Chapter 3

The Magnetic Field of the Earth

Introduction

Studies of the geomagnetic field have a long history, in particular because of its importance for **navigation**. The geomagnetic field and its variations over time are our most direct ways to study the **dynamics of the core**. The variations with time of the geomagnetic field, the secular variations, are the basis for the science of **paleomagnetism**, and several major discoveries in the late fifties gave important new impulses to the concept of plate tectonics. Magnetism also plays a major role in exploration geophysics in the search for ore deposits.

Because of its use as a navigation tool, the study of the magnetic field has a very long history, and probably goes back to the 12thC when it was first exploited by the Chinese. It was not until 1600 that William Gilbert postulated that the Earth is, in fact, a gigantic magnet. The origin of the Earth's field has, however, remained enigmatic for another 300 years after Gilbert's manifesto '*De Magnete*'. It was also known early on that the field was not constant in time, and the secular variation is well recorded so that a very useful historical record of the variations in strength and, in particular, in direction is available for research. The first (known) map of declination was published by Halley (yes, the one of the comet) in 1701 (the 'chart of the lines of equal magnetic variation', also known as the '*Tabula Nautica*').

The source of the main field and the cause of the secular variation remained a mystery since the rapid fluctuations seemed to be at odds with the rigidity of the Earth, and until early this century an external origin of the field was seriously considered. In a breakthrough (1838) Gauss was able to prove that almost the entire field has to be of **internal origin**. Gauss used spherical harmonics and showed that the coefficients of the field expansion, which he determined by fitting the surface harmonics to the available magnetic data at that time (a small number of magnetic field measurements at intervals of about 30° along several parallels - lines of constant latitude), were almost identical to the coefficients for a field due to a magnetized sphere or to a dipole. In fact, he also showed from a spectral analysis that the best fit to the observed field was obtained if the dipole was not purely axial but made an angle of about 11° with the Earth's rotation axis.

An outstanding issue remained: what causes the internal field? It was clear that the temperatures in the interior of the Earth are probably much too high to sustain permanent magnetization. A major leap in the understanding of the origin of the field came in the first decade of the twentieth century when Oldham (1906) and Gutenberg (1912) demonstrated the existence of a (outer) core with a very low viscosity since it did not seem to allow shear wave propagation (→ rigidity μ=0). So the rigidity problem was solved. From the cosmic abundance of metallic iron it was inferred that metallic iron could be the major constituent of the (outer) core (the seismologist Inge Lehmann discovered the existence of the inner core in 1936). In the 40's Larmor postulated that the magnetic field (and its temporal variations) were, in fact, due to the rapid motion of highly conductive metallic iron in the liquid outer core. Fine; but there was still the apparent contradiction that the magnetic field would diffuse away rather quickly due to ohmic dissipation while it was known that very old rocks revealed a remnant magnetic field. In other words, the field has to be sustained by some, at that time, unknown process. This lead to the idea of the **geodynamo** (Sir Bullard, 40-ies and 50-ies), which forms the basis for our current understanding of the origin of the geomagnetic field. The theory of *magneto-hydrodynamics* that deals with magnetic fields in moving liquids is difficult and many approximations and assumptions have to be used to find any meaningful solutions. In the past decades, with the development of powerful computers, rapid progress has been made in understanding the field and the cause of the secular variation. We will see, however, that there are still many outstanding questions.

Differences and similarities with Gravity

Similarities are:

- The magnetic and gravity fields are both potential fields, the fields are the gradient of some potential V, and Laplace's and Poisson's equations apply.

- For the description and analysis of these fields, spherical harmonics is the most convenient tool, which will be used to illustrate important properties of the geomagnetic field.

- In both cases we will use a reference field to reduce the observations of the field.

- Both fields are dominated by a simple geometry, but the higher degree components are required to get a complete picture of the field. In gravity, the major component of the field is that of a point mass M in the center of the Earth; in geomagnetism, we will see that the field is dominated by that of an axial dipole in the center of the Earth and approximately aligned along the rotational axis.

Differences are:

- In gravity the attracting mass m is positive; there is no such thing as negative mass. In magnetism, there are positive and negative poles.

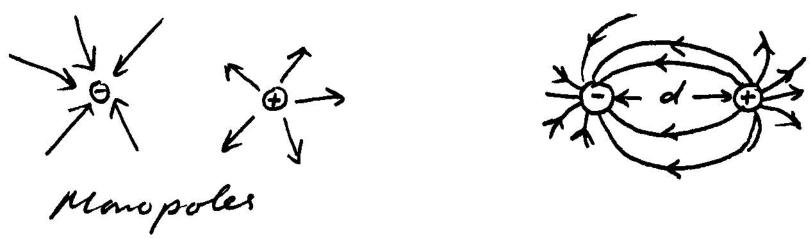


Figure 3.1:

- In gravity, every mass element dM acts as a monopole; in contrast, in magnetism isolated sources and sinks of the magnetic field H don't exist ($\nabla \cdot H = 0$) and one must always consider a pair of opposite poles. Opposite poles attract and like poles repel each other. If the distance d between the poles is (infinitesimally) small $\rightarrow$ dipole.
- Gravitational potential (or any potential due to a monopole) falls of as 1 over r, and the gravitational attraction as 1 over r^2. In contrast, the potential due to a dipole falls of as 1 over r^2 and the field of a dipole as 1 over r^3. This follows directly from analysis of the spherical harmonic expansion of the potential and the assumption that magnetic monopoles, if they exist at all, are not relevant for geomagnetism (so that the $l = 0$ component is zero).
- The direction and the strength of the magnetic field varies with time due to external and internal processes. As a result, the reference field has to be determined at regular intervals of time (and not only when better measurements become available as is the case with the International Gravity Field).
- The variation of the field with time is documented, i.e. there is a historic record available to us. Rocks have a 'memory' of the magnetic field through a process known as magnetization. The then current magnetic field is 'frozen' in a rock if the rock sample cools (for instance, after eruption) beneath the so called **Curie temperature**, which is different for different minerals, but about 500-600°C for the most important minerals such as magnetite. This is the basis for paleomagnetism. (There is no such thing as paleogravity!)

3.1 The main field

From the measurement of the magnetic field it became clear that the field has both internal and external sources, both of which exhibit a time dependence. Spherical harmonics is a very convenient tool to account for both components. Let's consider the general expression of the magnetic potential as the superposition of Legendre polynomials:

$$V_m(r,\theta,\varphi) = a\sum_{l=1}^{\infty}\sum_{m=0}^{l}\left\{\begin{array}{c}\left(\frac{a}{r}\right)^{l+1}[g_l^m\cos m\varphi + h_l^m \sin m\varphi] \\ \left(\frac{r}{a}\right)^{l}[g'^m_l\cos m\varphi + h'^m_l \sin m\varphi]\end{array}\right\} P_l^m(\cos\theta), \tag{3.1}$$

or, assuming **Einstein's summation convention** (implicit summation over repeated indices), we can write:

$$V_m(r,\theta,\varphi) = a\left\{I_l^m\left(\frac{a}{r}\right)^{l+1} + E_l^m\left(\frac{r}{a}\right)^{l}\right\}P_l^m(\cos\theta) \tag{3.2}$$

where I_l^m and E_l^m are the amplitude factors of the contributions of the internal and external sources, respectively. (Note that, in contrast to the gravitational potential, the first degree is $l = 1$, since $l = 0$ would represent a monopole, which is not relevant to geomagnetism.)

3.2 The internal field

The internal field has two components: [1] the crustal field and [2] the core field.

The crustal field

The spatial attenuation of the field as 1 over distance cubed means that the short wavelength variations at the Earth's surface must have a shallow source. Can not be much deeper than mid crust, since otherwise temperatures are too high. More is known about the crustal field than about the core field since we know more about the composition and physical parameters such as temperature and pressure and about the types of magnetization. Two important types of magnetization:

- **Remanent magnetization** (there is a field **B** even in absence of an ambient field). If this persists over time scales of $\mathcal{O}(10^8)$ years, we call this **permanent magnetization**. Rocks can acquire permanent magnetization when they cool beneath the Curie temperature (about 500-600° for most relevant minerals). The ambient field then gets frozen in, which is very useful for paleomagnetism.

- **Induced magnetization** (no field, unless induced by ambient field).

No mantle field

Why not in the mantle? Firstly, the mantle consists mainly of silicates and the average *conductivity* is very *low*. Secondly, as we will see later, fields in a low conductivity medium decay very rapidly unless sustained by rapid motion, but *convection* in the mantle is too *slow* for that. Thirdly, permanent magnetization is out of the question since mantle *temperatures* are too *high* (higher than the Curie temperature in most of the mantle).

The core field

The temperatures are too high for permanent magnetization. The field is caused by rapid (and complex) electric currents in the liquid outer core, which consists mainly of metallic iron. Convection in the core is much more vigorous than in the mantle: about 10^6 times faster than mantle convection (i.e, of the order of about 10 km/yr).
Outstanding problems are:

1. the energy source for the rapid flow. A contribution of radioactive decay of Potassium and, in particular, Uranium, can - at this stage - not be ruled out. However, there seems to be increasing consensus that the primary candidate for providing the driving energy is gravitational energy released by downwelling of heavy material in a **compositional convection** caused by **differentiation of the inner core**. Solidification of the inner core is selective: it takes out the iron and leaves behind in the outer core a relatively light residue that is gravitationally unstable. Upon solidification there is also latent heat release, which helps maintaining an adiabatic temperature gradient across the outer core but does not effectively couple to convective flow. The lateral variations in temperature in the outer core are probably very small and the role of thermal convection is negligible. Any aspherical variations in density would be annihilated quickly by convection as a result of the low viscosity.

2. the details of the pattern of flow. This is a major focus in studies of the geodynamo.

The knowledge about flow in the outer core is also restricted by observational limitations.

- the spatial attenuation is large since the field falls of as 1 over r^3. As a consequence effects of turbulent flow in the core are not observed at the surface. Conversely, the downward continuation of small scale features in the field will be hampered by the amplification of uncertainties and of the crustal field.

- the mantle has a small but non-zero conductivity, so that rapid variations in the core field will be attenuated. In general, only features of length scales larger than about 1500 km ($l < 12, 13$) and on time scales longer than 1 to 5 year are attributed to core flow, although this rule of thumb is *ad hoc*.

The core field has the following characteristics:

1. 90% of the field *at the Earth's surface* can be described by a **dipole** inclined at about 11° to the Earth's spin axis. The axis of the dipole intersects the Earth's surface at the so-called geomagnetic poles at about (78.5°N, 70°W) (West Greenland) and (75.5°S, 110°E). In theory the angle between the magnetic field lines and the Earth's surface is 90° at the poles but owing to local magnetic anomalies in the crust this is not necessarily the case in real life.

   The dipole field is represented by the degree 1 ($l = 1$) terms in the harmonic expansion. From the spherical harmonic expansion one can see immediately that the potential due to a dipole attenuates as 1 over r^2.

2. The remaining 10% is known as the **non-dipole** field and consists of a quadrupole ($l = 2$), and octopole ($l = 3$), etc. We will see that at the core-mantle-boundary the relative contribution of these higher degree components is much larger!

   Note that the relative contribution 90%↔10% can change over time as part of the secular variation.

3. The strength of the Earth's magnetic field varies from about 60,000 nT at the magnetic pole to about 25,000 nT at the magnetic equator. (1nT = $1\gamma = 10^{-1}$ Wb m^{-2}).

4. Secular variation: important are the westward drift and changes in the strength of the dipole field.

5. The field is probably not completely independent from the mantle. Core-mantle coupling is suggested by several observations (i.e., changes of the length of day, not discussed here), by the statistics of field reversals, and by the suggested preferential reversal paths.

Intermezzo 3.1 UNITS OF CONFUSION

The units that are typically used for the different variables in geomagnetism are somewhat confusing, and up to 5 different systems are used. We will mainly use the Système International d'Unités (*S.I.*) and mention the electromagnetic units (*e.m.u.*) in passing. When one talks about the geomagnetic field one often talks about **B**, measured in T (Tesla) (= $kg^{-1}A^{-1}s^{-2}$) or nT (nanoTesla) in S.I., or *Gauss* in e.m.u. In fact, **B** is the magnetic induction due to the magnetic field **H**, which is measured in Am^{-1} in S.I. or Oersted in e.m.u. For the conversions from the one to the other unit system: T = 10^4 G(auss) → 1nT = 10^{-5}G = 1γ (gamma). $\mathbf{B} = \mu_0\mathbf{H}$ with μ_0 the magnetic permeability in free space; $\mu_0 = 4\pi\times10^{-7}$ $kgmA^{-2}s^{-2}$ [=NA^{-2} = H(enry) m^{-1}], in S.I., and μ_0 = 1 G Oe in e.m.u. So, in e.m.u., **B** = **H**, hence the liberal use of **B** for the Earth's field. The magnetic permeability μ is a measure of the "ease" with which the field **H** can penetrate into a material. This is a material property, and we will get back to this when we discuss rock magnetism.
In the next table, some of the quantities are summarized together with their units and dimensions. There are only 4 so-called dimensions we need. These are (with their symbol and standard units) **mass** [M (kg)], **length** [L (m)], **time** [T (s)] and **current** [I (Ampère)].

Quantity	Symbol	Dimension	S.I. Units
force	$\mathbf{F}$	MLT^{-2}	Newton (N)
charge	q	IT	Coulomb (C)
electric field	$\mathbf{E}$	$MLT^{-3}I^{-1}$	N/C
electric flux	Φ_E	$ML^3T^{-3}I^{-1}$	N/C m^2
electric potential	V_E	$ML^2T^{-3}I^{-1}$	Volt (V)
magnetic induction	$\mathbf{B}$	$MT^{-2}I^{-1}$	Tesla (T)
magnetic flux	Φ_B	$ML^2T^{-2}I^{-1}$	Weber (Wb)
magnetic potential	V_m	$MLT^{-2}I^{-1}$	T m
permittivity of vacuum	ϵ_0	$M^{-1}L^{-3}T^4I^2$	$C^2/(N\ m^2)$
permeability of vacuum	μ_0	$MLT^{-2}I^{-2}$	Wb/(A m)
resistance	R	$ML^2T^{-3}I^{-2}$	Ohm (Ω)
resistivity	ρ	$ML^3T^{-3}I^{-2}$	Ωm

3.3 The external field

The strength of the field due to external sources is much weaker than that of the internal sources. Moreover, the typical time scale for changes of the intensity of the external field is much shorter than that of the field due to the internal source. Variations in magnetic field due to an external origin (atmospheric, solar wind) are often on much shorter time scales so that they can be separated from the contributions of the internal sources.

The separation is *ad hoc* but seems to work fine. The rapid variation of the external field can be used to study the (lateral variation in) conductivity in the Earth's mantle, in particular to depth of less than about 1000 km. Owing to the spatial attenuation of the coefficients related to the external field and, in particular, to the fact that the rapid fluctuations can only penetrate to a certain depth (the skin depth, which is inversely proportional to the frequency), it is difficult to study the conductivity in the deeper part of the lower mantle.

3.4 The magnetic induction due to a magnetic dipole

Magnetic fields are fairly similar to electric fields, and in the derivation of the magnetic induction due to a magnetic dipole, we can draw important conclusions based on analogies with the electric potential due to an electric dipole. We will therefore start with a brief discussion of electric dipoles.

On the other hand, our familiarity with the gravity field should enable us to deduce differences and similarities of the magnetic field and the gravity field as well. In this manner, we will start with the field due to a magnetic dipole — the simplest configuration in magnetics — in a straightforward analysis based on experiments, and subsequently extend this to the field induced by higher-order "poles": quadrupoles, octopoles, and so on. The equivalence with gravitational potential theory will follow from the fact that both the gravitational and the magnetic potential are solutions to Laplace's equation.

