Problem Set #2: Structure of Earth Materials Problem Set

Due 16 November 2004. Do the odd-numbered problems. The even-numbered problems are study questions.

1. Show that any 2^{nd}-rank tensor is centro-symmetric

2. Show, that in general, for a rotational transformation, $a_{ik}\,a_{jk}=1$ if i=j. *and* that $a_{ik}\,a_{jk}=0$ if $i\neq j$.
Hint: Write the new axes, x'_1 and x'_j, in terms of the old, and take the dot product.

3. Write the three transformation matrices for a rotation of angle θ around x1, x2, and x3.. Note: these are three separate transformations.

4. Show that the magnitude of a second-rank tensor property has a two-fold axis of symmetry around the 3 axis (where the principal directions are $\mathbf{e_1}$ $\mathbf{e_2}$ and $\mathbf{e_3}$). Hint: Show that

$$S(p')=S(p) \text{ where } p=\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}, \text{ and } p'=\begin{bmatrix} -l_1 \\ -l_2 \\ l_3 \end{bmatrix}$$

5. Exercise 1.3 Nye. Page 31-32.

[1] The electrical conductivity tensor of a certain crystal has the following components referred to axes x_1, x_2,x_3.

$$\sigma_{ij}=\begin{bmatrix} 25\cdot10^7 & 0 & 0 \\ 0 & 7\cdot10^7 & -3\sqrt{3}\cdot10^7 \\ 0 & -3\sqrt{3}\cdot10^7 & 13\cdot10^7 \end{bmatrix}$$

in m.k.s. units (ohm--1m-1). The axes are now transformed to a new set x'_1, x'_2,x'_3. given by the following angles:

$$\angle x_1'Ox_1=0°,\quad \angle x_2'Ox_2=30°,\qquad \angle x_2'Ox_3=60°,\qquad \angle x_3'Ox_3=30°$$

Draw up a table for the transformation [a_{ij}], and check that the sum of the squares of the transformation in each row and column is 1.

[2] Determine the values of the components, σ'_{ij} and comment on the result obtained.

[3] Draw on the new axes x'_2,x'_3 a section of the conductivity ellipsoid (representation quadric) in the plane x'_1 = 0, and notice that this is a principal section. Insert the old axes x'_2,x'_3 , on the drawing.

[4] Draw a radius vector OP in the direction whose cosines referred to the old axes are (0,½. (√3)/2). Measure the length of this radius vector and so find the electrical conductivity in this direction.

[5] Check the last result by using an analytical expression.

[6] Assume an electric field of 1 volt/m to be established in the direction OP. Calculate the components E_i along the x_1 axis, and hence calculate the components of current density j_i.

[7} Insert these components to scale on a vector diagram on the axes, x_1 x_2,x_3 , and hence, determine graphically the magnitude and direction of the resultant current density.

[8] Assuming the same electric field as in [6], repeat the calculation [6] and the construction [7] using the x'_i axes instead of the old axes, and use the values of the σ_{ij} found in [2]. Compare the result with that of [7].

[9] Compare the direction of the resultant current with that of the normal to the conductivity ellipsoid at the point P.

[10] Find graphically the component along OP of the resultant current density and hence find σ in this direction. Compare the value with those found in [4] and [5].

6. Suppose the conductivity tensor is

$$\sigma = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \bullet 10^7 (ohm \bullet m)^{-1}.$$

If the crystal axes are aligned with the coordinate axes for this representation, in which crystal class is this mineral? Calculate the current flux for E=[7.5, 4.5, 0].