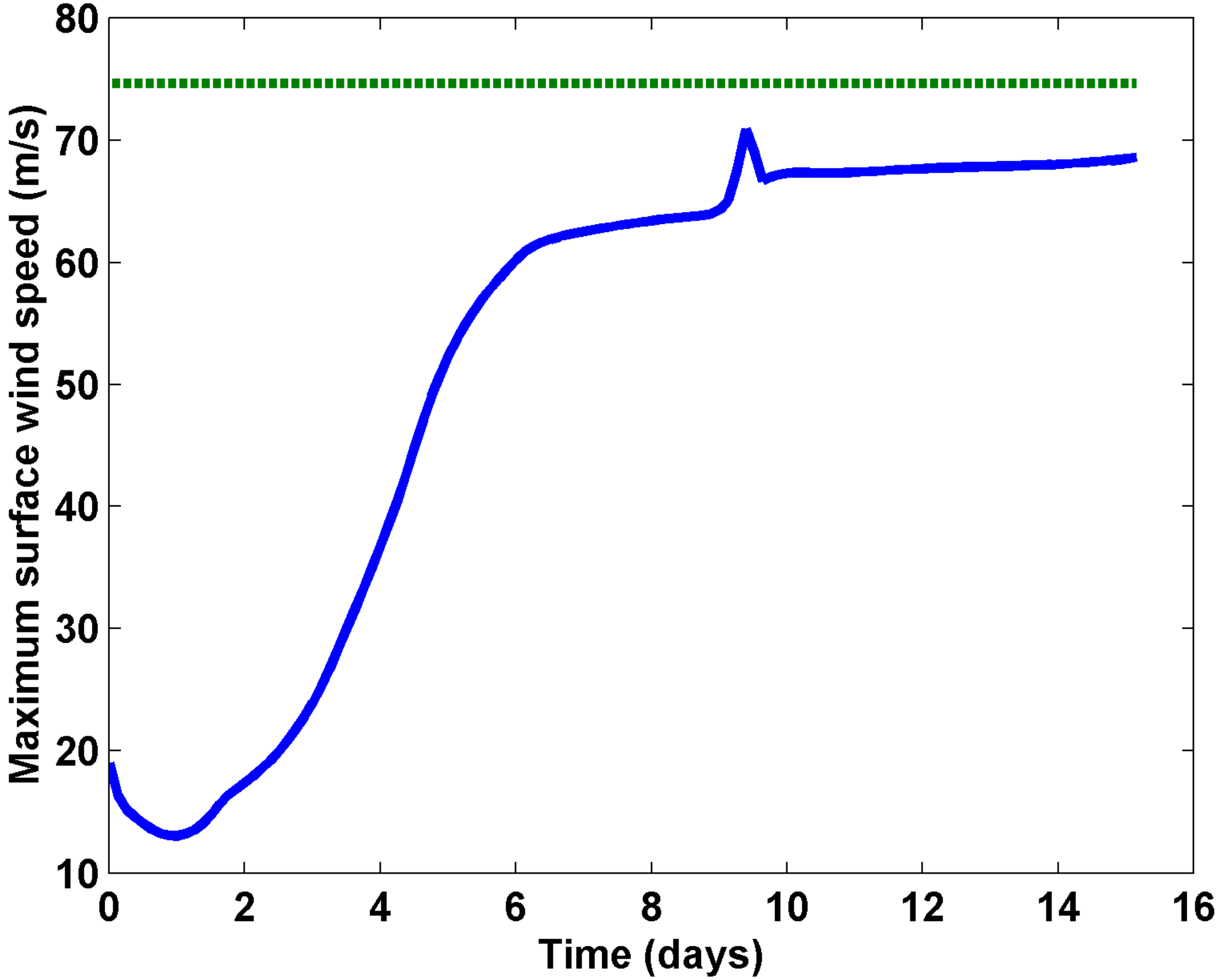
JANUARY MEAN



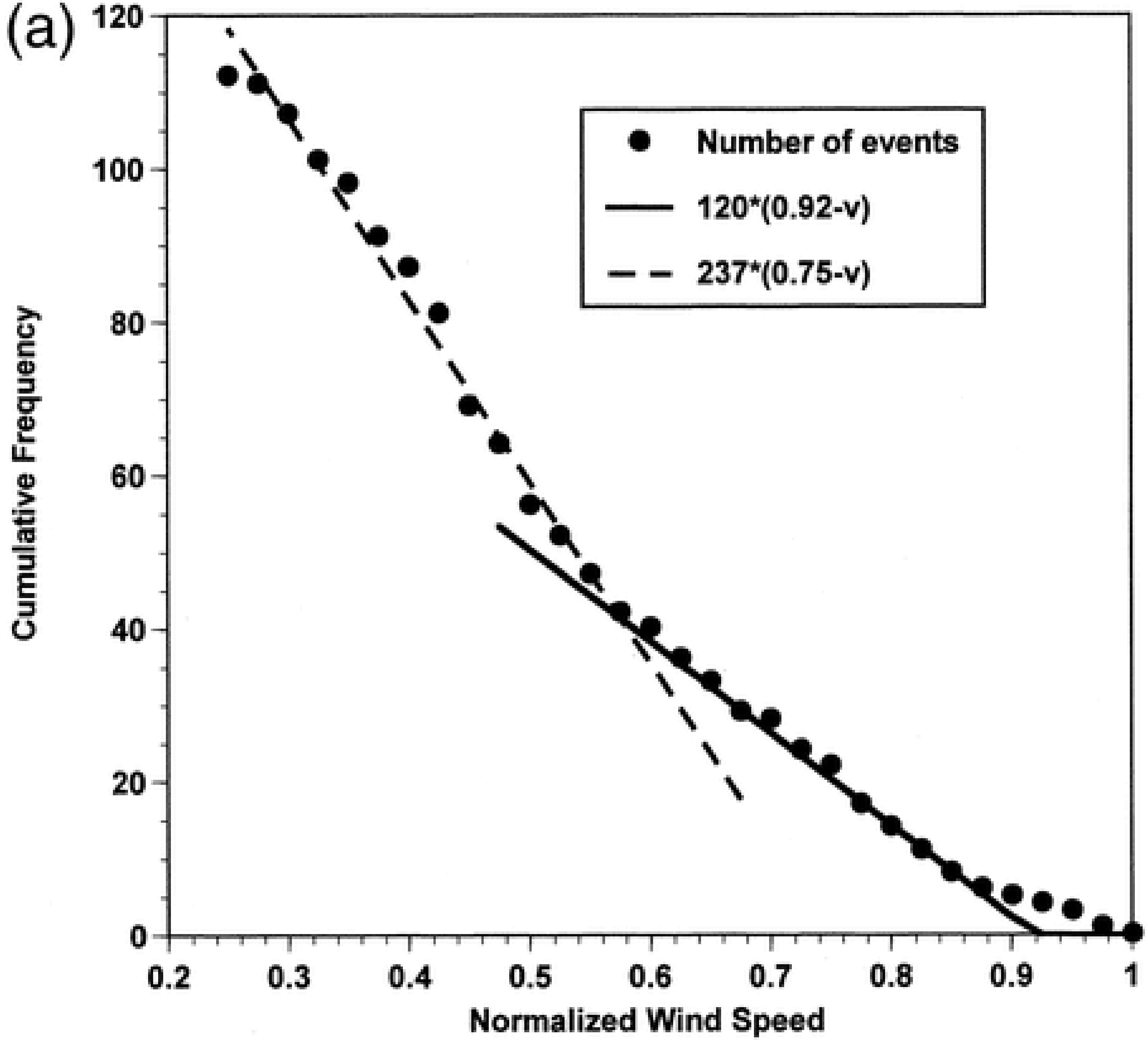
00 GMT 21 March 2008

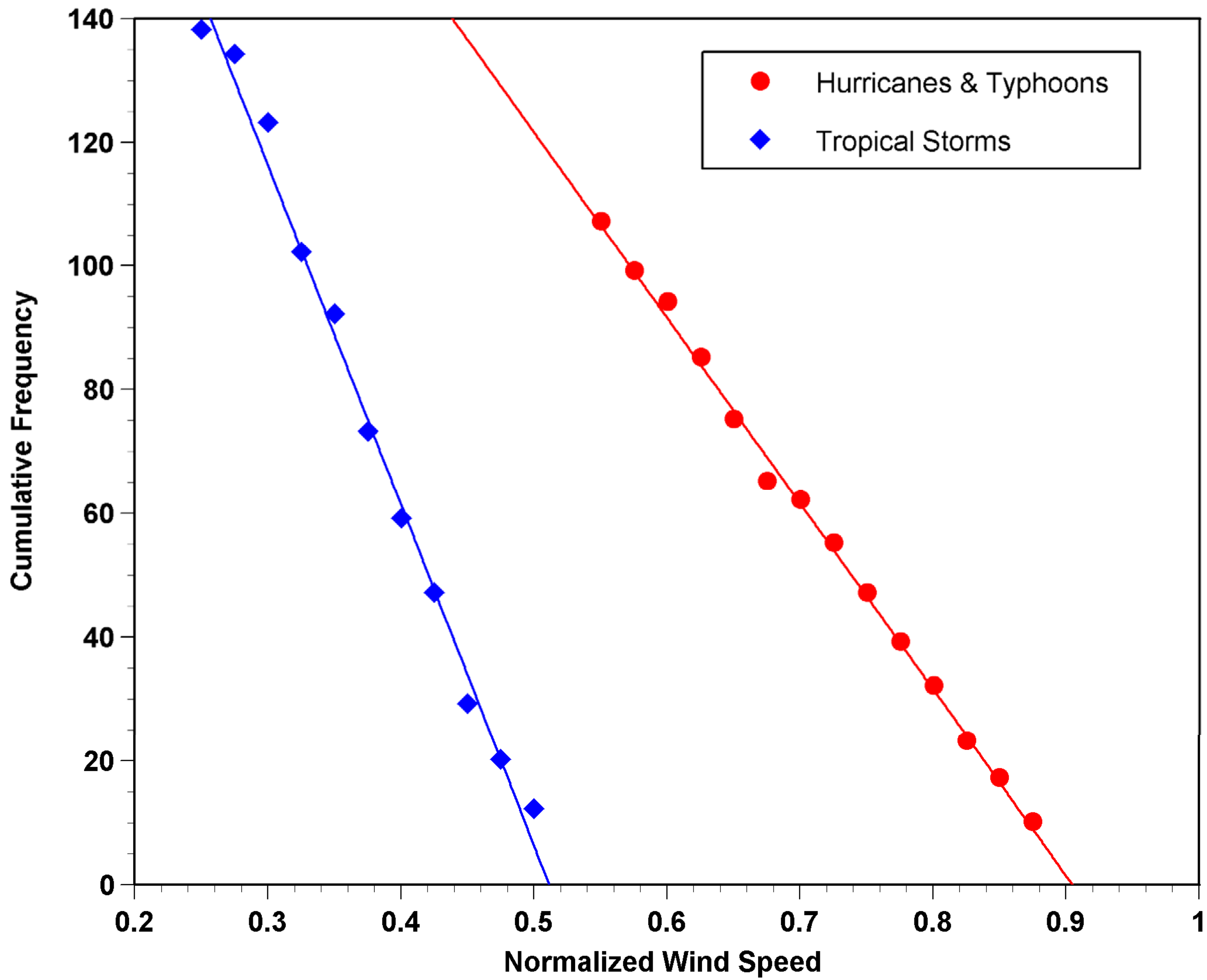


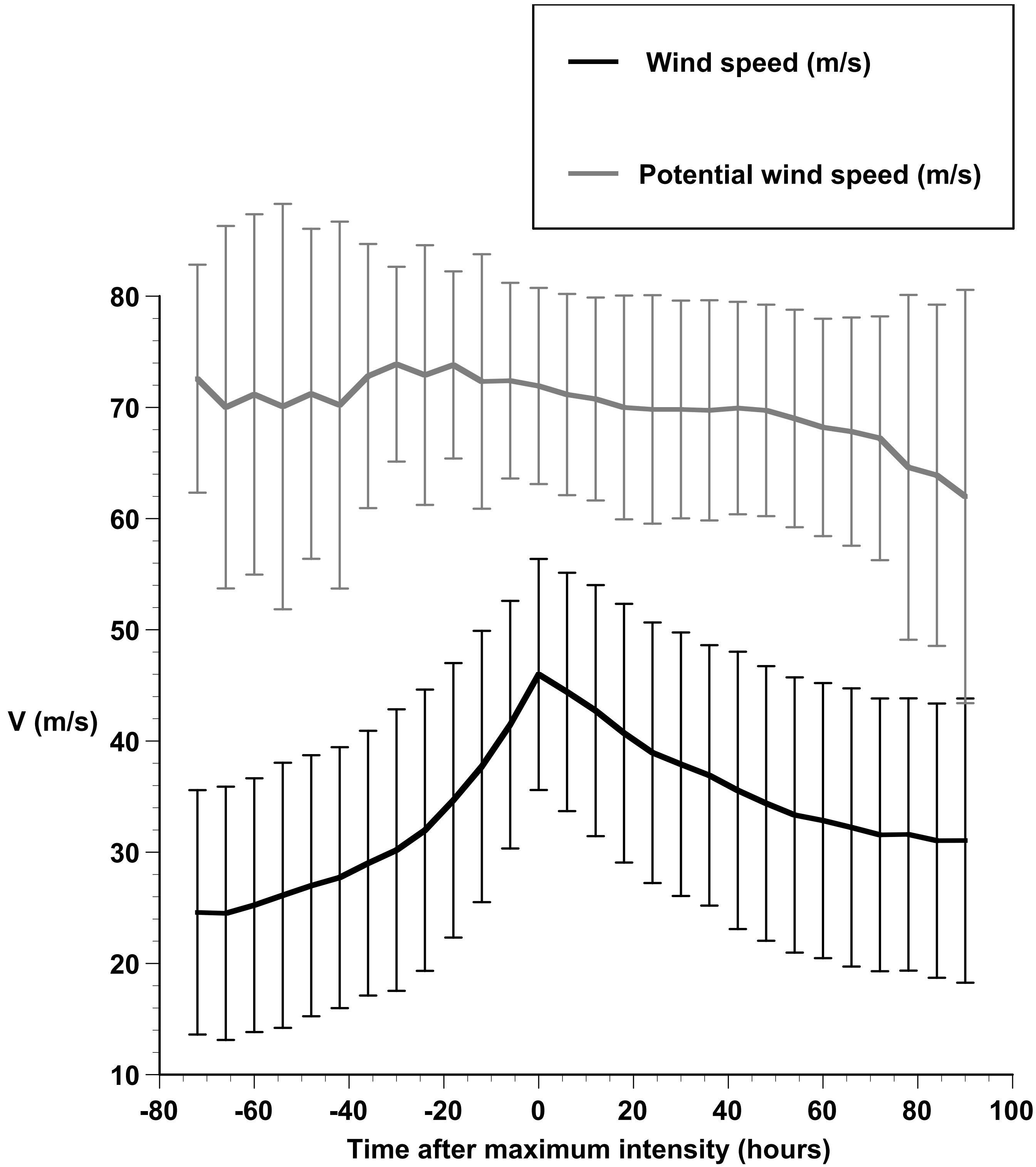
Numerical simulations