The electric field due to an electric dipole

The law obeyed by the force of interaction of point charges q (in vacuum) was established experimentally in 1785 by Charles de Coulomb. **Coulomb's Law** can be expressed as:

$$\mathbf{F} = K_e \frac{q_0 q}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\epsilon_0}\frac{q_0 q}{r^2}\hat{\mathbf{r}}, \tag{3.3}$$

where $\hat{\mathbf{r}}$ is the unit vector on the axis connecting both charges. This equation is completely analogous with the gravitational attraction between two masses, as we have seen. Just as we defined the gravity field $\mathbf{g}$ to be the gravitational force normalized by the test mass, the electric field $\mathbf{E}$ is defined as the ratio of the electrostatic force to the test charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}, \tag{3.4}$$

or, to be precise,

$$\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0}, \tag{3.5}$$

Now imagine two like charges of opposite sign $+p$ and $-p$, separated by a distance d, as in Figure 3.2. At a point P in the equatorial plane, the electrical fields induced by both charges are equal in magnitude. The resulting field is antiparallel with vector $\mathbf{m}$. If we associate a dipole moment vector m with this configuration, pointing from the negative to the positive charge and whereby $|\mathbf{m}| = \mathbf{d}p$, the field strength at the equatorial point P is given by:

$$E = K_e \frac{|\mathbf{m}|}{r^3} = \frac{1}{4\pi\epsilon_0} \frac{|\mathbf{m}|}{r^3}. \tag{3.6}$$

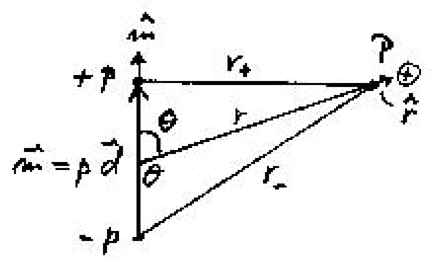


Figure 3.2:

Next, consider an arbitrary point P at distance r from a finite dipole with moment $\mathbf{m}$. In gravity we saw that the gravitational field $\mathbf{g}$ (the gravitational force per unit mass), led to the gravitational potential at point P due to a mass element dM given by $U_{\mathrm{grav}} = -GdMr^{-1}$. We can use this as an ad hoc analog for the derivation of the potential due to a magnetic dipole, approximated by a set of imaginary monopoles with strength p. To get an expression for the magnetic potential we have to account for the potential due to the negative ($-p$) and the positive ($+p$) pole separately.

With A some constant we can write

$$V_m = A\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = Ad\,\frac{\frac{1}{r_+} - \frac{1}{r_-}}{d} \tag{3.7}$$

and for small d

$$\frac{1}{d}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) \sim \frac{\partial}{\partial d}\left(\frac{1}{r}\right) \tag{3.8}$$

eq. (3.7) becomes:

$$V_m = Ad\,\frac{\partial}{\partial d}\left(\frac{1}{r}\right) \tag{3.9}$$

$\partial_d(1/r)$ is the directional derivative of $1/r$ in the direction of d. This expression can be written as the directional derivative in the direction of r by projecting the variations in the direction of $\mathbf{d}$ on $\mathbf{r}$ (i.e., taking the dot product between $\mathbf{d}$ or $\mathbf{m}$ and $\mathbf{r}$):

$$\frac{\partial}{\partial d}\left(\frac{1}{r}\right) = \frac{\partial}{\partial r}\left(\frac{1}{r}\right)\cos\theta = -\frac{1}{r^2}\cos\theta \tag{3.10}$$

Intermezzo 3.2 MAGNETIC FIELD INDUCED BY ELECTRICAL CURRENT

There is no such thing as a magnetic "charge" or "mass" or "monopole" that would make a magnetic force a law similar to the law of gravitational or electrostatic attraction. Rather, the magnetic induction is both defined and measured as the force (called the **Lorentz force**[a]) acting on a test charge q_0 that travels through such a field with velocity $\mathbf{v}$.

$$\mathbf{F} = q_0(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{3.11}$$

On the other hand, it is observed that electrical currents induce a magnetic field, and to describe this, an equation is found which resembles the electric induction to to a dipole. The idea of magnetic dipoles is born. In 1820, the French physicists Biot and Savart measured the magnetic field induced by an electrical current. Laplace cast their results in the following form:

$$\mathbf{dB}(P) = \frac{\mu_0}{4\pi} i \frac{\mathbf{dl} \times \hat{\mathbf{r}}}{r^2}. \tag{3.12}$$

An infinitesimal contribution to the magnetic induction $\mathbf{dB}$ due to a line segment $\mathbf{l}$ through which flows a current i is given by the cross product of that line segment (taken in the direction of the current flow) and the unit vector connecting the $\mathbf{dl}$ to point P.
For a point P on the axis of a closed circular current loop with radius R, the total induced field $\mathbf{B}$ can be obtained as:

$$B = \frac{\mu_0}{2\pi} i \frac{\pi R^2}{r^3} = \frac{\mu_0}{2\pi} \frac{|\mathbf{m}|}{r^3}. \tag{3.13}$$

In analogy with the electrical field, a dipole moment $\mathbf{m}$ is associated with the current loop. Its magnitude if given as $|\mathbf{m}| = \pi R^2 i$, i.e. the current times the area enclosed by the loop. $\mathbf{m}$ lies on the axis of the circle and points according in the direction a corkscrew moves when turned in the direction of the current (the way you find the direction of a cross product). Note how similar Eq. 3.13 is to Eq. 3.6: the simplest magnetic configuration is that of a dipole.
The definition of electric or gravitational potential energy is work done per unit charge or mass. In analogy to this, we can define the magnetic potential increment as:

$$dV = -\mathbf{B} \cdot \mathbf{dl} \quad \Leftrightarrow \quad \mathbf{B} = -\boldsymbol{\nabla} V \tag{3.14}$$

What is the potential at a point P due to a current loop? Using Eq. 3.12, we can write Eq. 3.14 as:

$$dV(P) = -\frac{\mu_0}{4\pi} i \oint \frac{\mathbf{dl}' \times \hat{\mathbf{r}}}{r^2} \cdot \mathbf{dl}. \tag{3.15}$$

Working this out (this takes a little bit of math) for a current loop small in diameter with respect to the distance r to the point P and introducing the magnetic dipole moment $\mathbf{m}$ as done above, we obtain for the magnetic potential due to a magnetic dipole:

$$V(P) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \cdot \hat{\mathbf{r}}}{r^2}. \tag{3.16}$$

[a] After Hendrik A. Lorentz (1853–1928).

Since $U_{\text{monopole}} \sim 1/r$ this expression means that the potential due to a dipole is the directional derivative of the potential due to a monopole. (Note that θ is the angle between the dipole axis $\mathbf{d}$ and OP (or $\mathbf{r}$) and thus represents the **magnetic co-latitude**.)

Just as the Newtonian potential was proportional to GdM, the constant A must be proportional to the strength of the poles, or to the magnitude of the magnetic moment $m = |\mathbf{m}| \rightarrow A = Cpd = Cm$. We have, in S.I. units,

$$V_m = \frac{\mu_0 m \cos\theta}{4\pi r^2} = \frac{\mu_0 \mathbf{m}\cdot\mathbf{r}}{4\pi r^3} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\cdot\hat{\mathbf{r}}}{r^2} \tag{3.17}$$

3.5 Magnetic potential due to more complex configurations

Laplace's equation for the magnetic potential

In gravity, the simplest configuration was the gravity field due to a point mass, or gravitational monopole. After that we went on and proved how the gravitational potential obeyed Laplace's equation. The solutions were found as spherical harmonic functions, for which the $l = 0$ term gave us back the gravitational monopole.

Magnetic monopoles have not been proven to exist. The simplest geometry therefore is the dipole. If we can prove that the magnetic field obeys Laplace's equation as well, we will again be able to obtain spherical harmonic solutions, and this time the $l = 1$ term will give us back the dipole formula of Eq. 3.16.

It is easy enough to establish that for a closed surface enclosing a magnetic dipole, just as many field lines enter the surface as are leaving. Hence, the total magnetic flux should be zero. At the north magnetic pole, your test dipole will be attracted, whereas at the south pole it will be repelled, and vice versa. Remember how this was untrue for the flux of the gravity field: an apple falls toward the Earth regardless if it is at the north, south or any other pole. Mathematically speaking, in contrast to the gravity field, the magnetic field is **solenoidal**. We can write:

$$\Phi_B = \oint_S \mathbf{B}\cdot\mathbf{dS} = 0. \tag{3.18}$$

Using Gauss's Divergence Theorem just like we did for gravity, we find that the magnetic induction is divergence-free and with Eq. 3.14 we obtain that indeed

$$\nabla^2 V = 0. \tag{3.19}$$

This equation is known in magnetics as Gauss's Law; we will encounter it again as a special case of the Maxwell equations. We have previously solved Eq.

3.19. The solutions are spherical harmonics, so we know that the solution for an internal source is given by (for $r \geq a$):

$$V = a \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left(\frac{a}{r}\right)^{l+1} P_l^m(\cos\theta)[g_l^m \cos m\varphi + h_l^m \sin m\varphi]. \tag{3.20}$$

Intermezzo 3.3 SOLENOIDAL, POTENTIAL, IRROTATIONAL

A **solenoidal** vector field **B** is divergence-free, i.e. it satisfies satisfies

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{3.21}$$

for every vector **B**. If this is true, then there exists a vector field **A** such that

$$\mathbf{B} \equiv \nabla \times \mathbf{A} \tag{3.22}$$

This follows from the vector identity

$$\boldsymbol{\nabla} \cdot \mathbf{B} = \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \mathbf{A}) = 0. \tag{3.23}$$

This vector field **A** is a **potential field**. For a function ϕ satisfying Laplace's equation $\boldsymbol{\nabla}\phi$ is solenoidal (and also irrotational). An **irrotational** vector field **T** is one for which the curl vanishes:

$$\boldsymbol{\nabla} \times \mathbf{T} = 0. \tag{3.24}$$

Reduction to the dipole potential

The potential due to a dipole is obtained from Eq. 3.20 by setting $l = 1$ and taking the appropriate associated Legendre functions:

$$V^D = \frac{a^3}{r^2}[g_1^0 \cos\theta + g_1^1 \cos\varphi \sin\theta + h_1^1 \sin\varphi \sin\theta]. \tag{3.25}$$

This is valid in Earth coordinates, with the z-axis the rotation axis.The coefficients are (g_1^0, g_1^1, h_1^1) and they represent one axial (g_1^0) and two equatorial components of the field. (with g_1^1 taken along the Greenwich meridian). At any point (r, θ, φ) outside the source of the field, i.e, outside the core, the dipole field can be composed as the sum of these three components.

Earlier, we had obtained Eq. 3.16, which we can write, still in *geographical* coordinates, as:

$$V = \frac{\mu_0}{4\pi} \frac{1}{r^2} \left[m_x \frac{x}{r} + m_y \frac{y}{r} + m_z \frac{z}{r}\right]. \tag{3.26}$$

Later, we will see how in a special case, we can take the dipole axis to be the z-axis of our coordinate system (*geomagnetic* coordinates or *axial dipole*

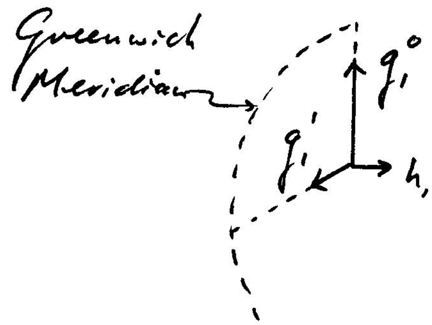


Figure 3.3:

assumption). Then, there is no longitudinal variation of the potential, m_z is the only nonzero component and the only coefficient needed is g_1^0. Comparing Eqs. 3.25 and 3.26 we see the equivalence of the **Gauss coefficients** g_l^m and h_l^m with the Cartesian components of the magnetic dipole vector:

$$\begin{cases} m_x &= \frac{4\pi}{\mu_0} a^3 g_1^1 \\ m_y &= \frac{4\pi}{\mu_0} a^3 h_1^1 \\ m_z &= \frac{4\pi}{\mu_0} a^3 g_1^0. \end{cases} \tag{3.27}$$

Obtaining the magnetic field from the potential

We've seen in Eq. 3.14 that the magnetic induction is the gradient of the magnetic potential. It is certainly more convenient to express the field in spherical coordinates. To this end, we remind the reader of the spherical gradient operator:

$$\boldsymbol{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\boldsymbol{\varphi}}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}. \tag{3.28}$$

In other words, the three components of the magnetic induction in terms of the magnetic potential are given by:

$$\begin{aligned} B_r &= -\frac{\partial}{\partial r}V \\ B_\theta &= -\frac{1}{r}\frac{\partial}{\partial \theta}V \\ B_\varphi &= -\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}V \end{aligned} \tag{3.29}$$

We remind that $\hat{\mathbf{r}}$ points in the direction of increasing distance from the origin (outwards from the Earth), $\hat{\boldsymbol{\theta}}$ in the direction of increasing θ (that is, southwards) and $\hat{\boldsymbol{\varphi}}$ eastwards. See Figure 3.4.

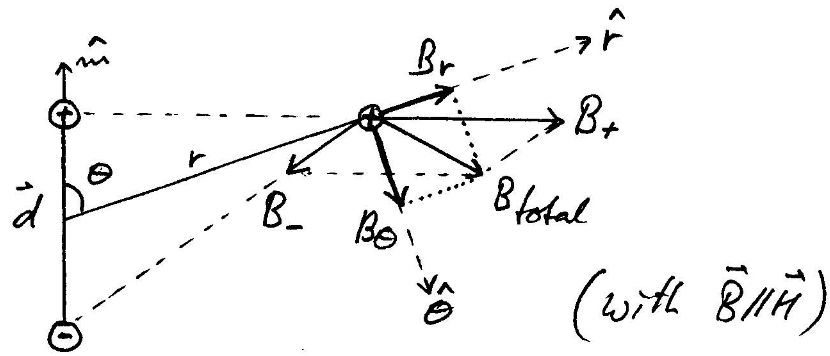


Figure 3.4:

Geographic and geomagnetic reference frames

It is useful to point out the difference between geocentric (or geographic) and geomagnetic reference frames.

In a geomagnetic reference frame, the dipole axis coincides with the z coordinate axis. Since the dipole field is axially symmetric, it is now symmetric around the z-axis. This implies that there is no longitudinal variation: no φ-dependence. The components of the field can be described by zonal spherical harmonics — in the upper hemisphere, field lines are entering the globe, and they are leaving in the lower hemisphere. Only one Gauss coefficient is necessary: $\mathcal{G}_1^0$ or $\mathcal{M}'_z$ describe the dipole completely (curled letters used for dipole reference frame). See Figure 3.5(A).

In Figure 3.5(B), the dipole is placed at an angle to the coordinate axis. To describe the field, we need more than one spherical harmonic: a zonal and a sectoral one. Longitudinal φ-variation is introduced: $m \neq 0$. We need three Gauss coefficients to describe the dipole behavior: g_1^0, g_1^1 and h_1^1. Of course the dipole itself hasn't changed: its magnitude is now $[(g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2]^{(1/2)} = (\mathcal{G}_1^0)^2$. Compared to the dipole reference frame, all we have done is a spherical harmonic rotation, resulting in a redistribution of the magnitude of the dipole over three instead of one Gauss coefficients.

The angle the magnetic induction vector makes with the horizontal is called the **inclination** I. The angle with the geographic North is the **declination** D. In a dipole reference frame the declination is indentically zero.

Let's use Eqs. 3.16 and 3.29 to calculate the components of a dipole field *in the dipole reference frame* for a few special angles.

$$V = \frac{\mu_0}{4\pi}\frac{|\mathbf{m}|\cos\theta}{r^2}, \tag{3.30}$$

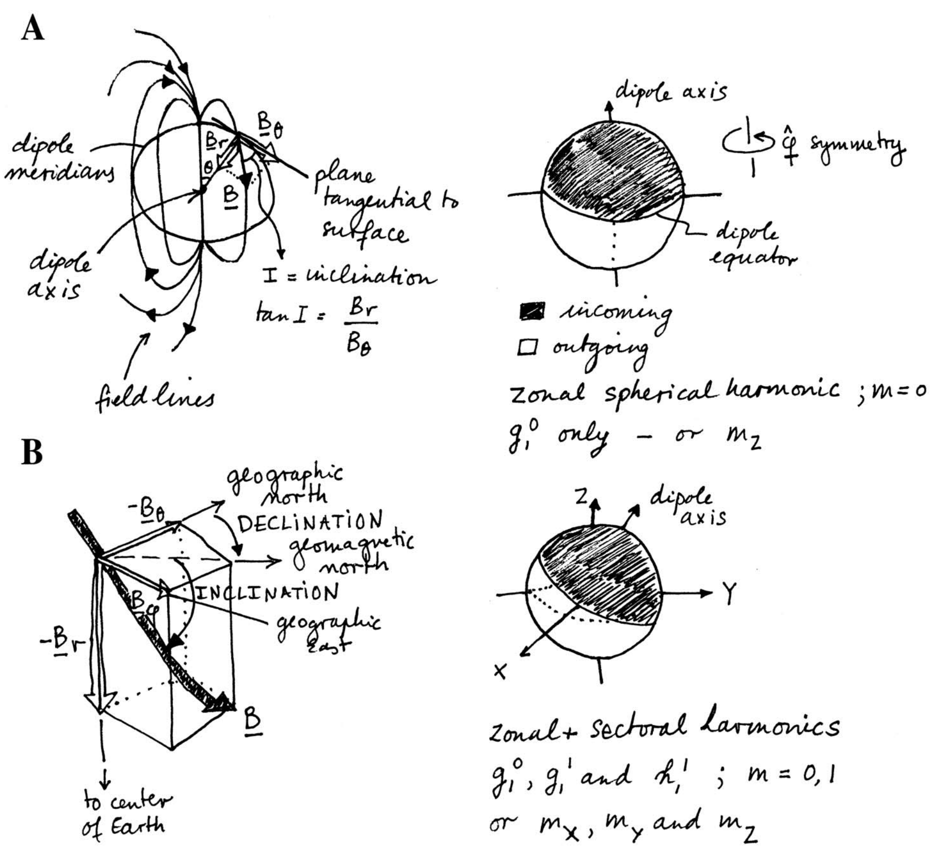


Figure 3.5: Geographic and geomagnetic reference frames and how to represent the dipole with spherical harmonics coefficients in both references systems.

from which follows that

$$\begin{cases} B_r &= \frac{\mu_0}{4\pi}\frac{|\mathbf{m}|}{r^3} 2\cos\theta = 2B_0\cos\theta \\ B_\theta &= \frac{\mu_0}{4\pi}\frac{|\mathbf{m}|}{r^3} \sin\theta = B_0\sin\theta \\ B_\varphi &= 0. \end{cases} \tag{3.31}$$

$B_0 = (\mu_0|\mathbf{m}|)/(4\pi a^3) = 3.03\times10^{-5}$T $(= 0.303$ Gauss) at the surface of the Earth.

