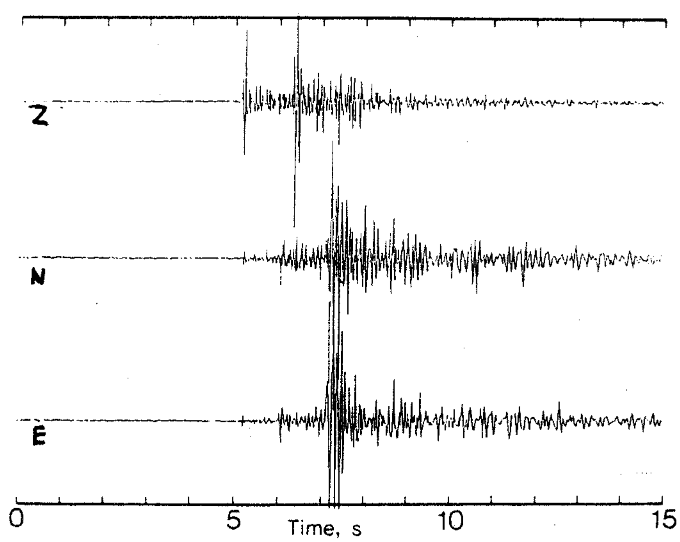
Essentials of Geophysics 12.201/501

Problem Set 5

Due Friday, December 5.

1. A three-component seismogram reveals the arrival of several phases between about 5 and 8 s (see below). Which of these arrivals are the P, S, and Sp arrivals? Can you see any other phases? If so, explain what it/they could be. Is the mantle part of S_p an SH or an SV wave?



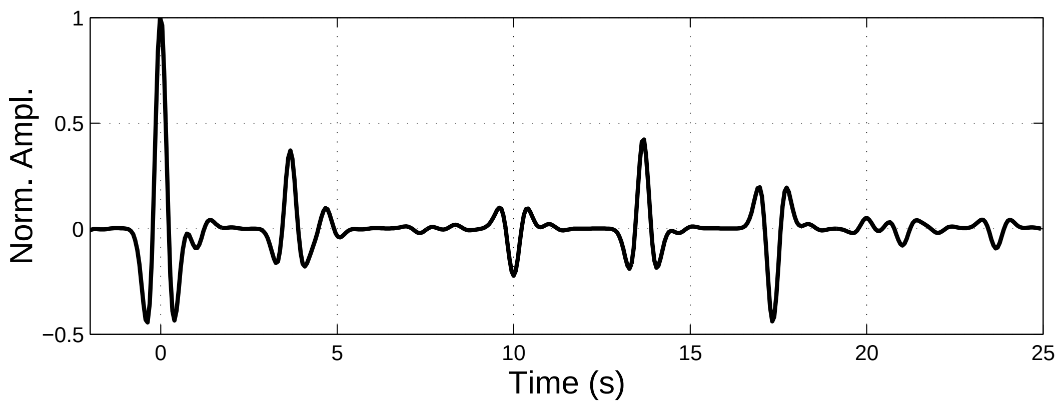
(NB The three components of the seismograms refer to the component of ground motion that is recorded: 'Z' records the vertical component of ground motion, whereas 'N' and 'E' record the North-South and East-West components of ground motion, respectively.

2. (a) Can Rayleigh waves exist in a half space? And Love waves? Why (not)? If they do, characterize their dispersion.

   (b) What type of surface wave is more likely to be caused by a nuclear explosion, a Rayleigh or a Love wave? Explain your answer.

   (c) Consider the sensitivity kernels of the surface waves as discussed in class. Are deep earthquakes, say deeper than 400 km, efficient in generating fundamental mode Rayleigh waves that can be used to study crustal structure? Can shallow earthquakes generate fundamental modes, higher modes, or both? Explain!

3. The following seismogram (calculated with a method known as the **reflectivity method**) shows the displacement due to a P-wave coming in from a deep and far earthquake. The component shown is horizontal component of motion in the vertical plane containing the source and the station.



   (a) Can you identify the phases?

   (b) Assuming the Earth under the station is characterized by a homogeneous mantle and a crust on top of it, can you infer the crustal thickness, if you also know that the P-wave velocity in the crust 6 is km/s, and the crustal Poisson ratio is 1/4?

   (c) Why did we use the horizontal seismogram rather than the vertical?

4. GRAD STUDENTS ONLY

   This exercise is based on the following textbooks:

   Lay & Wallace, Modern Global Seismology, 1995.

   Bullen & Bolt, An introduction to the theory of seismology, 1985.

   Červený, Molotkov & Psencik, Ray methods in seismology, 1977.

   (a) We have seen the wave equation

$$\nabla^2\Phi = \frac{1}{c^2(\mathbf{x})}\frac{\partial^2\Phi}{\partial t^2} \tag{1}$$

We can solve this PDE by assuming a functional form

$$\Phi(\mathbf{x}, t) = A(\mathbf{x})e^{i\omega(W(\mathbf{x})/c_0 - t)} \tag{2}$$

where $W(\mathbf{x})/c_0$ may be replaced by $\mathbf{k}\cdot\mathbf{x}$ (which is the way we obtained it before) and c_0 is some reference velocity.

$$W(\mathbf{x}) = \text{constant} \tag{3}$$

defines the position of the wavefront at a particular time.

(b) Substitute eq. 2 into eq. 1 and write out $\partial^2\Phi/\partial x_i^2$ separated into their real and imaginary parts. Equate the real and the imaginary parts - you should get two equations.

(c) Rearrange the terms of the real-part equation; show that for very high frequencies (as $\omega \to \infty$), this equation can be reduced to

$$(\nabla W)^2 = \frac{c_0^2}{c^2(\mathbf{x})} \tag{4}$$

This equation is known as the **eikonal equation** — the approximation we made is a **high-frequency approximation**: the eikonal equation will approximate the wave equation well if the fractional change in velocity gradient over one seismic wavelength is small compared to the velocity.

We can now replace

$$p_i = \frac{\partial W}{\partial x_i} \tag{5}$$

where p_i are the components of the **slowness vector**. Note that in general, p_i will be normal to the wavefront and tangent to the ray. From the general theory of partial differential equations, it can be shown that the so-called 'characteristic curves' for eq. 4 are given by

$$\frac{dx_i}{dt} = c^2 p_i \tag{6}$$

$$\frac{dp_i}{dt} = -\frac{1}{c}\frac{\partial c}{\partial x_i} \tag{7}$$