Relationship between potential intensity (PI) and intensity of real tropical cyclones


(a)
Number of events
120*(0.92-v)
237*(0.75-v)
Cumulative Frequency
Normalized Wind Speed
0
20
40
60
80
100
120
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

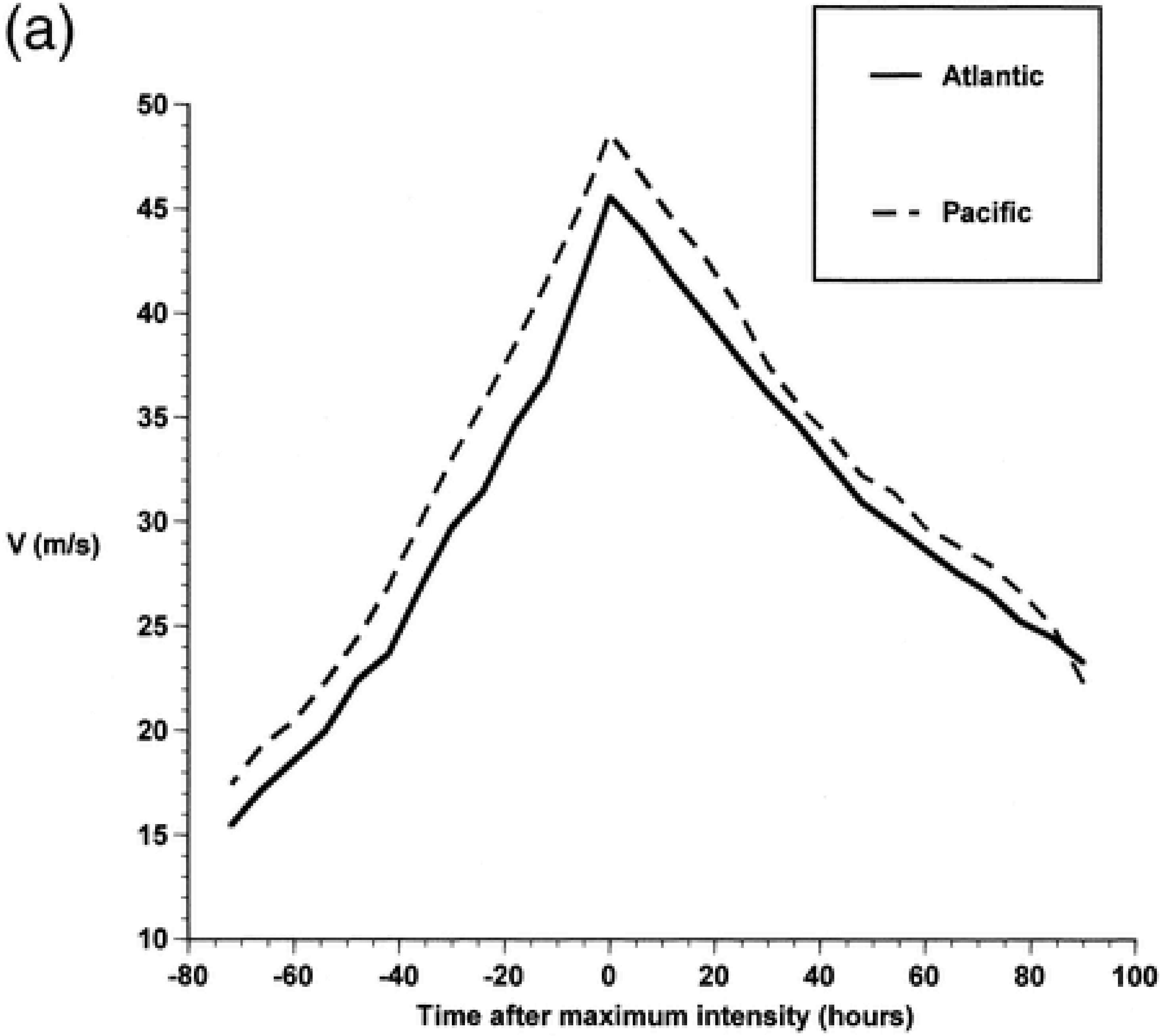

Hurricanes & Typhoons
Tropical Storms
Cumulative Frequency
Normalized Wind Speed
0
20
40
60
80
100
120
140
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