So for the magnetic North Pole, Equator and South Pole, respectively, we get the field strengths summarized in Table 3.1.

So the field at the magnetic equator is half that at the magnetic poles, and at the North Pole it points radially inward, but outward at the geographic South Pole.

	θ	B_r	B_θ	B_φ
North Pole	0	$2B_0$	0	0
Equator	$\pi/2$	0	B_0	0
South Pole	0	$-2B_0$	0	0

Table 3.1: Field strengths at different latitudes in terms of the field strength at the equator.

In geomagnetic studies, one often uses $Z = -B_r$ and $H = -B_\theta$ and $E = B_\varphi$. The magnetic North Pole is in fact close to the geographic South Pole! The expression for inclination is often given as:

$$\tan I = \frac{Z}{H} = 2\frac{\cos\theta}{\sin\theta} = 2\tan^{-1}\theta = 2\cot\theta = 2\tan\lambda_m. \tag{3.32}$$

with λ_m the magnetic latitude ($\lambda_m = 90° - \theta$) at which the field line crosses the point $r = a$.

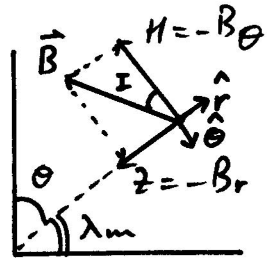


Figure 3.6:

3.6 Power spectrum of the magnetic field

The power spectrum I_l at degree l of the field $\mathbf{B}$ is defined as the scalar product $\mathbf{B}_l \cdot \mathbf{B}_l$ averaged over the surface of the sphere with radius a. In other words, the definition of I_l is:

$$I_l = \frac{1}{4\pi a^2}\int_0^{2\pi}\int_0^{\pi} \mathbf{B}_l \cdot \mathbf{B}_l \, a^2 \sin\theta \, d\theta d\varphi, \tag{3.33}$$

which, with, as we've seen $\mathbf{B}_l$ equal to

$$\mathbf{B}_l = -\boldsymbol{\nabla}\left[a\left(\frac{a}{r}\right)^{l+1}\sum_{m=0}^{l}(g_l^m\cos m\varphi + h_l^m \sin m\varphi)P_l^m(\cos\theta)\right] \tag{3.34}$$

From this, the power spectrum for a particular degree l is given by

$$I_l = (l+1)\sum_{m=0}^{l}[(g_l^m)^2 + (h_l^m)^2]. \tag{3.35}$$

The root mean square (r.m.s) **field strength** at the Earth's surface for degree l is defined as $B_l = \sqrt{I_l}$ and the *total* r.m.s. field is given by

$$B = \left\{\sum_{l=1}^{\infty} I_l\right\}^{\frac{1}{2}} = \left\{\sum_{l=1}^{\infty}(l+1)\sum_{m=0}^{l}[(g_l^m)^2 + (h_l^m)^2]\right\}^{\frac{1}{2}} \tag{3.36}$$

I_l can be plotted as a function of degree l:

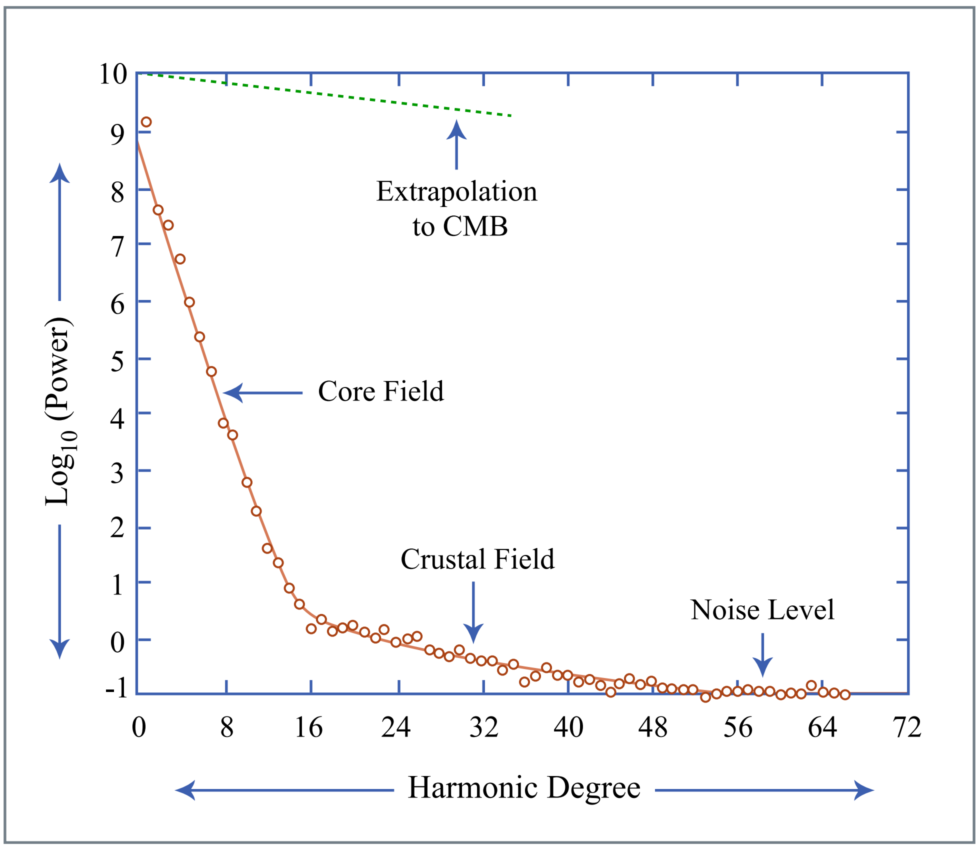


Figure by MIT OCW.
Figure 3.7:

The power spectrum consists of two regimes. Up to degree l = 14-15 there is a rapid roll-off of the mean square field with degree l . The field is obviously predominated by the lower degree terms, and such a so called "red" spectrum is, of course, consistent with the spherical harmonic expansion for internal sources, see Eq. 3.20. This part of the spectrum is due to the core field. To be more precise, the core field dominates the spectrum up to degree $l = 14$-15. At higher degrees the core field is obscured by a flat "white-ish" spectrum where the power no longer seems to depend on the degree, or, alternatively, the wavelength of the causative anomalies. This part of the spectrum must be due to sources close to the observation points; it relates to the crustal field. We will see that this field is important even, or, in particular, when one wants to study the magnetic field at the surface of the outer core, the core mantle boundary (CMB).

3.7 Downward continuation

In order to study core dynamics or the geodynamo one wants to know what the magnetic field is close to its source, i.e., at the CMB.

From Eq. 3.20, we deduce that

$$V_l(r) = V_l(a)\left(\frac{a}{r}\right)^{l+1} \tag{3.37}$$

$$\mathbf{B} = -\boldsymbol{\nabla} V_l(r) = \mathbf{B}_l(a)(l+1)\left(\frac{a}{r}\right)^{l+2} \tag{3.38}$$

$$\tag{3.39}$$

So for the power spectrum, logically,

$$I_l(r) = I_l(a)\left(\frac{a}{r}\right)^{2l+4} \tag{3.40}$$

with $I_l(a)$ the power at Earth's surface ($r = a$).

Let's look at some numbers to illustrate the effect of downward continuation (see Table 3.2). Consider the (r.m.s.) field strength at the equator at both the Earth's surface ($r = a$) and at the CMB ($r = 0.54a$ (so that $a/r = 1.82$) (Use eq. (3.36)).

	Surface (nT)	CMB (nT)
dipole (l=1)	42,878	258,493
quadrupole (l=2)	8,145 (19% of dipole)	89,367 (35% of dipole)
octopole (l=3)	6,079 (14% of dipole)	121,392 (47% of dipole)

Table 3.2: Field strengths at the surface and the core mantle boundary.

In other words, if the spectrum of the core field is "red" at the Earth's surface, it is more "pink-ish" at the CMB because the higher degree components are preferentially amplified upon downward continuation. However, the amplitude of the higher degree components (the value of the related Gauss coefficients) is small and, consequently, the relative uncertainty in these coefficients large. Upon downward continuation these uncertainties are — of course — also amplified, so that at the CMB the higher degree components are large but uncertain and the observational constraints for them are increasingly weak! We can now also understand why the crustal field poses a problem if one wants to study the core field at the CMB for degrees $l > 14$: these high degree components will be strongly amplified upon downward continuation and for high harmonic degrees the core field at the CMB will be contaminated with the crustal field!

3.8 Secular variation

Secular variation is loosely used to indicate slow changes with time of the geomagnetic field (declination, inclination, and intensity) that are (probably) due to the changing pattern of core flow. The term secular variation is commonly used for variations on time scales of 1 year and longer. This means that there is some overlap with the temporal effects of the external field, but in general the variations in external field are much more rapid and much smaller in amplitude so that confusion is, in fact, small. From measurements of the components H, Z, and E, at regular time intervals one can also determine the time derivatives $\partial_t g_l^m = \dot{g}_l^m$ and $\partial_t h_l^m = \dot{h}_l^m$, and, if need be, also the higher order derivatives. The values of g_l^m and h_l^m averaged over a particular time interval along with the time derivatives $\dot{g}_l^m$ and $\dot{h}_l^m$ determine the International Geomagnetic Reference field (IGRF), which is published in map and tabular form every 5 year or so.

Temporal variations in the internal field are modeled by expanding the Gauss coefficients in a Taylor series in time about some epoch t_e, e.g.,

$$\begin{aligned} g_e^m(t) &= g_e^m(t_e) + \left(\frac{\partial g}{\partial t}\right)\Bigg|_{t_e} (t - t_e) \\ &+ \left(\frac{\partial^2 g}{\partial t^2}\right)\Bigg|_{t_e} \frac{(t - t_e)^2}{2!} + \text{higher-order terms} \end{aligned} \tag{3.41}$$

Most models include only the first two terms on the right-hand side, but sometimes it is necessary to include the third derivative term as well, for distance, in studies of **magnetic jerks**.

Similar to the mean square of the surface field, we can define a mean square value of the variation in time of the field at degree l:

$$\dot{I}_l = (l+1) \sum [(\dot{g}_l^m)^2 + (\dot{h}_l^m)^2] \tag{3.42}$$

and the relaxation time τ_l for the degree l component as

$$\tau_l = \left(\frac{I_l}{\dot{I}_l}\right)^{\frac{1}{2}} \tag{3.43}$$

There are at least three important phenomena:

1. Change in the **strength** of the dipole. We can infer that for the dipole, the coefficients $\dot{g}_l^m$ and $\dot{h}_l^m$ are all of opposite sign than those of the main field. This indicates a weakening of the dipole field. From the numbers in

Table 7.1 of Stacey and eq. (3.43) we deduce that the relaxation time of the dipole is about 1000 years; in other words, the current rate of change of the strength of the dipole field is about 8% per 100yr. Note that this represents a "snapshot" of a possibly complex process, and that it does not necessarily mean that we are headed for a field reversal within 1000yr.

2. Change in **orientation** of the main field: the orientation of the best fitting dipole seems to change with time, but on average, say over intervals of several tens of thousands of years, it can be represented by the field of an *axial* dipole. For London, in the last 400 year or so the change in declination and inclination describes a clockwise, cyclic motion which is consistent with a westward drift of the field.

3. Westward drift of the field. The westward drift is about $0.2°a^{-1}$ in some regions. Although it forms an obvious component of the secular variation in the past 300-400 years, it may not be a fundamental aspect of secular variation for longer periods of time. Also there is a strong regional dependence. It is not observed for the Pacific realm, and it is mainly confined to the region between Indonesia and the "Americas".

Cause of the secular variation

The slow variation of the field with time is most likely due to the reorganization of the lines of force in the core, and not to the creation or destruction of field lines. The variation of the strength and direction of the dipole field probably reflect oscillations in core flow. The westward drift has been attributed to either of two mechanisms:

1. differential rotation between core and the mantle

2. hydromagnetic wave motion: standing waves in the core that slowly migrate westward, but without differential motion of material.

Like many issues in this scientific field, this problem has not been resolved and the cause of the secular variations are still under debate.

3.9 Source of the internal field: the geodynamo

Introduction

Over the centuries, several mechanisms have been proposed, but it is now the consensus that the core field is caused by rapid and complex flow of highly conductive, metallic iron in the outer core. We will not give a full treatment of the complex issues involved, but to provide the reader with some baggage with which it is easier to penetrate the literature and to follow discussions and presentations.

Maxwell's Equations

Maxwells Equations describe the production and interrelation of electric and magnetic fields. A few of them we've already seen (in various forms). In this section, we will give Maxwell's Equations in vector form but derive them from the integral forms which were based on experiments.

Two results from vector calculus will be used here. The first we already know: it is **Gauss's theorem** or the **divergence theorem**. It relates the integral of the divergence of the field over some closed volume to the flux through the surface that bounds the voume. The divergence measures the sources and sinks within the volume. If nothing is lost or created within the volume, there will be no net flux through its surface!

$$\int_V \boldsymbol{\nabla} \cdot \mathbf{T}\, dV = \int_{\partial V} \hat{\mathbf{n}} \cdot \mathbf{T}\, dS \tag{3.44}$$

A second important law is **Stokes' theorem**. This law relates the curl of a vector field, integrated over some surface, to the line integral of the field over the curve that bounds the surface.

$$\int_S \boldsymbol{\nabla} \times \mathbf{T}\, dS = \int_{\partial S} \mathbf{T} \cdot \mathbf{dl} \tag{3.45}$$

1. The magnetic field is solenoidal

   We have already seen that magnetic field lines begin and end at the magnetic dipole. Magnetic "charges" or "monopoles" do not exist. Hence, all field lines leaving a surface enclosing a dipole, reenter that same surface. There is no magnetic flux (in the absence of currents and outside the source of the magnetic field):

   $$\Phi_B = \oint \mathbf{B} \cdot \mathbf{dS} = 0 \tag{3.46}$$

   Rewriting this with Eq. 3.44 gives Maxwell's first law:

   $$\boxed{\boldsymbol{\nabla} \cdot \mathbf{B} = 0} \tag{3.47}$$

2. Electromagnetic induction

   An empirical law due to Faraday says that changes in the magnetic flux through a surface induce a current in a wire loop that defines the surface.

   $$\frac{d}{dt}\Phi_B = \frac{d}{dt}\oint \mathbf{B} \cdot \mathbf{dS} = -\oint \mathbf{E} \cdot \mathbf{dl}, \tag{3.48}$$

   which can be rewritten using Eq. 3.45 to give Maxwell's second law:

   $$\boxed{\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}} \tag{3.49}$$

3. Displacement current

We've seen that a time-dependent magnetic flux induces an electric field. The reverse is also true: a time-dependent electric flux induces a magnetic field. But a current by itself was also responsible for a magnetic field (see Eq. 3.12). Both effects can be combined into one equation as follows:

$$\mu_0 \left(i + \epsilon_0 \frac{d}{dt} \Phi_E \right) = \oint \mathbf{B} \cdot \mathbf{dl} \tag{3.50}$$

The term i is the conductive "regular" current. The term $\epsilon_0 \frac{d}{dt} \Phi_E$, where ϵ_0 is the permittivity of free space and has units of farads per meter $= A^2 s^4 kg$ m^3, also has the dimensions of a current and is termed "displacement" current. Instead of current i we will now speak of the current *density* vector $\mathbf{J}$ (per unit of surface and perpendicular to the surface) so that $\int_S \mathbf{J} \cdot \mathbf{ds} = i$. We can also also use the definition of the electric flux (analogous to Eq. 3.46) and write the **electric displacement vector** as $\mathbf{D} = \epsilon_0 \mathbf{E}$. Then, again using Stokes' Law (Eq. 3.45) and defining $\mathbf{B} = \mu_0 \mathbf{H}$, we can write Maxwell's third equation:

$$\boxed{\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t} \mathbf{D}} \tag{3.51}$$

4. Electric flux in terms of charge density

Remember how we obtained the flux of the gravity field in terms of the mass density. In contrast, the flux of the magnetic field was for a closed surface enclosing a dipole. For a closed surface enclosing a charge distribution, the flux through that surface will be related to the electrical charge density contained in the volume! This is a manifestation of the potential (rather than solenoidal) nature of the electric field.

We write

$$\oint_S \mathbf{E} \cdot \mathbf{dS} = \frac{q}{\epsilon_0}, \tag{3.52}$$

which, with the help of Gauss's theorem (Eq. 3.19) transforms easily to Maxwell's fourth law:

$$\boxed{\boldsymbol{\nabla} \cdot \mathbf{D} = \rho_E} \tag{3.53}$$

It's interesting to note that, in the absence of conduction or displacement current, the magnetic field is both irrotational and solenoidal (divergence-free): $\boldsymbol{\nabla} \times \mathbf{B} = 0$ and $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$. In that case, there is actually a theorem that says that $\mathbf{B}$ should be harmonic, satisfying $\nabla^2 \mathbf{B} = 0$. Hence, the Maxwell equations imply Laplace's equation: they are more general.

Ohm's Law of Conduction

A last important law is due to **Ohm**: it describes the conduction of current in an electromagnetic field. Experimentally, it had been verified that a force called the Lorentz force was exerted on a charge moving in an electric and magnetic field, according to:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{3.54}$$

This can be transformed into **Ohm's law** which is obeyed by all materials for which the current depends linearly upon the applied potential difference. Here σ is the conductivity, in 1/(Ohm m).

$$\boxed{\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}).} \tag{3.55}$$

Intermezzo 3.4 Scalar potential for the magnetic field

For a formal derivation of the relationship between the field and the potential we have to consider two of **Maxwell's Equations**

$$\nabla \times \mathbf{H} = \mathbf{J} + \partial_t \mathbf{D} \tag{3.56}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3.57}$$

where $\mathbf{H}$ is the magnetic field, $\mathbf{B}$, the induction, $\mathbf{J}$ the electric current density and $\partial_t \mathbf{D}$ the electric displacement current density. We will use this in the discussion of the geodynamo, but for the study of the magnetic field outside the core we make the following approximations. Ignoring electromagnetic disturbances such as lightning, and neglecting the conductivity of Earth's mantle, the region outside the Earth's core (and in the atmosphere up to about 50 km) is often considered an electromagnetic vacuum, with $\mathbf{J} = 0$ and $\partial_t \mathbf{D} = 0$, so that the magnetic field is rotation free ($\nabla \times \mathbf{H} = 0$). This means that $\mathbf{H}$ is a conservative field in the region of interest and that a scalar potential exists of which $\mathbf{H}$ is the (negative) gradient (but watch out for normalization constants — we'll actually define the potential starting from the magnetic induction $\mathbf{B}$ rather than from the field $\mathbf{H}$ by saying that $\mathbf{B} = -\boldsymbol{\nabla} V_m$), where $\mathbf{B} = \mu_0 \mathbf{H}$. With (3.57) it follows that such a potential potential must satisfy Laplace's equation ($\nabla^2 V_m = 0$) so that we can use spherical harmonics to describe the potential and that we can use up- and downward continuation to study the behavior of the field at different positions r from Earth's center.

The Magnetic Induction Equation

In geomagnetism an important simplification is usually made, known as the **magnetohydrodynamic (MHD) approximation**: electrons move according to Ohm's law (steady state), which means that $\partial_t \mathbf{D} = 0$. Now,

$$\boldsymbol{\nabla} \times \mathbf{H} = \sigma(\mathbf{E} + \mu_0 \mathbf{v} \times \mathbf{H}) \tag{3.58}$$

We apply the rotation operator to both sides and use the vector rule $\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times () = \boldsymbol{\nabla}\boldsymbol{\nabla} \cdot () - \nabla^2()$. With the help of this and the Maxwell equations, Eq. 3.58 can be rewritten as:

$$\boxed{\frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{H}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{H}} \tag{3.59}$$

This rephrasing of Maxwell's equations is probably one of the most important equations in dynamo theory, the **magnetic induction equation**. We can recognize identify the two terms on the right hand side as related to flow (advection) and one due to diffusion. In other words, the temporal change of the magnetic field is due to the inflow of new material, which induces new field, plus the variation of the field when it's left to decay by Ohmic decay.

It is interesting to discuss the two end-member cases corresponding to this equation, when either of the two terms goes to zero, that is.

1. Infinite conductivity: the frozen flux

   Suppose that either the flow is very fast (large $\mathbf{v}$) or that the conductivity σ is very large (or both) so that the advection term dominates in eq. (3.59).

   $$\partial_t \mathbf{H} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{H}) \tag{3.60}$$

   It is important to realize that $\mathbf{H}$ and $\mathbf{v}$ are so-called *Eulerian* variables: they specify the magnetic and velocity fields at fixed points in space: $\mathbf{H} = \mathbf{H}(\mathbf{r}, t)$ and $\mathbf{v}(\mathbf{r}, t)$. The partial derivative is not connected to a physical body. Now let's consider any area S bounded by a line C. The surface moves about with the velocity field $\mathbf{v}$. Consider the flux integrals of both sides of Eq. 3.60:

   $$\int_S \frac{\partial}{\partial t} \mathbf{H} \cdot \mathbf{n}\, dS = \int_S \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{H}) \cdot \mathbf{n}\, dS \tag{3.61}$$

   Using Stokes' theorem and the non-commutativity of the vector product, we obtain:

   $$\int_S \frac{\partial}{\partial t} \mathbf{H} \cdot \mathbf{n}\, dS + \int_C \mathbf{H} \cdot (\mathbf{v} \times \mathbf{dl}) = 0 \tag{3.62}$$

   Using a relationship known as Reynold's theorem, we can transform Eq. 3.62 into:

   $$\frac{d}{dt} \int_S \mathbf{H} \cdot \mathbf{n}\, dS = 0 \tag{3.63}$$

   This equation is called the **frozen-flux equation**. For any surface moving through a highly conductive fluid, the magnetix flux Φ_B always stays constant. Note that the derivative is a material derivative: it describes the variation of the flux through a *moving* surface *while it is moving*! The field lines do not move with respect to the flowing material: there is **no change** in the electromagnetic field within a perfect conductor. This is one of the fundamental approximations used to make problems in dynamo theory tractable and it underlies many computational and theoretical developments in geomagnetism. While it simplifies the very complex magneto-hydrodynamic theory, it is now known that it is probably not correct. In particular, if one wants to describe effects on a somewhat *longer time scale*, say longer than several tens of years, *one has to account for diffusion*. However, for the description of relatively fast processes the application of the "frozen-flux" approximation is appropriate.

2. No flow: diffusion (decay) of the field

Suppose that either there is no flow ($\mathbf{v} = \mathbf{0}$) or that the conductivity σ is very low. In both cases the diffusion term in eq. (3.59) ($(\mu_0\sigma)^{-1}\nabla^2\mathbf{H}$) will control the temporal variation in $\mathbf{H}$. Effectively, eq. (3.59) can be rewritten as the (vector) **diffusion equation**

$$\partial_t\mathbf{H} = (\mu_0\sigma)^{-1}\nabla^2\mathbf{H} \tag{3.64}$$

which means that H (=$|\mathbf{H}|$) decays exponentially with time at a rate $(\mu_0\sigma)^{-1} = \tau^{-1}$, where τ is the decay time of the field. The decay time τ increases with conductivity σ, but unless we consider a superconductor, the field will decay. For the earth, this case would represent the situation that the main field is due to some primordial field and that no core flow is involved. For realistic numbers, the geomagnetic field would cease to exist after several tens of thousands of years.

This is a very important conclusion, since it means that the magnetic field has to be ***sustained!*** *because otherwise it dies out relatively quickly (on the geological time scale). This is one of the primary requirements of a* ***geodynamo****: it has to sustain itself! (by means of scenario 2)*

The diffusion equation also shows that the depth to which the ambient field can penetrate into conducting material is a function of frequency. This is an important concept if one wants to use fluctuating fields to constrain the conductivity or if one studies the propagation of changes in core field through the conducting mantle and crust.

Consider a magnetic field that varies over time with a certain frequency ω (in practice we would use a Fourier analysis to look at the different frequencies), diffusing into a half-space with constant conductivity σ. It is straightforward to show that a solution of the vector diffusion equation is

$$\mathbf{H} = \mathbf{H}_0 e^{-\frac{z}{\delta}} e^{i\left(\omega t - \frac{z}{\delta}\right)} \tag{3.65}$$

with

$$\delta = \left(\frac{2}{\omega\sigma\mu_0}\right)^{\frac{1}{2}} \tag{3.66}$$

the **skin depth**, the depth at which the amplitude of the field has decreased to 1/e of the original value. The skin depth is large for low frequency signals and/or low conductivity of the half-space. Rapidly fluctuating fields do not penetrate into the material. (I gave the example of swimming in still pond: on a winter day you would - probably - not consider to go swimming even on a nice day with a day time temp of, say, 20°C, whereas you might on a summer day with the same temperature. The point is that the short period fluctuations controled by night-day cycles do not penetrate deep into the water; the temperature of the water that makes you decide whether or not to go for a swim depend more on the long period variations due to the changing seasons.) The rapid fluctuations of the external field are used to study σ in the upper mantle (z $<$ 1000 km) and the core field is used to study σ in the lower mantle.

Geodynamo

So we have a large volume of highly conducting liquid (metallic iron) that moves rapidly in the Earth's outer core. The basic idea behind the geodynamo is that the rapid motion of part of the liquid in an ambient magnetic field generates a current that induces a secondary magnetic field which is largely carried along in the fluid low ("frozen flux") and which reinforces the original field. In principle, this concept can be illustrated by **Faraday's disk generator**.

Excess of the light constituent in the outer core is released at the inner core boundary by progressive freezing out of the inner core. The resulting buoyancy drives compositional convection in the outer core, and the combination of convection and rotation produces the complex motion needed for self-excited dynamo action. The rotation effectively stretches the poloidal field into toroidal field lines (the ω-effect). Most geodynamo models require a strong toroidal field, about 0.01 T (or 100 Gauss), even though this field cannot be observed at the Earth's surface. These toroidal field lines are warped up or down due to the radial convective flow (assuming "frozen flux"): as a result of the Coriolis force this results in helical motion, which, in fact, recreates a poloidal component from a toroidal one (this is know as the α-effect). The rotation controls the motion in such a way that the dipole field is stronger than any other poloidal component and, averaged over a sufficient time, coincides with the Earth's rotation axis.

Intermezzo 3.5 SPHEROIDAL, TOROIDAL, POLOIDAL

The following will help understanding the terminology. An arbitrary vector field **T** on the surface of a unit sphere can be represented in terms of three scalar fields U, V and W as follows:

$$\mathbf{u} = \hat{\mathbf{r}}U + \boldsymbol{\nabla}_1 V - \hat{\mathbf{r}} \times \boldsymbol{\nabla}_1 W, \tag{3.67}$$

where the operator $\boldsymbol{\nabla}_1$ is the dimensionless surface gradient on the unit sphere, formed by taking the projection of the "real" gradient onto the plane tangent to the surface.

A vector field of the form $\hat{\mathbf{r}}U + \boldsymbol{\nabla}_1 V$ is said to be **spheroidal** (i.e. having both radial and tangential components) whereas one of the form $-\hat{\mathbf{r}} \times \boldsymbol{\nabla}_1 W$ is said to be **toroidal**. A toroidal field is purely tangential. It resembles a torus which is purely circular about the z-axis of a sphere (i.e., follows lines of latitude). The curl of a toroidal field (its rotation), is, by definition, **poloidal**. A poloidal field resembles a magnetic multipole which has a component along the z-axis of a sphere and continues along lines of longitude.

3.10 Crustal field and rock magnetism

From spherical harmonic analysis it is clear that short wavelength magnetic anomalies must have a shallow origin. Within the outer few km of the Earth are rocks with minerals that have ferromagnetic properties. The study of these rocks and their magnetization has two important applications in geophysics:

- These rocks distort the magnetic field due to the core and the induced local field can be used to investigate crustal structures. (Note that we have seen that the downward continuation of the crustal field obscures the higher degree components of the core field at the CMB).
- Some of these rocks exhibit permanent magnetization (= remanent magnetization with very long, i.e. > 108 yr, relaxation times) and effectively provide an invaluable record of the history of the magnetic field and the relative motion of tectonic units → **paleomagnetism**.

Before discussing paleomagnetism we need to know some of the basics of rock **magnetism** in order to study the local field. In particular:

- What are the possible sources of magnetization and what are the conditions that result in a strong, stable magnetization?
- What are the important rock types and minerals?
- What are the essential aspects of sample preparation before any accurate paleomagnetic measurements can be done? (I will discuss this only briefly.)

The physics of the magnetization of an assemblage of rocks is not simple. Traditionally, the French have played a major role in research of magnetism, and L. Néel was awarded a Nobel prize for his pioneering theoretical work on rock magnetism.

3.11 Magnetization

Strength of magnetization: permeability and susceptibility

Let's start from one of Maxwell's equations. Remember that

$$\nabla \times \mathbf{H} = \mathbf{J}_{\text{mac}} + \frac{\partial}{\partial t}\mathbf{D}, \tag{3.68}$$

where $\mathbf{H}$ is the magnetic field strength and $\mathbf{J}_{\text{mac}}$ and $\mathbf{D}$ are the macroscopic current and displacement current densities, respectively. Let's forget about the displacement current density for a moment (like in the magnetohydrodynamic assumption). The displacement current is usually spread out over large areas and hence $\mathbf{D}$ and also $\frac{\partial}{\partial t}\mathbf{D}$ can be neglected.

In a vacuum, the only current one needs to worry about is the macroscopic current. In real materials (such as rocks), the atoms and molecules that make up the substance are like little magnetic dipoles with a dipole moment **m**: a hydrogen atom, for instance is little more than the primitive current loop we started the definition of the magnetic dipole moment with. The **magnetization** of a substance is defined as the volume density of all these little dipole vectors:

$$\mathbf{M} = \lim_{\Delta V \to 0} \frac{\sum_{\Delta V} \mathbf{m}}{\Delta V}. \tag{3.69}$$

One can express the density of microscopic molecular currents $\mathbf{J}_{\mathrm{mol}}$ through this magnetization vector as follows:

$$\boldsymbol{\nabla} \times \mathbf{M} = \mathbf{J}_{\mathrm{mol}}. \tag{3.70}$$

Hence we can rewrite Eq. 3.68 with both the macroscopic and microscopic current densities as follows (using $\mathbf{B} = \mu_0 \mathbf{H}$):

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J}_{\mathrm{mac}} + \mu_0 \boldsymbol{\nabla} \times \mathbf{M}, \tag{3.71}$$

which leads to

$$\boldsymbol{\nabla} \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{J}_{\mathrm{mac}}. \tag{3.72}$$

Comparison with Eq. 3.68 shows that the field strength in a material is given with respect to the macroscopic currents as:

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}, \tag{3.73}$$

It is customary to associate the magnetization not with the magnetic induction, but with the field strength. It is assumed that this relationship exists:

$$\mathbf{M} = \mathbf{X} \cdot \mathbf{H}. \tag{3.74}$$

The Greek capital letter chi (**X**) is used for the magnetic susceptibility tensor. It relates the three components of the internal magnetization to the applied field, hence it is a second-order tensor (a matrix). This is usually a complex function, as **X** may depend on anything — temperature, grain size, **H**, strain and so on. It can be negative, too. If **X** is a tensor, then **H** and **M** need not be collinear. Usually, the assumption of magnetic isotropy is made, and Eq. 3.74 is approximated by a scalar relationship:

$$\mathbf{M} = \chi \mathbf{H}. \tag{3.75}$$

Now **H** and **M** are collinear, but the magnitudes and sense are regulated by the value and sign of χ. Because both **H** and **M** have dimensions of the field, χ is a dimensionless constant: the **magnetic susceptility**.