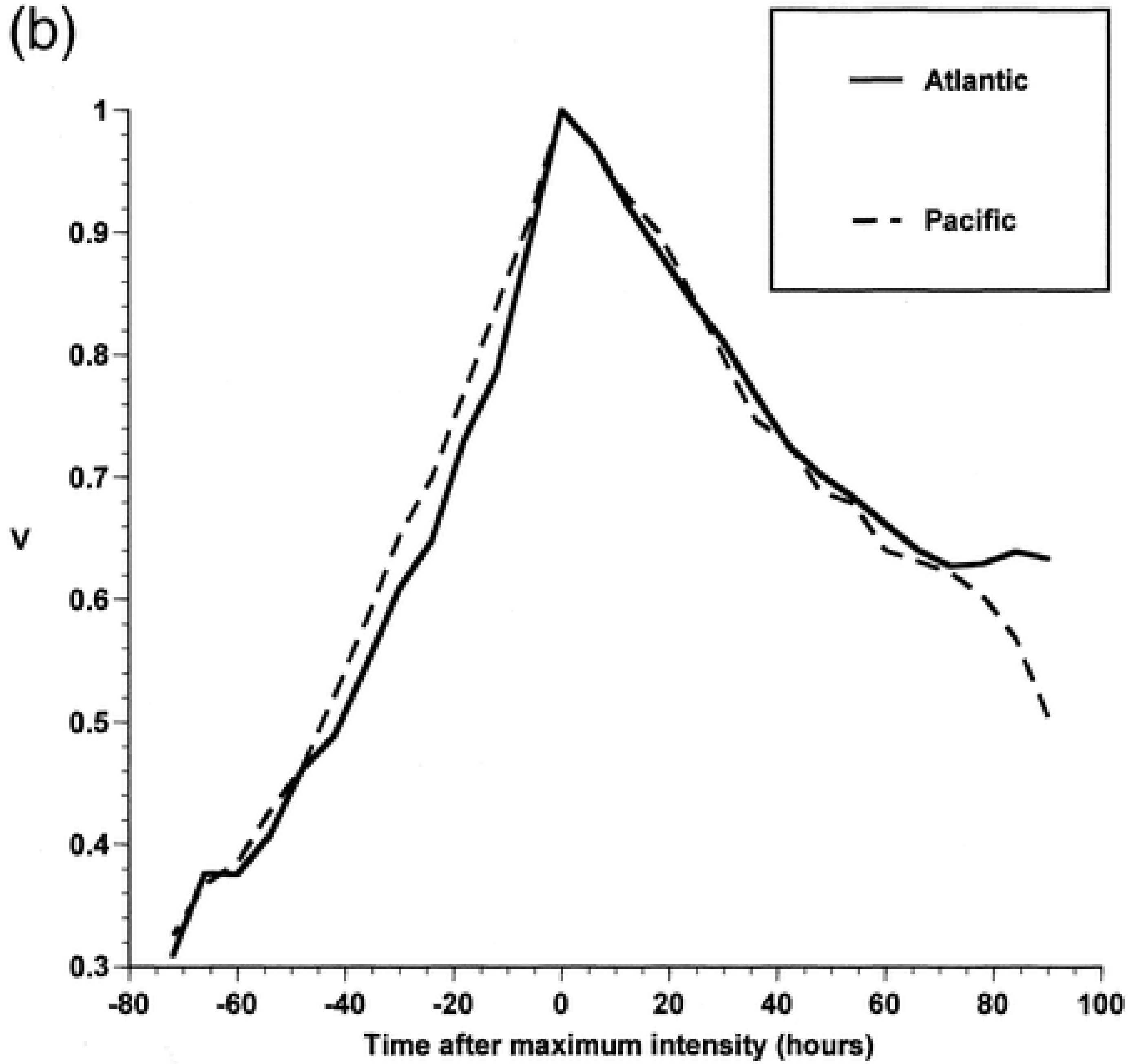
Wind speed (m/s)
Potential wind speed (m/s)
V (m/s)
80
70
60
50
40
30
20
10
-80
-60
-40
-20
0
20
40
60
80
100
Time after maximum intensity (hours)

Evolution with respect to time of maximum intensity

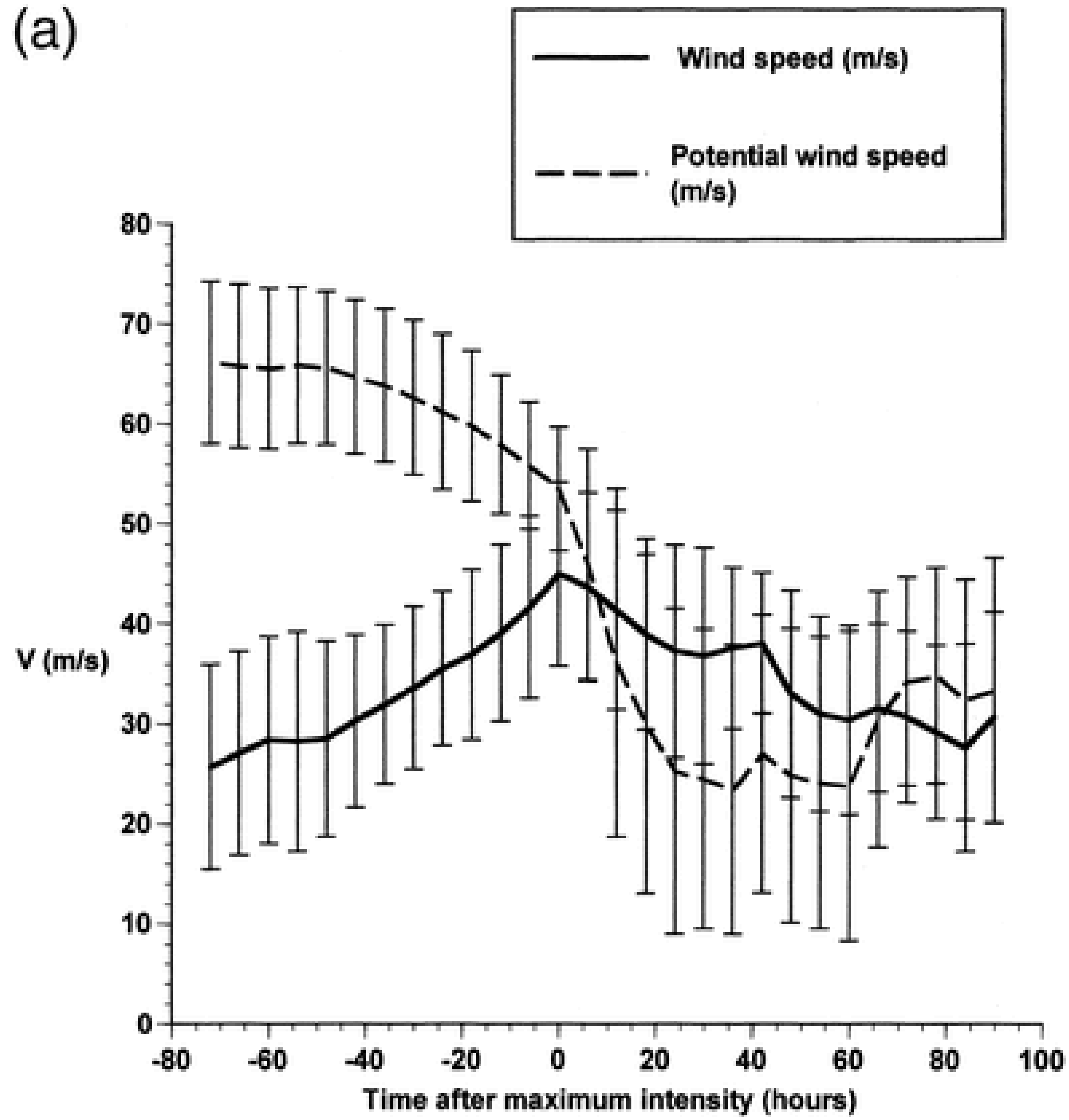
(a)



Evolution with respect to time of maximum intensity, normalized by peak wind

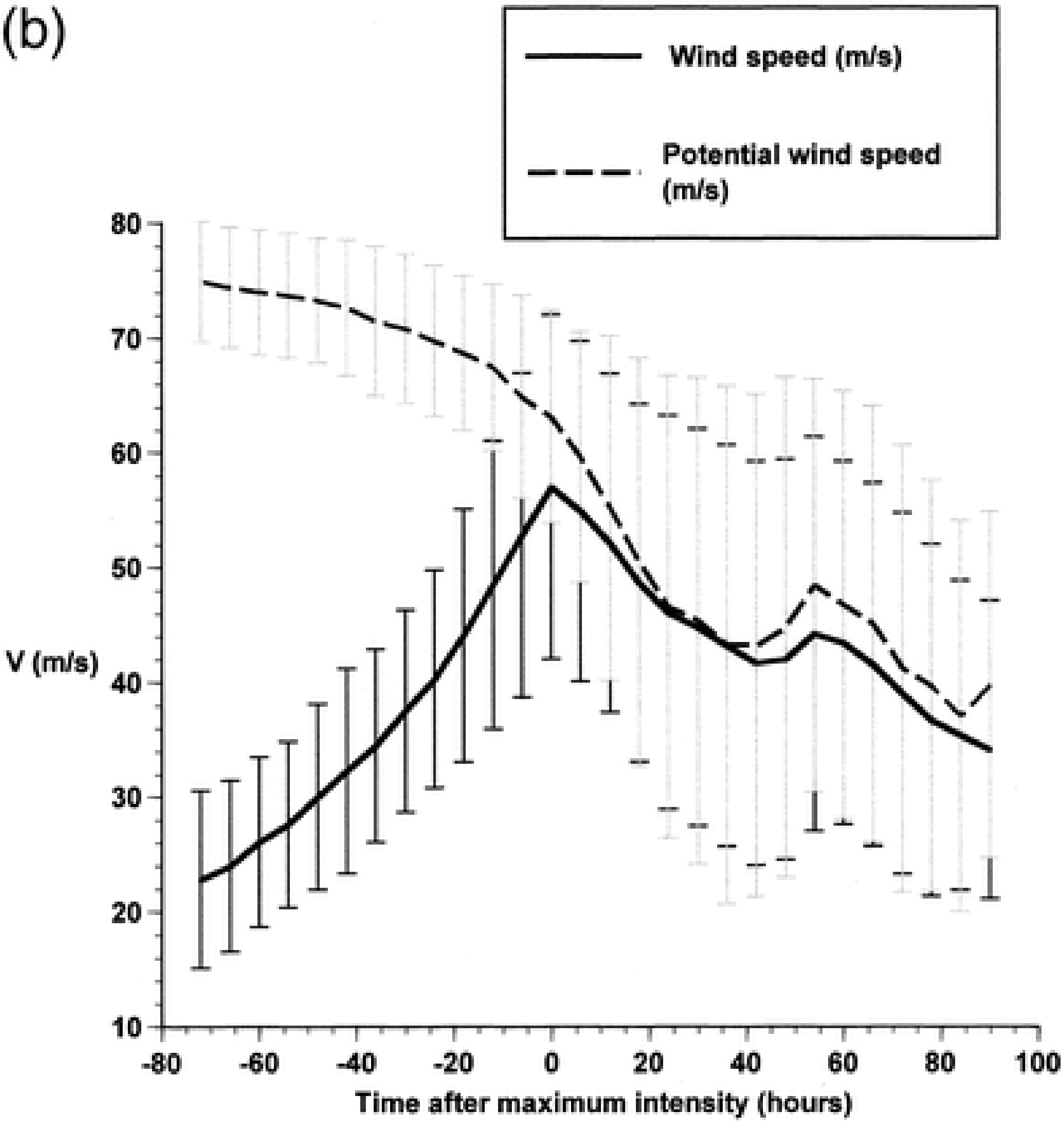


Evolution curve of Atlantic storms whose lifetime maximum intensity is limited by declining potential intensity, but not by landfall