So Eq. 3.73 reduces to

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0(1+\chi)} = \frac{\mathbf{B}}{\mu_0\mu}. \tag{3.76}$$

A quantity μ was defined which is called the **relative permeability** of the substance. In vacuum, $\mu = 1$, $\chi = 0$ and $\mathbf{M} = 0$ — no bound charges exist in free space.

When a magnetic body is placed in an external magnetic field $\mathbf{H}$ the density of the field lines *inside* the body depends on the strength of $\mathbf{H}$ and on the magnetization $\mathbf{M}$ induced by $\mathbf{H}$. Thus, the magnetic susceptibility χ indicates the ease with which a magnetic body can be magnetized in an external field.

Types of magnetization

The magnetization of a mineral is controlled by intrinsic (i.e., material dependent) magnetic moments of electrons spinning about their axes (*spin dipole moments*) or the motion of electrons in their orbits about the atomic nuclei (*orbital dipole moments*). There are several types of spin interactions that give rise to different magnetic effects. The following is a brief summary.

Remanent or Induced? (Königsberger ratio)

When we talk about magnetization, we can broadly identify two types:

1. Induced magnetization, $\mathbf{M}_I$, which occurs only if an ambient field $\mathbf{H}$ is present and decays rapidly if this external field is removed. This induced field is very important in ore exploration.

2. Remanent magnetization, $\mathbf{M}_R$, which is the part of initial magnetization that remains after the external field disappears or changes in character. $\mathbf{M}_R$ forms the record of the past field and is the type of magnetization that makes paleomagnetism work.

In natural rocks, the ratio between the two is known as the **Königsberger ratio** Q

$$Q = \frac{|\mathbf{M}_R|}{|\mathbf{M}_I|} \tag{3.77}$$

Rocks with high Q tend to be magnetically stable and are good recorders of the ancient geomagnetic field. (I just remark at this stage that the strength of $|\mathbf{M}_R|$ does not only depend on the composition (more precisely the type of magnetic minerals) but also on the grain size, and whether the magnetic particles consist of single or multiple grains). Rock types with high Q are most of the mafic rocks, such as basalt and gabbro, and also granite. Limestone, for instance, typically has a very small Königsberger ratio.

Examples of *induced magnetization* are **diamagnetism** and **paramagnetism**. The fields are weak and decay rapidly when the external field is removed (by means of diffusion).

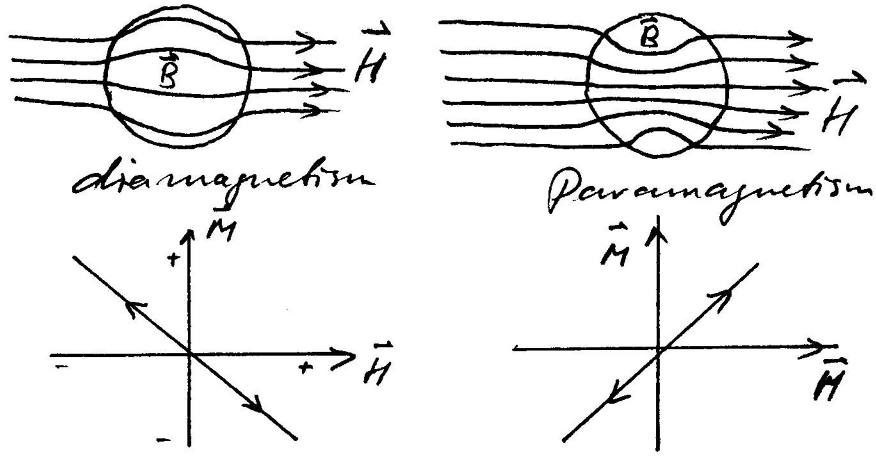
DIAMAGNETISM

All materials tend to repel the magnetic field lines of force so that the density of field lines within the body ($\sim |B_i|$) is smaller than outside ($|B_o|$):

$$B_i = \mu_0(1+\chi)|\mathbf{H}| < B_o = \mu_0|\mathbf{H}| \Rightarrow (1+\chi) < 1 \Rightarrow \chi < 0 \quad (3.78)$$

This effect, which is controlled by *orbital dipole moments* can be explained by **Lenz's Law**, which states that the field produced by a conductor moving into a magnetic field tends to oppose the external field. Here, the induced field produced by the electron orbits tends to oppose the external field. Even though all materials are diamagnetic, in some the effect is completely overshadowed by much stronger effects such as ferrimagnetism.

Diamagnetism is a weak effect $|\chi| < 10^{-6}$ so that $B_{\text{induced}} \approx B_{\text{free space}}$.



Owing to diffusion, the original state is quickly restored when the external field **H** would be removed unless the conductivity σ is very large (superconductors) so that the diffusion (which scales as $1/\sigma$, see **magnetic induction equation**) can be neglected.

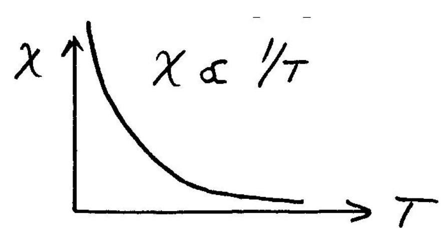
PARAMAGNETISM

Intrinsic paramagnetism is relevant for only a small class of materials, but most magnetic minerals are paramagnetic above the curie temperature (see below). Paramagnetic minerals tend to concentrate the lines of force so that the induced internal field **B** is larger than the external field **H**.

$$B_i = \mu_0(1+\chi)|\mathbf{H}| > B_o = \mu_0|\mathbf{H}| \Rightarrow (1+\chi) > 1 \Rightarrow \chi > 0 \quad (3.79)$$

Electrons spin in opposite directions giving rise to *spin dipole moments* in opposite directions. The spins are arranged in pairs so that the net effect of the magnetic dipoles is zero. However, if the number of orbital electrons is odd there is a small net magnetic moment that can be aligned with the external field. This is known as paramagnetism. Atoms with an even number of electrons tend to be diamagnetic and those with an odd number tend to be both paramagnetic and diamagnetic. Like diamagnetism, the effect is small: $|\chi| < 10^{-4}$ so that $B_{\text{induced}} \approx B_{\text{free space}}$.

The susceptibilities of para- and dia- magnetic minerals are virtually independent of the ambient field (which gives a linear behavior in the above diagrams) but paramagnetism is strongly temperature dependent because thermal fluctuations tend to disorient the alignment with the applied field (competition between thermal energy that tends to destroy alingment and magnetic energy that tends to create it). The **Curie Law** states that paramagnetic susceptibility is inversely proportional to the absolute temperature.



Ferro- and ferri- magnetism

This type of magnetism can result in a remanent magnetization which remains even after the ambient field **H** changes at a later time. The susceptibility of ferro- and ferri- magnetic minerals depends strongly on the applied external field, but in general $\chi \gg 0$. In ferromagnetic minerals the electron spins line up spontaneously. Perfect line-up occurs only in a few metals, such as metallic iron in the Earth's core, and alloys; in other minerals, such as magnetite, the line-up is not complete which results in **ferrimagnetism**.

Which minerals are important?

The most important rock-forming minerals with magnetic properties are

magnetite $Fe_3O_4 = Fe_2^{3+}Fe^{2+}O_4$
hematite $Fe_2O_3 = Fe_2^{3+}O_3$ (can be formed from magnetite by oxidation)
ilmenite $FeTiO_3$

The magnetic properties of these magnetic minerals and the continuous series of solid solutions between them can be conveniently displayed in **ternary diagram** of the $FeO - TiO_2 - Fe_2O_3$ system:

$$Fe_3O_4 = FeO + Fe_2O_3;$$
$$FeTiO_3 = FeO + TiO_2;$$
$$\text{ulvöspinel } Fe_2TiO_4 = FeO + FeTiO_3$$

The titanomagnetite solid solution series A from magnetite→ulvöspinel consists of strongly magnetic cubic oxides. The titanohematite series B (from

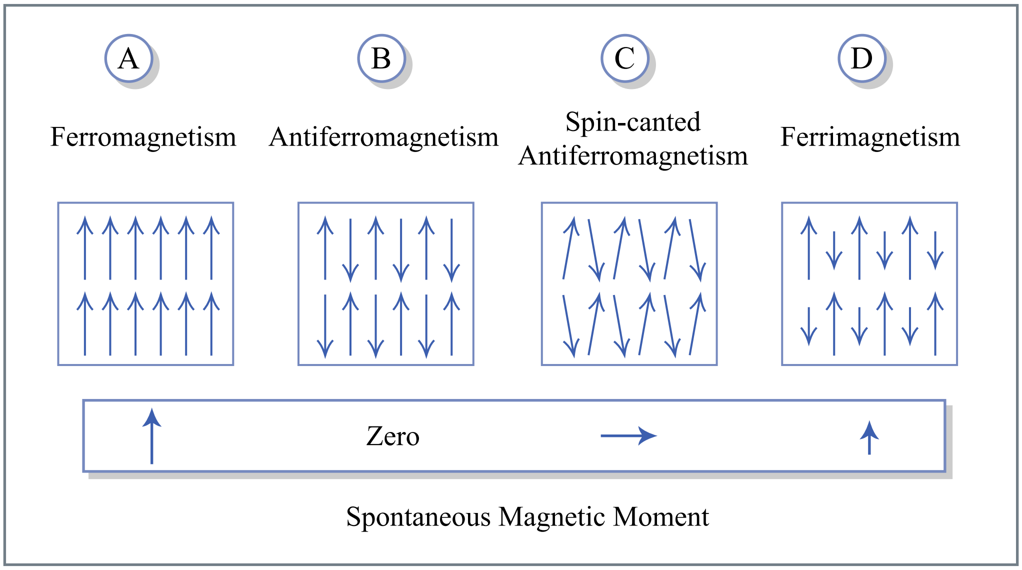


Figure by MIT OCW.

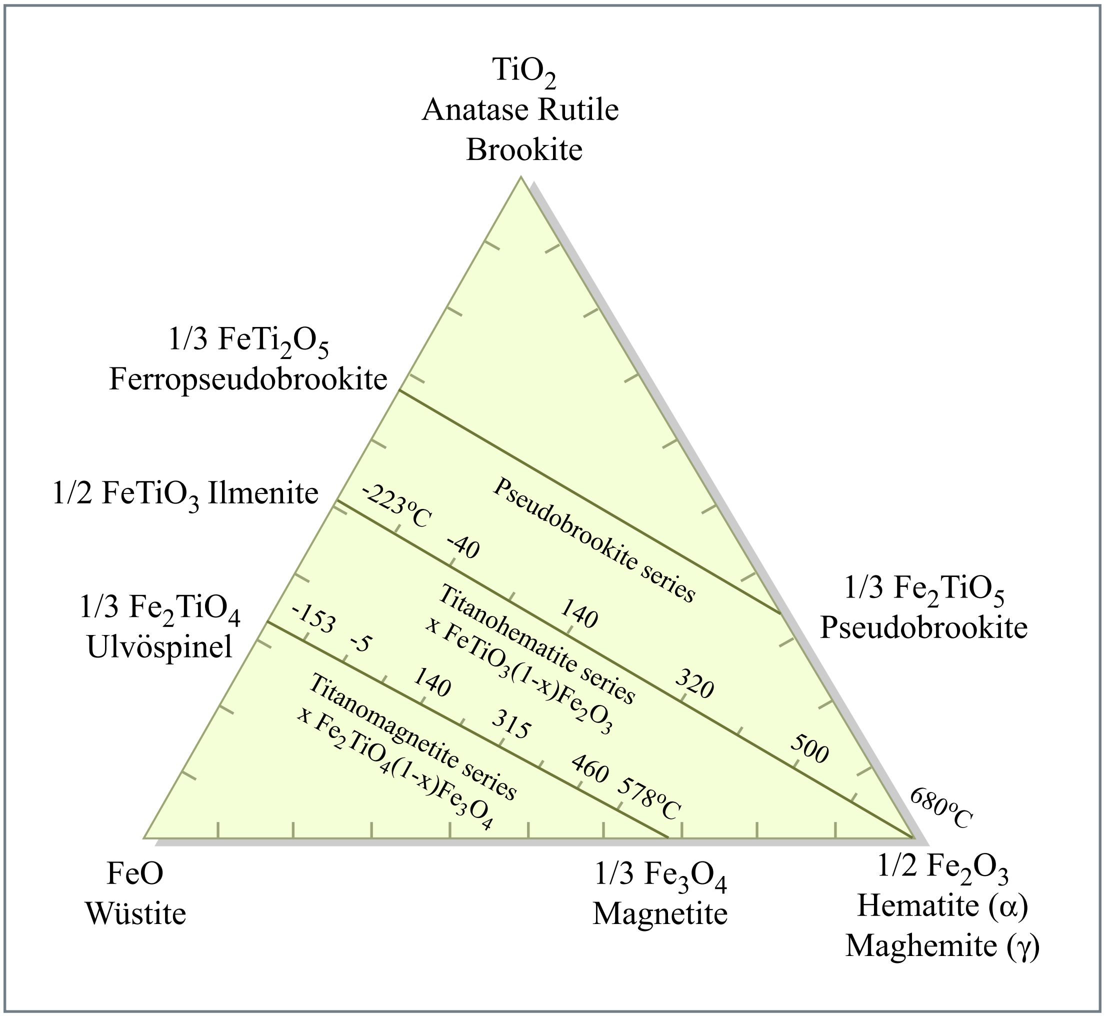
ilmenite→hematite) consists of weakly magnetic rhombohedral minerals. Some important properties of the minerals in the ternary diagram are: Curie temperatures decrease from right to left; generally, the susceptibility increases from right to left. Even though, for instance, magnetite has a lower susceptibility than, say ulvöspinel , it is more suitable for paleomagnetic studies, because uit maintains its remanence up to much higher temperatures.

Intermezzo 3.6 MAGNETIC HYSTERESIS

When a ferromagnetic body (or an assemblage of asymmetric single grains) is placed in an ambient field the magnetization will initially increase linearly with the strength of the ambient field. This linear part of the magnetization is reversible and the sample will return to its original state when the ambient field is removed. The initial susceptibility is then defined as $\chi = (dM/dH)|_{t=0}$. If the external field increases in strength saturation will occur, for instance because no more magnetic domains within a rock can be aligned, and the magnetization $\mathbf{M}$ rotates into the direction of the ambient field $\mathbf{H}$. This part of the process in non-reversible: if the ambient field is removed, there may be some relaxation (the reversible part) but a remanent magnetization $\mathbf{M}_R$ will remain. If the external field would change directions a hysteresis the magnetization would follow a hysteresis curve. The strength of the magnetization depends on grain size and temperature: the larger the single grains and the lower the temperature, the wider the hysteresis curve and the stronger ("harder") the magnetization. See Figure 3.8.

Magnetic domains and influence of grain size

In bulk magnetic material, say, magnetite, the regions of spontaneous magnetization (magnetic domains) are arranged in patterns to form paths of magnetic



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flux closure with neighboring domains, so that no net effect is observed. This is known as the *demagnetized* state. The size of the single domains in such agglomerates depends on susceptibility and are larger in hematite than in, say, the titanomagnetites (see below). When placed in an ambient field **H** the boundaries between the domains may migrate in such a way as to enlarge the domains that are favorably oriented with respect to the external field, and this may cause magnetic remanence. The process of cell boundary migration is (energetically speaking) an easier process than the realignment of the direction of magnetization and, as a consequence, large *multidomain grains* are magnetically "soft" (i.e. not stable over long periods of time and sensitive to later changes in the orientation and strength of the ambient field).

In contrast, *single domain grains* have the same direction of spontaneous magnetization throughout, and if they are not in contact with neighboring grains they cannot form closed loops and they can more easily be magnetized to saturation.