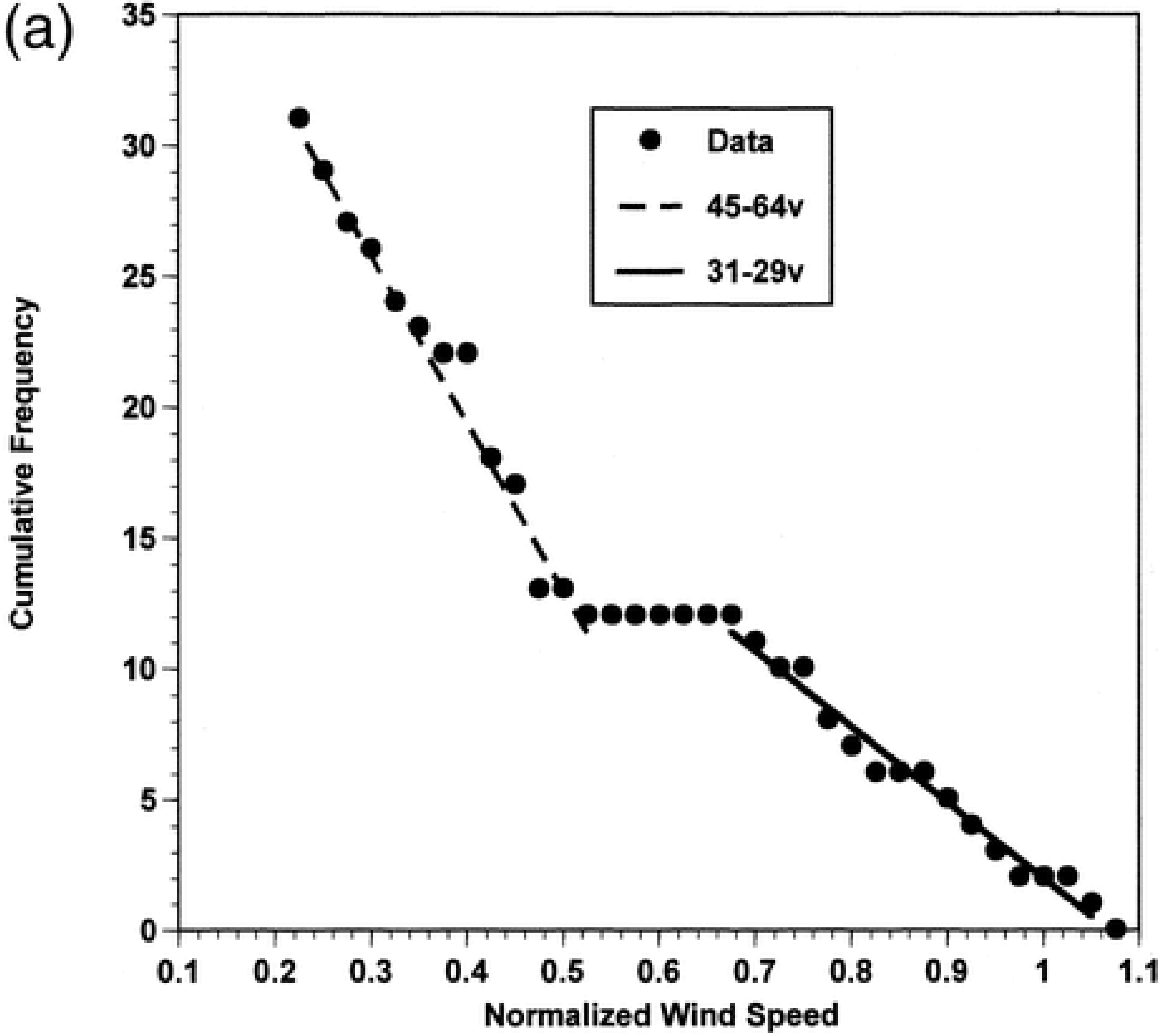
Evolution curve of WPAC storms whose lifetime maximum intensity is limited by declining potential intensity, but not by landfall

(b)

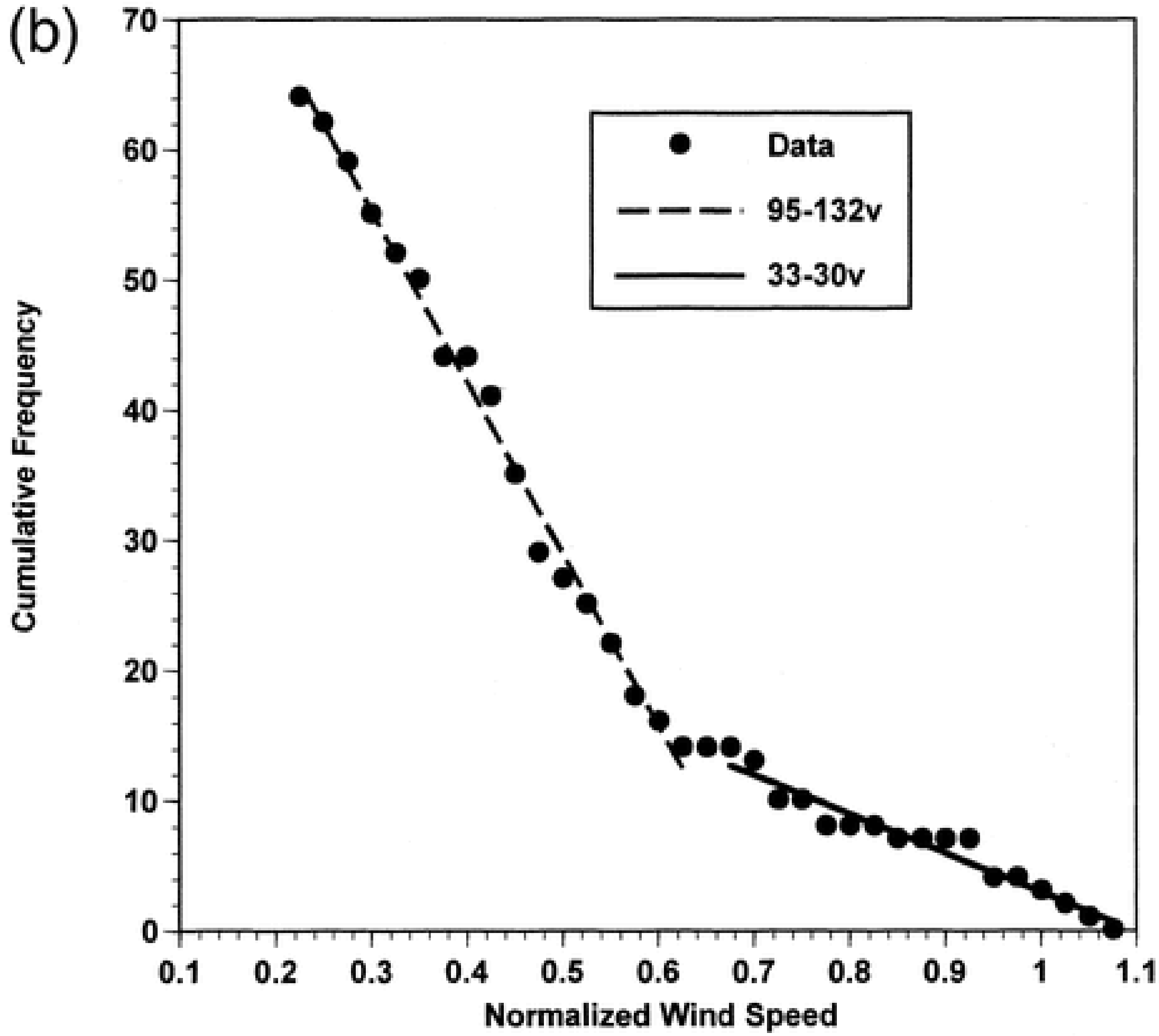


CDF of normalized lifetime maximum wind speeds of North Atlantic tropical cyclones of tropical storm strength (18 m s−1) or greater, for those storms whose lifetime maximum intensity was limited by landfall.