At high temperatures (or for very small single grains) the kinetic energy, or the thermal agitation, of the system can be too large for any kind of cooperative

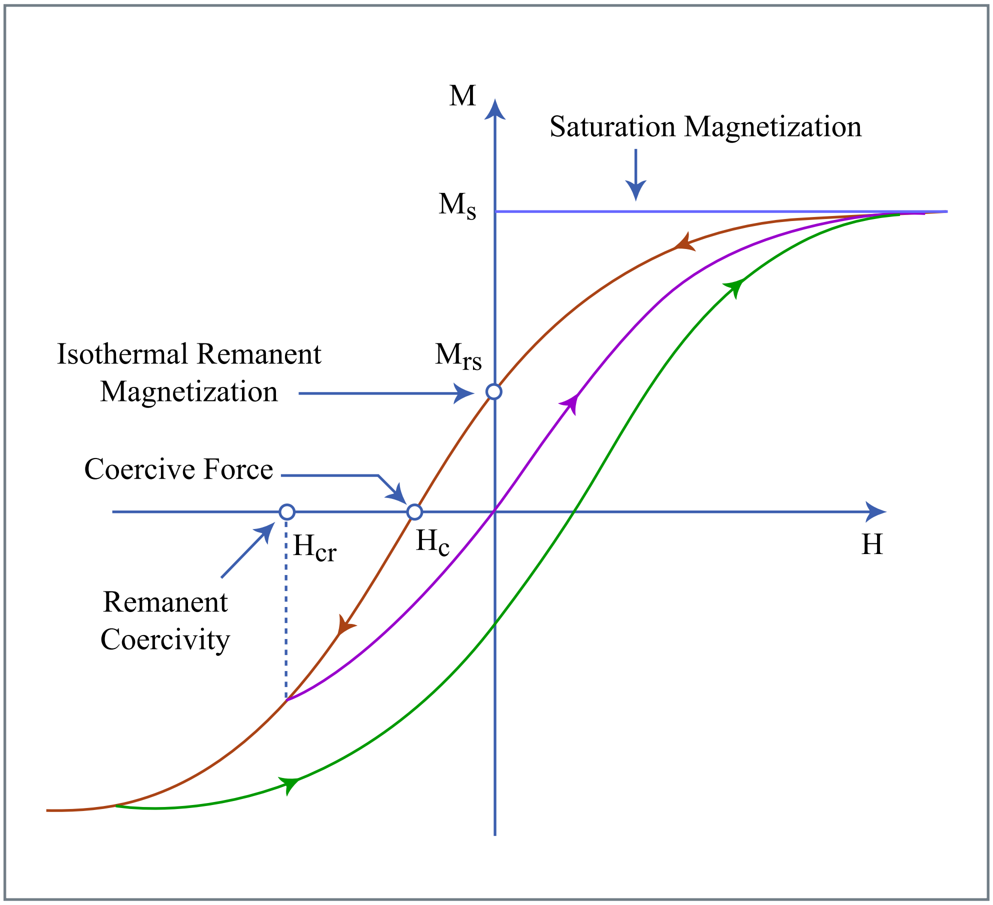


Figure by MIT OCW.

Figure 3.8: Magnetic hysteresis.

process to be effective and the electron spins are not aligned but cancel out. In this state the rocks are *paramagnetic*, and magnetization can only be induced by an external field. If the assemblage is given a magnetization at time t_0 the (induced) magnetization decays a e^{-t}/τ_0 where τ_0, the relaxation time, varies directly as grain volume (d^3) and inversely as temperature T. In other words, the relaxation time becomes shorter exponentially with increasing temperature. When the relaxation time τ_0 is very large (say 10^8 year) the remanence is said to be permanent. The dependence of the relaxation time on temperature and the grain size of single grain particles can be illustrated schematically:

When the rock sample cools, the relaxation time and the susceptibility (Curie's Law) increase (the width of the hysteresis curve increases) and there comes a point, at $T = T_c$, the Curie temperature (after Pierre Curie), at which the thermal agitation is no longer large enough to prevent alignment of the magnetic moments. At this point the assemblage of grains acquires a magnetization in the direction of the ambient field, which upon further cooling becomes frozen in. This is known as **ThermoRemanent Magnetization** (TRM).

The transition from a state of paramagnetism to TRM is not instantaneous and typically occurs over a certain temperature range. The process of acquiring TRM consists of a series of partial thermo remanent magnetizations, each

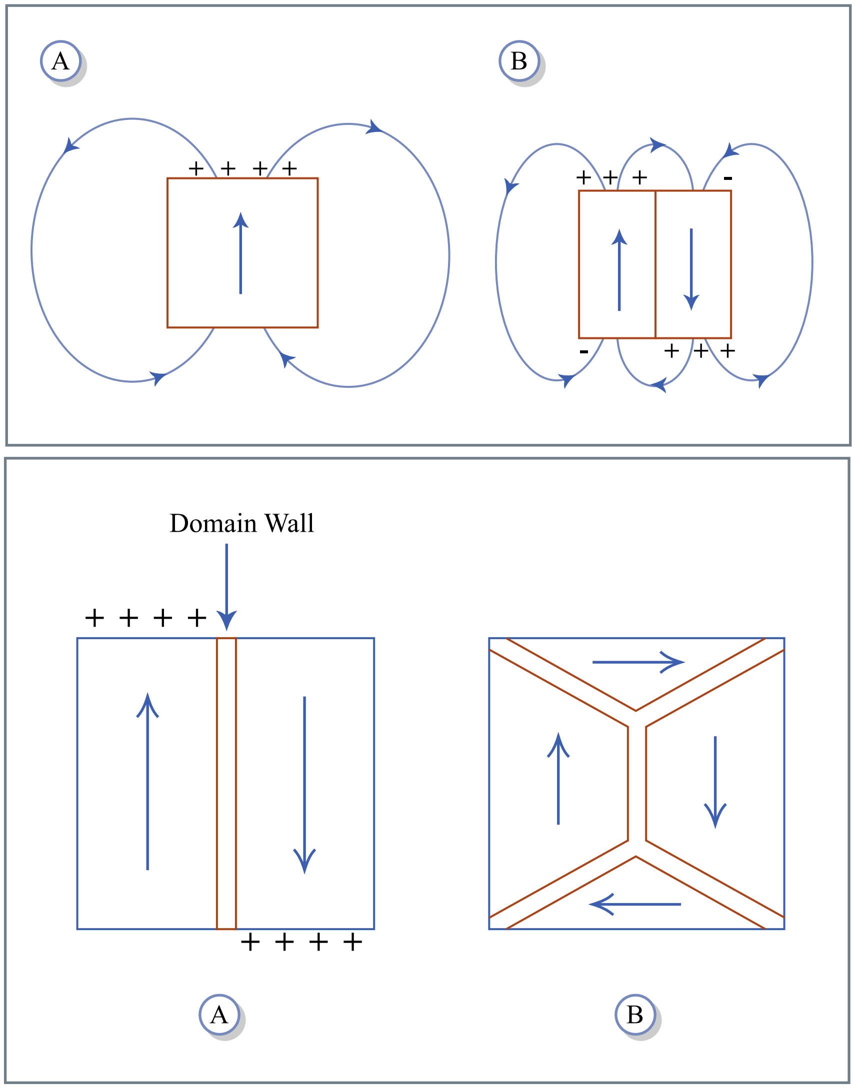


Figure by MIT OCW.

Figure 3.9: Magnetic domains in grains.

acquired in a given temperature range, say, $T_1 < T < T_2$, and each preserving a record of the ambient field in that time interval. Reheating to T_1 would not destroy that part of the remanent magnetization, but reheating to T_2 would. This, in fact, underlies the principle of **thermal cleaning**. For most rocks containing a single, magnetic constituent the partial TRM acquired at about 50° below the Curie point is dominant. If the assemblage consists of a series of minerals with different Curie points the initial remanence can easily be overprinted with secondary components reflecting the field direction at some later time. For several minerals along the solid solution lines, the ternary system described above gives values of the Curie temperature Tc above which ferro- or ferri- magnetism is not possible. In general the Curie temperature decreases from right to left, i.e., hematite to ilmenite and from magnetite to ulvöspinel. For hematite T_c=680°, for magnetite T_c=580°, and for metallic iron (core!) T_c=770°.

It is important to realize that the strongly magnetic minerals do thus not necessary result in a strong and stable (= long lasting) magnetization since their Curie temperatures can be so small that the original magnetization can easily

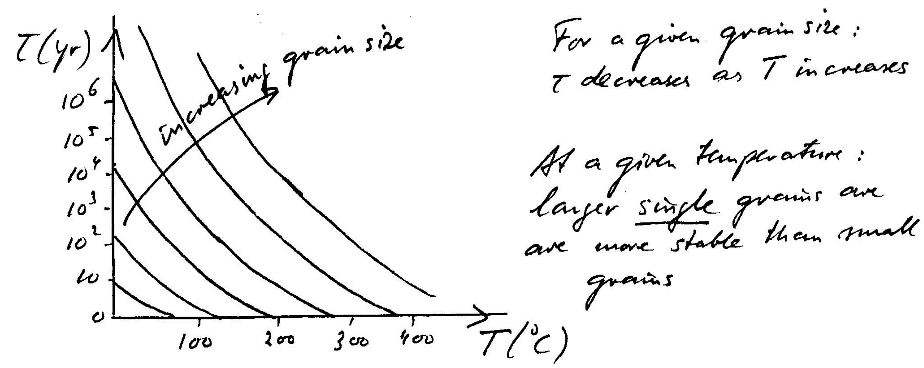


Figure 3.10: Temperature dependence of Curie temperature.

be altered during later thermal events (for some magnetic minerals this can happen at room temperature). Conversely, the weakly magnetic minerals may, in fact, produce very stable magnetization. Weak and strong refers to the value (small, large) of the magnetic susceptibility; stability refers to the time scales over which the magnetization can last, and this is more a function of grain size and the composition-dependent Curie and blocking temperatures.

3.12 Other types of magnetization

TRM – ThermoRemanent Magnetization

In terms of later overprints, it is good to realize that the direction of TRM can be reset if the temperature is raised to above the relevant Curie temperature during some later thermal event. Upon cooling a new TRM is frozen in. Often, however, reheating will also reset the dating clocks since the closure temperatures for minerals typically used for radio-isotope dating are often lower than the Curie temperatures for the iron oxides. (This is generally true for the minerals involved in K–Ar and Rb-Sr dating – hornblende (530°), muscovite (350°), biotite (280°), apatite (∼350°), but U-Pb dating is more borderline with the closure temperatures for several important minerals exceeding 600°, for instance Zircon at 750°). As a consequence, the data can still be used for paleomagnetism; the magnetization just relates to a later thermal event.

DRM – Detrital Remanent Magnetization

When igneous rock is eroded and the magnetic constituent is deposited in sufficiently still water the magnetic grains that carry TRM from previous events may become aligned in the ambient field. Since this type of magnetization is in fact based on previous TRM it can be a stable and 'strong' magnetization which can be useful, provided that the time of deposition can be determined accurately. There are, however, several complications, for instance, the change in any lineation direction due to compaction may result in an underestimation if the inclination I and this would underestimate the paleolatitudes.

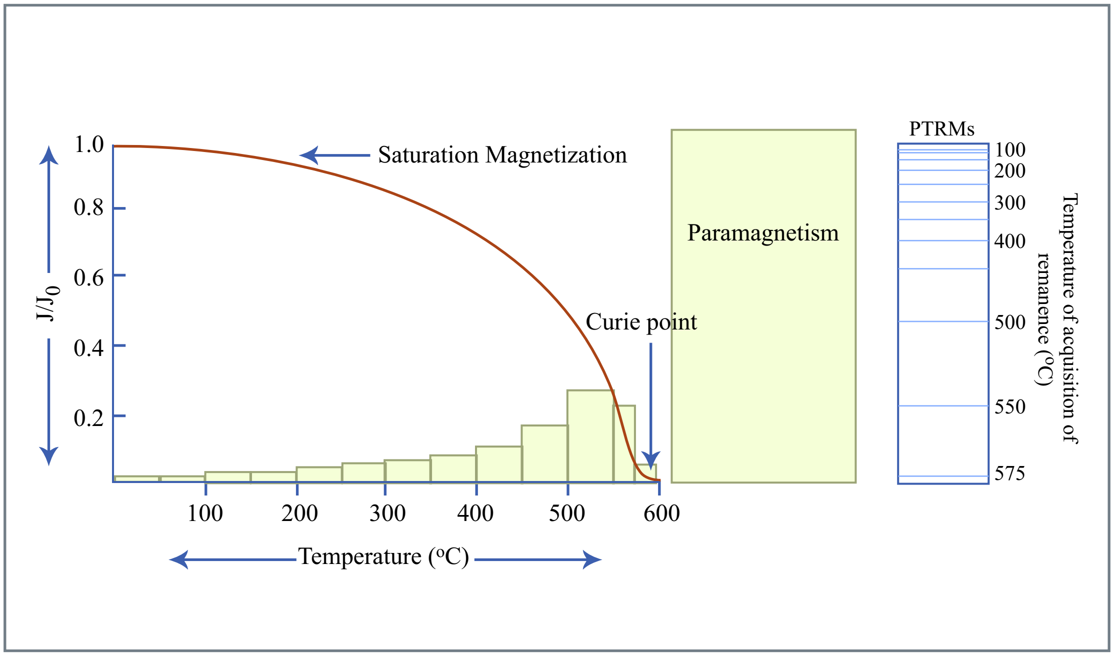


Figure by MIT OCW.

Figure 3.11: Saturation magnetization.

CRM – Chemical Remanent Magnetization

Some ferro- or ferri- magnetic minerals may be formed long after the rocks first cooled below the Curie temperature. For instance, magnetite may oxidize to hematite which then can settle as cement in the matrix between the grains. Conversely, magnetite may be formed from hematite in a reducing environment. Upon the chemical reactions and renewed deposition the minerals are re-magnetized in the then ambient field. In these examples the CRM is very strong. Since the acquisition of CRM does not involve any reheating up to the Curie temperature (which is typically above the blocking temperature for radioisotope dating!) it is not possible to put a time tag on and it can not be used for paleomagnetism.

IRM = Isothermal Remanent Magnetization

Exposure of a magnetized sample to very strong field even without increasing the temperature (for long enough time) can result in the re-arrangement of part of the initial TRM. IRM is characterized by a very narrow hysteresis curve so that the magnetization is relatively “weak”. An important source for IRM are lightning strikes.

VRM – Viscous Remanent Magnetization

Results from the slight reorganization of the magnetic moment in some grains if the grain is exposed to an ambient field for a very long time. The external field can ”diffuse” into the sample, in particular if both μ and χ are large.

3.13 Magnetic cleaning procedures

For paleomagnetic purposes, the overprints by the other types of magnetization are thus a nuisance; if the secondary remanence is ”soft” their effects can be removed by techniques collectively known as "magnetic cleaning". Cleaning is based on the principle that the soft components are destroyed while the strong original TRM or DRM is preserved. This can not always be guaranteed and, as a result, the **intensity** of the field after cleaning may be unreliable.

- Alternating field cleaning (a.f. cleaning). Makes use of hysteresis properties of ferro- and ferri- magnetic minerals. The strength of a particular magnetization is determined by the width of the hysteresis curve, which represents the field of opposed direction that would have to be applied to reduce the remanence to zero. The field is often referred to as the *coercive force* H_c (see hysteresis curve). In a.f. cleaning one exposes the field sample to an alternating field with diminishing strength and the sample is rotated in all directions. This process wipes out all components with a strength H_c that is less than the maximum field applied. Of course, this only works if the primary component is stronger than that! This process must be performed in a laboratory set up where one compensated for the external Earth field in order to avoid remagnitization along $\mathbf{H}_E$.

- Thermal cleaning. Makes use of Curie's law and Curie temperatures. The field sample is heated to a particular temperature to destroy magnetization of minerals with Curie temperatures smaller than that value. This is, of course, only useful if the component you're interested in has a higher Curie temperature than the temperature that is applied.

- Chemical cleaning: leaching or dissolving certain components of the rock, such a the hematite rich matrix in a sand stone.

In general, one applies progressively stronger cleaning agents to the sample until the remaining field is stable and no longer changes in direction. The intensity of the original TRM/DRM will be obscured in the process!

3.14 Paleomagnetism

A primary objective of paleomagnetism is to determine the history of the geomagnetic field (for a variety of purposes!) and what is needed is (1) "hard" (+stable) magnetization and (2) information about the time at which that magnetization was acquired. So, of the above mechanisms only TRM and DRM are useful since they have hard magnetization and the timing of acquisition of the RMs can be determined (radio-isotope dating and stratigraphy, respectively).

Field practice: orientation of the sample

One of the primary objectives is the measurement of the *inclination* and *declination.* One has to know where these angles – measured in a lab – refer to. Before the rock sample is taken from the field one, therefore, has to make sure that its orientation in space at the sample site is known exactly. The current

North direction must be marked on the sample, and also the horizontal level. If it is clear that the sample is taken from a rock setting that has been deformed one must also mark the relation between the reference horizontal and the original horizontal, or the orientation of the stratification (in sedimentary rocks). For example, if a sample is taken from marine strata that have been folded, one must measure the dip of the bedding plane. One must then also apply techniques that are well developed in structural geology to correct for the later deformation and retrieve the original inclination.