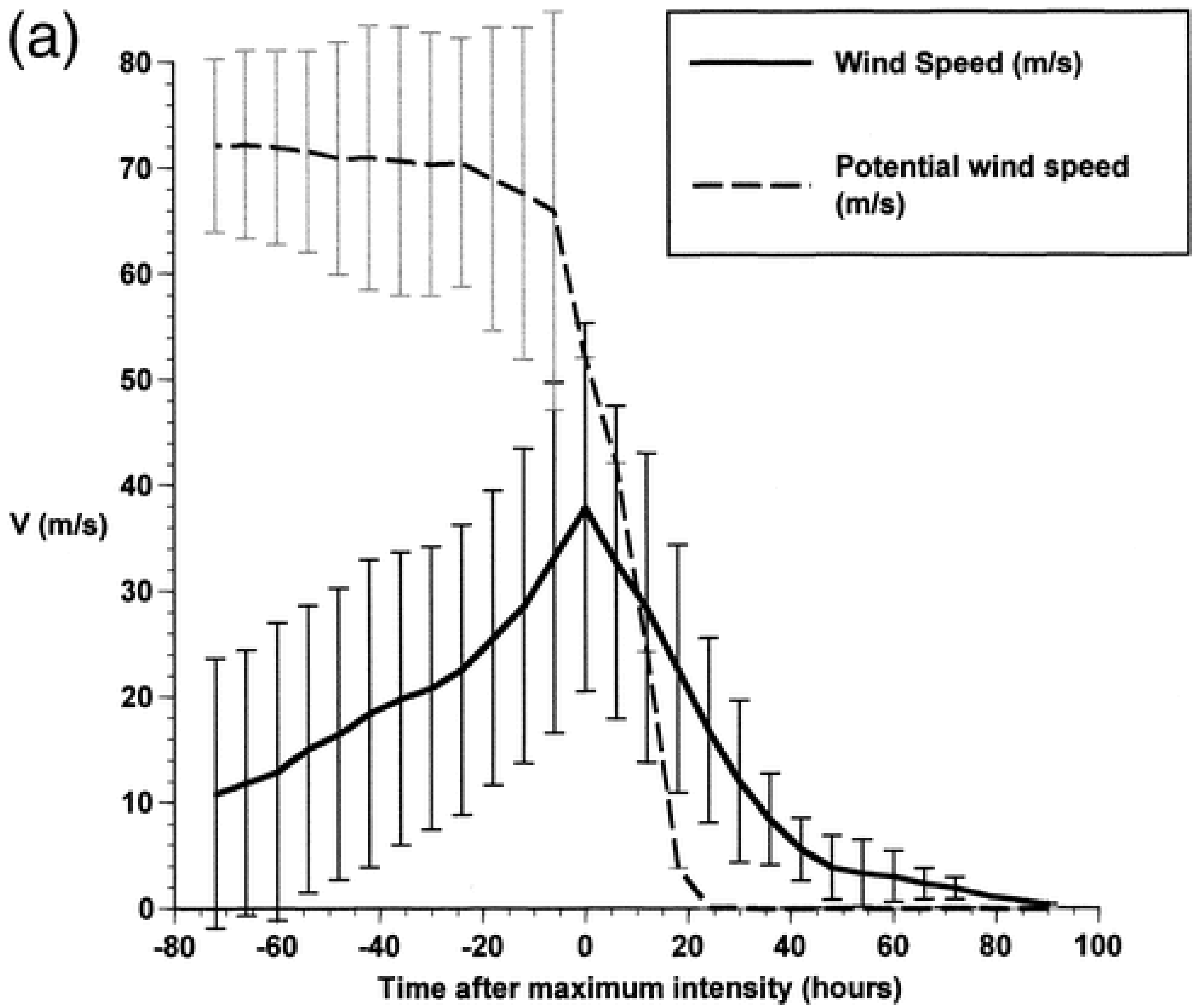
(a)