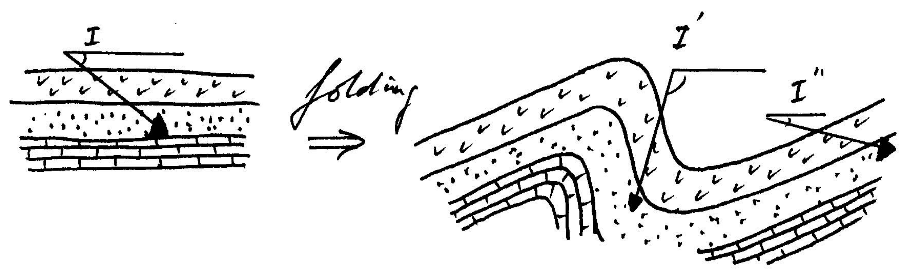


Figure 3.12: Paleomagnetic orientations.

Ideally, many samples are taken in a sedimentary or volcanic sequence so that the measurements represent, in fact, a time span of several thousands of years. In this way the effects of secular variation can be averaged out and one can apply the so called "axial dipole assumption", see below. Core drilling perpendicular to the stratification is an effective way to do that, but the original azimuth required to determine the declination may not be preserved, for instance in a core of a deep marine sediment, so that only the inclination can be used.

Presentation of inclination and declination

The angular information can be used in two ways.

- Bauer plots: these are plots (named after Bauer) of the angle of inclination vs. declination. We have used them to illustrate the secular variation for London.
- Virtual Geomagnetic Pole (VGP) plots.

  From the measured inclination I and declination D we can calculate (1) the magnetic co-latitude (i.e., the angular distance from the paleo-pole to the sampling site) and (2) the position of the paleomagnetic pole and the virtual (or apparent) geomagnetic pole (VGP). Let's see how we can do that.

From the derivation of the potential of an axial dipole we know that the magnetic co-latitude θ_m can be calculated from the basic equation for paleo-

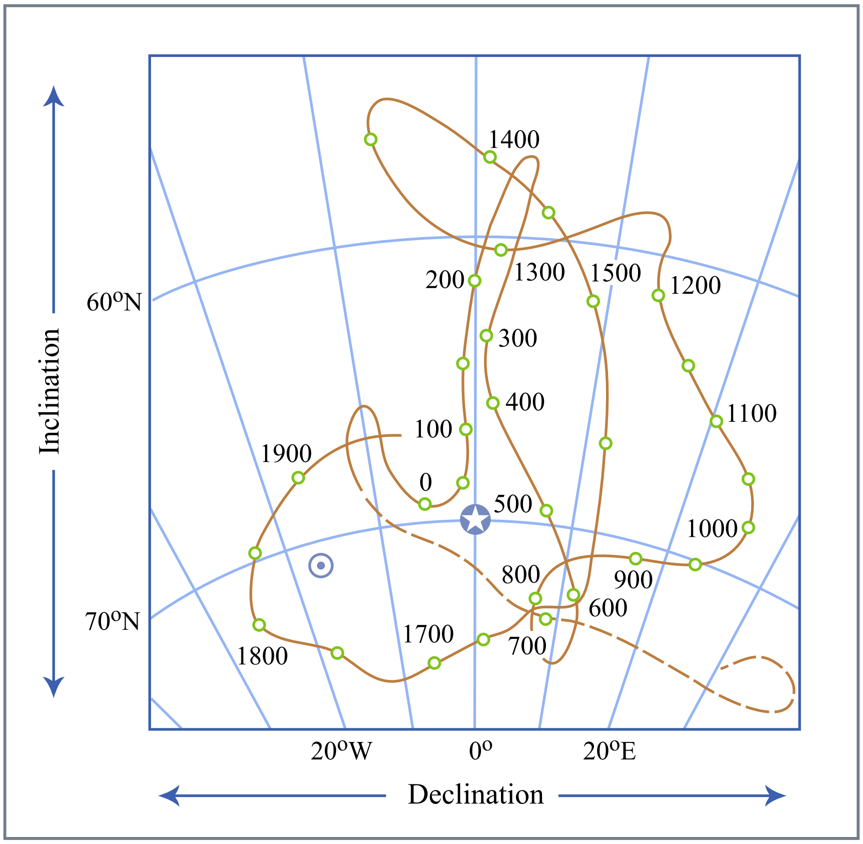


Figure by MIT OCW.

Figure 3.13: Bauer plot.

magnetism:

$$\tan I = 2\cot\theta_m = 2tan\lambda_m \tag{3.80}$$

where λ_m is the magnetic latitude ($\theta_m = 90°$ - λ_m). We can also calculate the location of the pole by measuring out the angular distance θ_m in the direction of D from North.

This can be done by means of stereographic projections, but we can also use the sine and cosine rules on a sphere (see Intermezzo).

Intermezzo 3.7 COSINE AND SINE RULES ON A SPHERE

Let a, b, and d be the angular distances between 3 points and A, B, and D the angles as indicated in Figure 3.15.
The **cosine rule** for a and A is:

$$\cos a = \cos b \cos d + \sin b \sin d \cos A \tag{3.81}$$

and the **sine rule**:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin d}{\sin D} \tag{3.82}$$

(Note that we used (d,D) instead of the (c,C) to be consistent with the use of D for declination.)

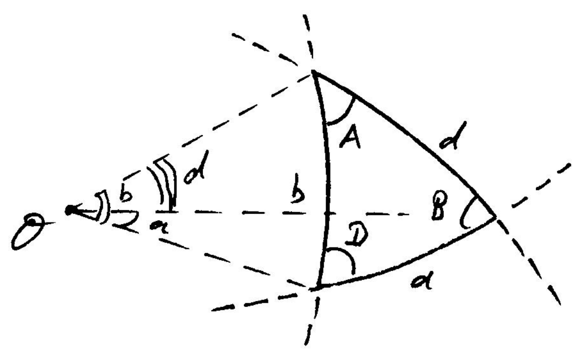


Figure 3.14: Cosine and sine rules on a sphere.

Let φ, θ, λ be the longitude, co-latitude, and latitude of a point, and subscripts p and s indicate the paleopole and the sample site, respectively (θ_m and φ_m as above). Then we can apply the cosine rule for D:

$$\cos\theta_p = \cos\theta_m \cos\theta_s + \sin\theta_m \sin\theta_s \cos D \tag{3.83}$$

and

$$\theta_p = \arccos\{\cos\theta_m \cos\theta_s + \sin\theta_m \sin\theta_s \cos D\} \tag{3.84}$$

with

$$0 < \theta < 180^\circ \tag{3.85}$$

The application of the sine rule gives the longitude of the paleopole relative to the longitude of the sample site:

$$\frac{\sin\theta_m}{\sin(\varphi_p - \varphi_s)} = \frac{\sin\theta_p}{\sin D} \tag{3.86}$$

$$\varphi_p - \varphi_s = \arcsin\left\{\frac{\sin D \sin\theta_m}{\sin\theta_p}\right\}$$

(Note that arcsin is double valued: one always has to check that one uses the correct solution).

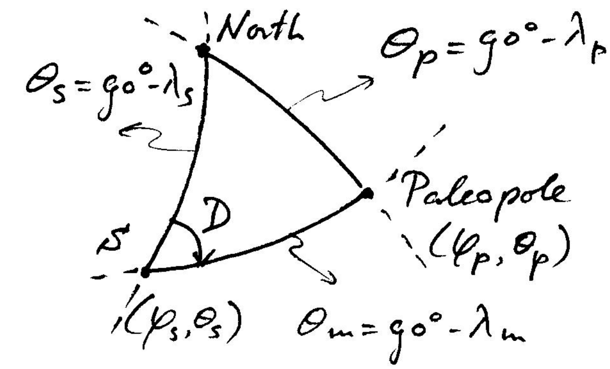


Figure 3.15: Location of paleopole.

The coordinate (φ_p,θ_p) is the apparent position of the pole for a magnetic field if the field is dipolar. The pole determined from a single site is called the **Virtual Geomagnetic Pole** (VGP). An important result from paleomagnetism is that the non-dipole field almost completely averages out over extended periods of time (10,000 yr +) and that the average of all the VGPs from rocks of about the same age cluster together about the rotation pole. The is summarized in the **Axial Dipole Assumption**: if we consider sufficiently long time intervals the average VGP represents the **paleopole** or the **paleogeographic pole**.

It was soon realized that the paleogeographic poles did not coincide with the current rotation pole and that the poles determined from samples of a different age do not necessarily overlap. This was initially interpreted as evidence that the poles must move, hence the (obsolete) name Polar Wander. We now know that the poles are fixed and that the pole paths describe the relative motion of the landmass relative to the pole. A rotation of both the pole and the sample site (continent) about a rotation pole at the present-day equator at and angle of 90° from (φ_p,θ_p) gives the situation at the time of magnetization (assuming normal direction of the field). For each separate tectonic unit, measurements from samples that are magnetized at different times determines give a series of VGPs which define a **pole path** , or **apparent polar wander** (APW) path.

Image removed due to copyright concerns.
See paleogeographic map on the book:
Stacey, F. D. Physics of the Earth. 3rd ed.
Brisbane: Brookfield Press, 1992. ISBN: 0646090917.

Figure 3.16:

Note that on VGP diagrams one plots the VGP position relative to a fixed landmass but in reality the poles are the same and the landmass moves!

The observation that the pole paths do not agree for different continents demonstrates that the continents must have moved relative to one another → **plate tectonic motion**. Conversely, the difference in pole paths can be used to reconstruct these movements, and this is one of the basic principles of plate reconstruction.

Ambiguities in determination of VGPs and in reconstructing past plate motion

The position of the VGP can thus be determined from measurements of the inclination and declination. There are, however, two important ambiguities.

1. The position of the VGP (φ_p,θ_p) is invariant for variation in longitude of the sample site. The landmass can be anywhere along a small circle at an angular distance of θ_m about the paleopole. Since many earth systems (for instance, climate) exhibit a high degree of rotational symmetry and, thus, primarily zonal variation, this ambiguity is difficult to solve.

2. The declination in the sense used above is poorly defined. In addition to rotation about the paleopole (which does not effect the calculation of the VGPs) the sample site may have been subject to rotation around a vertical axis nearby or through the land mass. In the latter case the uncertainty in

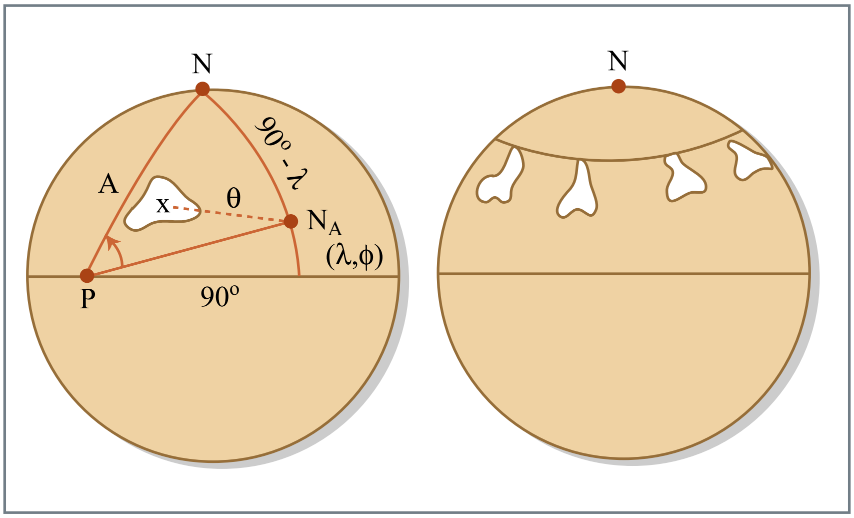


Figure by MIT OCW.

Figure 3.17: Apparent polar wander paths.

the true declination means that the VGP can – in principle – be anywhere along a small circle at an angular distance of θ_m about the sample site. The true paleopole can then be found by the intersection of many such small circles and VGP estimates. The same applies to data from drill cores since one does not know the declination.

Once the paleopole is known from many observations, this ambiguity disappears, and from the known pole one can determine the magnetic (co-)latitude, or, in other words, the N-S motion of the sample site *and* the relative rotation about any axis other than the paleopole.

3.15 Field Reversals

For Earth, a **reversal** can be defined as a *globally* observed *change in sign* of the gaussian coefficient g_1^0 that is *stable* over long ($>$ 5kyr) periods of time.

What's the relevance of studying field reversals?

- The alternating fields are recorded as magnetic anomalies in newly created ocean floor. This provides us with a fantastic dating device (the magnetic reversal time scale) and a very powerful tool to track past plate motions.
- It may give us important clues and constraints for our understanding of core dynamics, the very origin of the main field, and the possible mechanisms of core-mantle coupling.

- The magnetic field creates a protective shield for radiation from space (solar wind, for instance) and changes in intensity of the field may also change the effectiveness of this protection. Major events in the evolutionary process have been linked to variations in paleo-intensity.

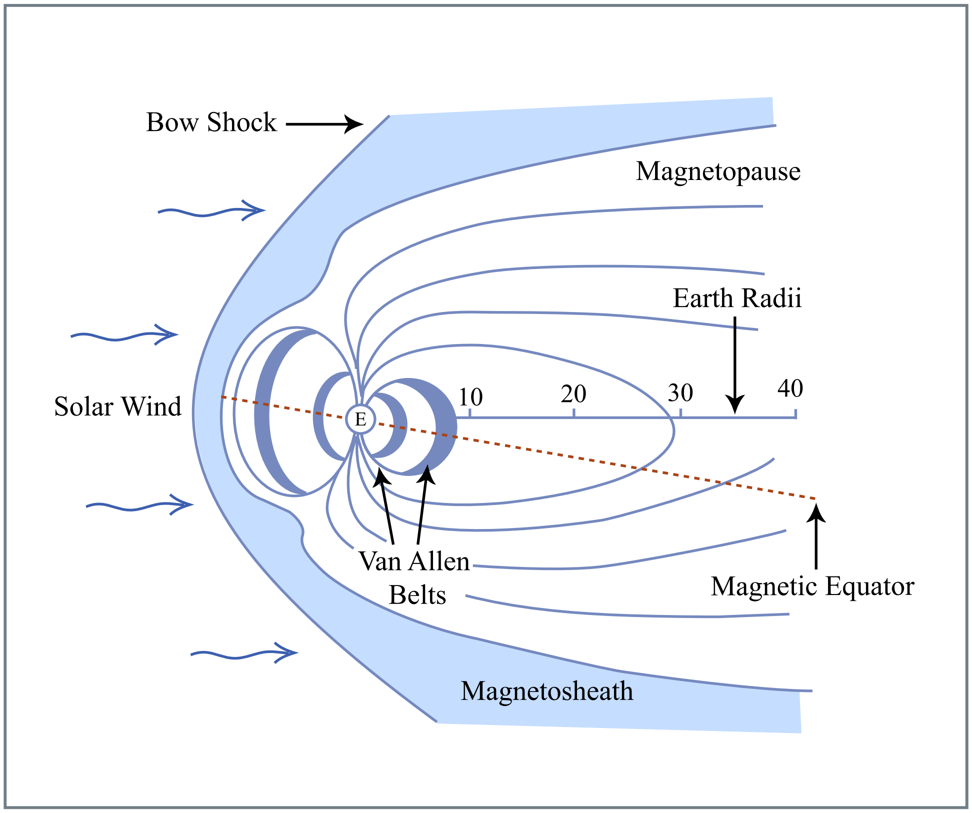


Figure by MIT OCW.

Figure 3.18: The magnetosphere.

- For navigation purposes it is important to know what's going on with the field, for instance whether it is going through a more rapid-than-average change.

Discovery of field reversals: a historical note

TRM was well known in the 19th century. Evidence for field reversals was first reported in 1906 when the French physicist Bernard Brunhes discovered that the direction of magnetization in a lava flow and the adjacent baked clay was opposite of that of the current main field. Brunhes concluded that the field must have reversed, but that explanation was not generally accepted. Recall that at that time the origin of the main field was still a mystery. In fact, it was in the same year 1906 that Oldham demonstrated the existence of a liquid core from seismological observations, and that discovery triggered the development of geodynamo theory as we now know it. Later, in 1929 Matuyama published similar observations as Brunhes, but these also attracted little attention. This continued until well into the 50ies, by which time there was ample observational evidence for field reversals.