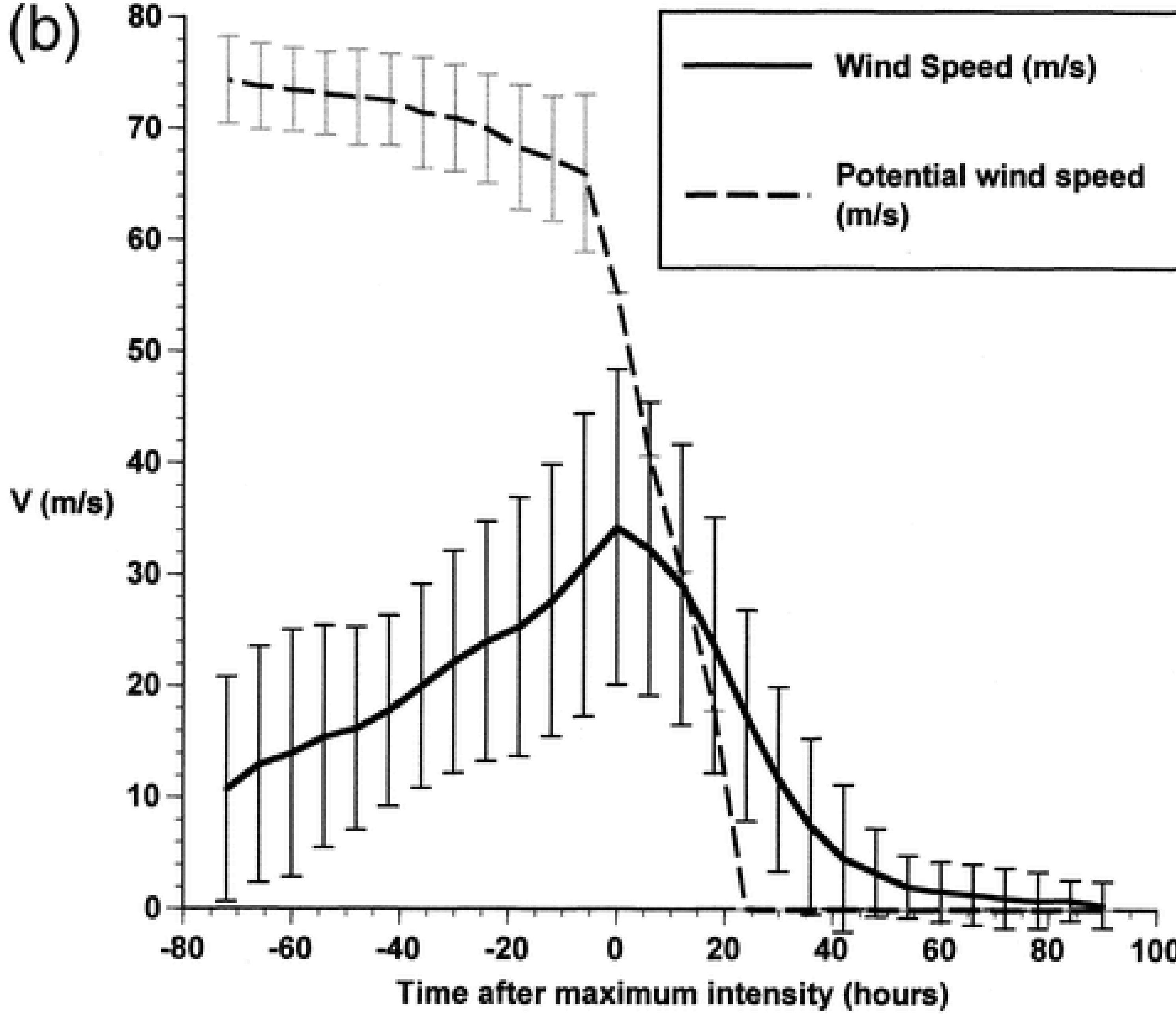
CDF of normalized lifetime maximum wind speeds of Northwest Pacific tropical cyclones of tropical storm strength (18 m s−1) or greater, for those storms whose lifetime maximum intensity was limited by landfall.



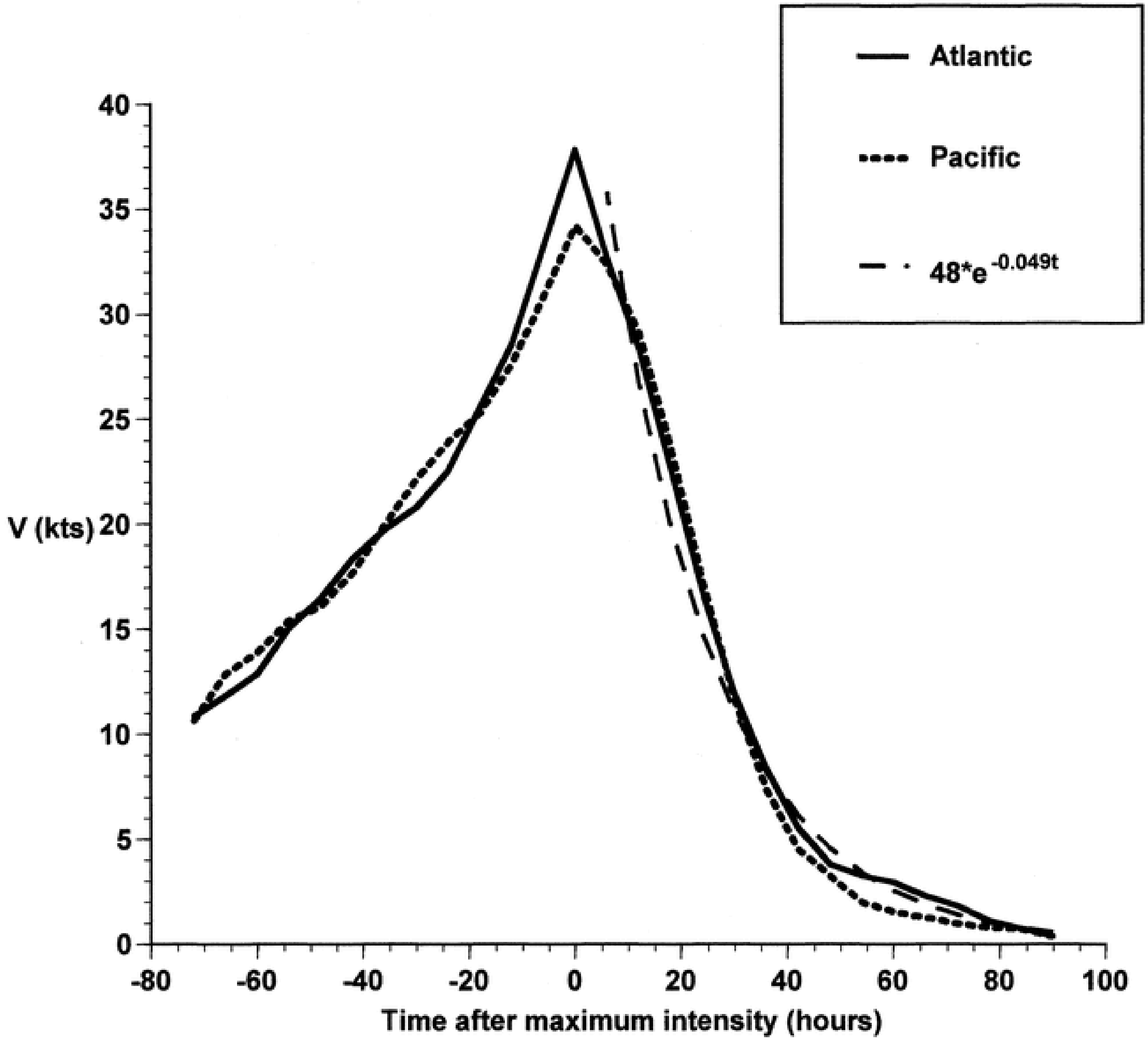
Evolution of Atlantic storms whose lifetime maximum intensity was limited by landfall



Evolution of Pacific storms whose lifetime maximum intensity was limited by landfall



Composite evolution of landfalling storms



12.103 Science and Policy of Natural Hazards
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