However, the necessity for field reversals was debated and a major proponent of an alternative explanation was the French physicist Néel who showed that under certain conditions ferri-magnetic minerals could, in principle, be self-reversing. Self-reversal can happen in a situation where you have a solid solution of magnetic minerals (or two at least) with different Curie temperatures (think of the hematite-ilmenite series). Then, when the sample cools the mineral with the highest Curie temperature, say mineral A, freezes in the ambient main field while the minerals with lower Curie temperatures are still in paramagnetic state. If these paramagnetic minerals are very close to mineral A, the remanent field of A can induce a field in the other minerals that may be opposite of the direction of the main field. This secondary field becomes frozen in when mineral B cools below its own Curie temperature, etc. etc. In case the mineral with the lower Curie temperature has a larger susceptibility (and this is indeed the case for the minerals along the solid solution lines in the ternary diagram!) the reversed field may dominate the combination of the ambient field and the induced field in mineral A, and the net effect would be opposite in direction of the main field without requiring a reversal of that main field. Even though it has been shown experimentally that self-reversal is possible, it requires very special physical conditions and it is NOT a plausible mechanism to explain the by then (mid 50-ies) overwhelming evidence that reversals are quite common.

Study of baked clays (or thermal aureoles of igneous bodies in general) at many sites world wide demonstrated that the direction of magnetization in the igneous body of the same age and the contact aureole were often the same but did not coincide with that of the adjacent non-metamorphosed rocks. It was found that the rocks showing reversals grouped in specific age intervals, and this argument became even more convincing when accurate radiometric techniques such as K–Ar dating were applied. The reversals appeared to be rather global phenomena and could be traced in volcanic rocks as well as in marine sediments. There appeared to be no chemical difference between the rocks the exhibited normal and reversed orientations.

By the time of the early 60-ies several important discoveries associated with sea floor spreading resulted in the general acceptance of field reversals.

Outstanding questions

1. What happens to the dipolar field during reversals? Does the dipole field decay to almost zero before it is regenerated in the opposite direction, or is the dipole field perhaps weakened but still prominent during reversals? The measurement of paleointensity of the main field is difficult and prone to large uncertainties (see also notes on "magnetic cleaning"). Yet, it is now well established that during reversals the intensity of the field reduces by at least a factor of 3. Recall that in the present-day field the dipole

makes up for about 90% of the field and that the non-dipole component is only about 10% . So although the field reduces to about 30% of its original strength, the dipolar component cannot be neglected during reversals. These numbers are, however, not yet well constrained. Some data (and numerical models) suggest a reduction during reversal to about 10% of the original strength, which would either imply that the dipole component has become negligible or that if there still is a dipolar field also the non-dipole components must significantly reduce in strength. The observational evidence is problematic because [1] the non-dipole field at the surface is so much weaker than it is at the CMB and [2] because a reversal is a relatively short phenomenon (say 5,000 yr) and it is hard to get the time resolution in volcanic or sedimentary strata.

2. A closely related question is: can a non-dipole field produce continuous VGP paths during reversals? Are VGP paths random or are there are preferred longitudes along which the VGPs move from the one pole to the other, and what are the implications of the answer to this question for our understanding of core dynamics and core-mantle coupling?

Mechanisms of reversals

The mechanisms of field reversals are still subject of spirited debate, but it is generally accepted that they are controlled by core dynamics. Some important constraints (and, indeed, the need for core-mantle coupling) can be inferred directly from general properties of the geodynamo and from studying the (statistical) behavior of the field during and in between reversals.

1. It is important to realize that the geodynamo does not care about the sense of the field. The magnetic induction equation $\partial\mathbf{H}/\partial t = \nabla \times (\mathbf{v} \times \mathbf{H}) + (4\mu_0\sigma)^{-1}\nabla^2\mathbf{H}$ is invariant to a change in sign of $\mathbf{H}$.

   We have argued before that in the core we can largely ignore the diffusion term, and it is clear that the (sign) of the changes in magnetic field depend entirely on *how* the turbulent flow field $\mathbf{v}$ interacts with the magnetic field $\mathbf{H}$.

2. The typical relaxation time of core processes is much longer than the time interval in which most reversals take place. If the field is not sustained by dynamo action, for instance if the velocity $\mathbf{v}$ in the above equation goes to zero, the field will die out in several 10^4 years. The rate of reversals must mean that the core field is *actively* destructed and regenerated. Part of dynamo theory is the complex interaction of and conversion between the poloidal and (unseen) toroidal field. Differential (radius dependent) rotation in the outer core stretches the poloidal field into a toroidal field; radial flow owing to compositional convection distorts the purely toroidal field (and the coriolis force adds extra complexity in the form of helical motion) which gives a component along the poloidal field. It is conceivable

that turbulence produces local changes in the direction of the helicity of the toroidal field and thus the poloidal component, which may trigger a reversal.

3. Reversals are not simply extreme cases of secular variation even though the distinction is vague. Growth, decay, and drift of the non-dipole field and of the axial dipole are continuous, albeit possibly random, processes. Reversals, however, occur over time scales that are longer than the typical secular variations, but much shorter than the average time interval between reversals ($> 10^5$ yr). In other words, statistically speaking reversals do not occur as frequently as one might expect from the continuous (secular) fluctuations of the main field. It resembles a chaotic system that switches back and forth between two quasi-stable states. This behavior is nicely illustrated by measurements of the inclination, declination, and intensity during a reversal that characterizes the lower boundary of the Jaromillo event (a short N event in the Reversed Matuyama epoch documented near a Creek in Jaromillo, Mexico:

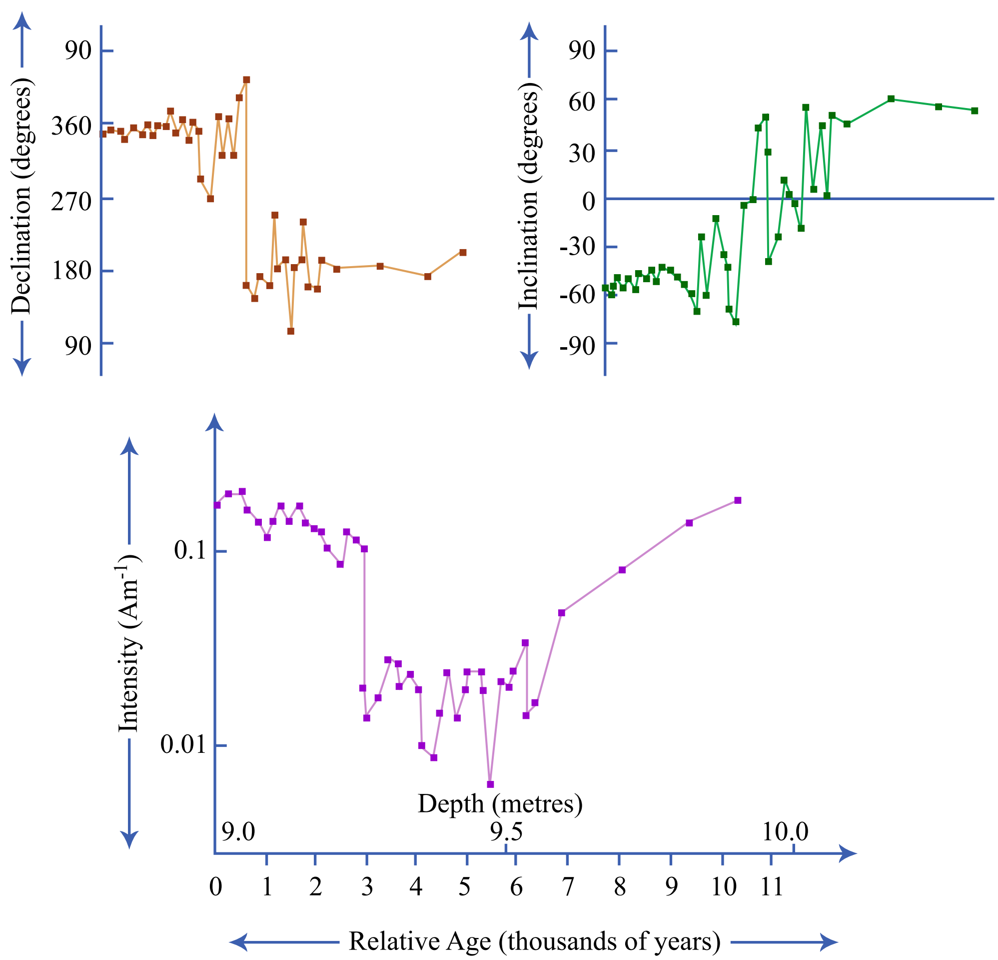


Figure 3.20. Detailed record of a reversal from a rapidly deposited deep-sea sediment core. Samples were "cleaned" in a 10^{-2} T alternating field before measurement. This reversal marks the lower boundary of the Jaramillo polarity event.

Figure by MIT OCW.

3.16 Qualitative arguments that explain the need for core-mantle coupling

There are compelling reasons to believe that the mantle and the dynamic processes in the mantle have a strong influence on reversals (and thus perhaps on core dynamics in general). It is probable that core flow is too turbulent and chaotic to be able to organize systematic patterns in field reversals. Reversals seem to be a random process in the sense that [1] N and R directions of the Earth's magnetic moment occur equally often and [2] the probability that a reversal happens in the next Δt years is independent of the time since the last reversal. However, as mentioned above, the average time interval between reversals is much larger than expected from the continuous fluctuations in the field. Moreover, in the last 5 year several papers have been published that presented evidence for preferred reversal paths of VGPs along meridians through the 'Americas' and through east Asia. The selection of the same VGP path over and over again seems to require that the core has some 'memory' about what happened many thousands of years ago, which is not very likely.

In contrast, the mantle, with its high viscosity has such 'memory' since the mantle structure does not change much over periods that are smaller than 1 Myr. For this reason, investigators have invoked a strong influence of (or coupling with) the mantle. This coupling could be mechanical (topography on core mantle boundary) but that is not very likely. Alternatively, the coupling could be electro-magnetical; in this mechanism flux out of the core would be concentrated in regions where the conductivity of the lowermost mantle is lowest (see dynamo equation above!) and swept away from regions of high conductivity.

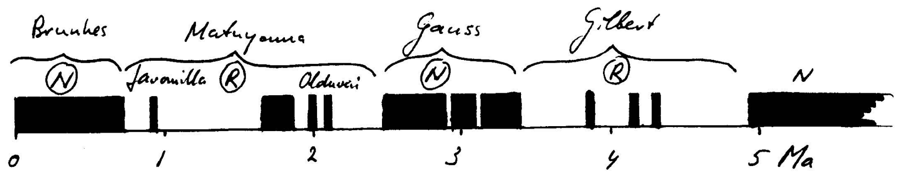
Several investigators have related the patches of high and low mantle conductivity to lateral variation in the thickness of the so called D" layer and this structure is likely to be influenced by flow in the entire mantle. This depicts a complex system where mantle convection controls the lateral variation in thickness of the D" layer (thin beneath major downwellings (slabs) – Americas and east Asia – and thicker beneath upwellings – plumes (central Pacific, Hawaii)) and thus – if D" is anomalously conductive – lateral variation in mantle conductivity that produces torques on the core and creates 'windows' for flux bundles from the core into the mantle. The research in this exciting field is very much ongoing and no conclusive models has been developed yet.

3.17 Reversals: time scale, sea floor spreading, magnetic anomalies

We have talked in some detail about the implications of the reversals of the magnetic field for our understanding of core dynamics and core-mantle coupling. This is a field of active research: rapid progress is being made, but many outstanding questions remain to be answered. About 30-35 years ago (first half of the 60-ies) the reversals of the main field played a central role in the development of the concept of sea floor spreading, which resulted in the acceptance of plate tectonics.

To appreciate the line of thoughts it is good to realize that by the end of the 50-ies there was no agreement on the issue of field reversals, even though an increasing number of scientists had accepted the idea. At the same time, continental drift was still hotly debated. Since the idea was first clearly postulated by Alfred Wegener in 1912 it had been advocated by many scientists, for example the South African DuToit, but, in particular, in the US the concept met with continued resistance. A central issue was the driving force and the mechanism of **continental drift**; one did not believe it possible that continents would plow their way through the mafic and strong oceanic lithosphere.

1. Initially, the evidence for reversals was mainly based on land samples of volcanic rocks. It was recognized that rocks with the same magnetic polarity grouped into distinct time intervals, and the boundaries of these time windows could be dated accurately with radiometric methods. The first Geomagnetic Polarity Time Scale (GPTS) was based on K–Ar and ^{40}Ar/^{39}Ar dating of rocks less than 5 Ma old. Based on the first time scales one defined **epochs** as relatively long periods of time of mostly one polarity, and **events** as relatively short excursions or reversals of the field within the epochs. The first (= most recent) epochs are named after some of the pioneers in geomagnetism (Brunhes, Matuyama, Gauss, and Gilbert); events are typically named after the location where it was either first discovered or where it is best represented (Jaromilla creek, Mexico; Olduvai gorge, Tanzania). The current epoch is referred to as "normal" (or 'negative' since the magnetic moment direction is opposite of the angular moment direction). As more data became available the time tables were refined and it became clear that the distinction between epochs and events is not so useful.



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2. In the early 60-ies, marine expeditions and geomagnetic surveys resulted in the first detailed maps of magnetic anomalies at the ocean floor. A "stripe" pattern formed by alternating strength of the magnetic field (the anomalies were made visible after subtracting the values of the reference field (IGRF)) was documented for the Pacific sea floor off the coast of Oregon, Washington, and British Columbia (Raff & Mason, 1961), and for the Atlantic just south of Iceland. The origin of this pattern was, however, not known. Explanations included the lateral variation in susceptibility

owing to variations in composition or changes in stress due to buckling ('folding') of the oceanic lithosphere (it was known since the large 1906 earthquake in San Francisco that changes in the stress field can influence magnetization).

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See Figure 3.7 on Geol. Soc. Am. Bull. 72 (1961): 1267-1270.

3. At about the same time, Dietz (1961) and Hess (1962) postulated the concept of **sea floor spreading**. Not as an explanation for the magnetic anomalies, but rather as a way out of the long standing problem that continental drift seemed unlikely because continents cannot move through the oceanic lithosphere. Dietz and Hess realized that the oldest parts of oceanic sea floor were less than about 200 Ma old, which is more than an order of magnitude less than the age of the oldest continental rocks. This suggests that oceanic lithosphere is being recycled more efficiently than continental material. These scientists proposed that sea floor is generated at mid oceanic ridges (where pillow lavas had been dredged and high heat flow measured) and then spreads away carrying the continents with them. So the continents move along with the ocean floor, not through it!!

   In 1963 Vine and Matthews and, independently, Morley, put these ideas and developments (1-3) together and correctly interpreted the magnetic anomalies at the sea floor: new ocean floor is created at mid oceanic ridges $\rightarrow$ the basalt cools below the curie temperature and freezes in the direction of the then current magnetic field (normal (N) or reversed (R)) → the N-R pattern spreads away from the ridge. In this way the ocean floor acts as a tape recorder of the polarity of the Earth's magnetic field in the past. Initially, the time table based on land volcanics was correlated with the stripe pattern in the oceans, but this correlation was restricted to the past 5 Ma. Later, paleontological data (the fossil record) was used (along with radiometric calibration of land volcanics)) to extend the reversal time scale to ocean floor of Jurassic age (∼160 Ma). This was first done in the south Atlantic where the time scale was extended to 80 Ma by assuming that the rate of sea floor spreading was constant in that time interval, an assumption that was later justified). With the increasing amount of data, the time scale is continually being updated. The time scale is very important for two main reasons:

- Using the GPTS allows the calculation of sea floor spreading rates by measuring the distance between identified lineations.
- Identification of the magnetic pattern of the sea floor and matching with the time scale can be used to date a piece of oceanic crust.

3.18 Magnetic anomaly profiles

In order to exploit the time scales (for instance for points (a) and (b) above) one must be able to identify and interpret magnetic anomalies, or, more specifically for marine surveys, magnetic profiles due to a succession of magnetic anomalies with reverse polarity. In the interpretation one usually tries to find a combination of N and R 'blocks' that best matches the observed profile. This is easier said than done. A visual inspection of any of the profiles used as illustrations in, for instance, it can be demonstrated that the relationship between the simplified block models and the corresponding magnetic profiles is far from trivial. The magnetic profiles look more complex than you would expect from the simple block models, and this is true not only for real data but also for the synthetic (computer generated) profiles.

In the first place, the real sea floor is not magnetized in the form of regular geometric bodies. The geological processes at work in a mid oceanic ridge environment are complex and so are the resulting patterns of magnetization. In particular, the creation of new ocean floor in the so called **emplacement zone** is an irregular process that is not necessarily symmetric across the ridge axis. Also the spreading rate has an effect in that fast spreading (Pacific rise) results in wide zones with the same magnetization which facilitates identification, whereas slow spreading (Atlantic) produces narrow anomalies with poor separation of the anomalies, which complicates the analysis.

But even for a regular alternation of N and R polarity, say in a computer model, the magnetic profiles can be complicated. This is mainly due to the dipole character of the magnetic field as opposed to the monopoles in gravity, which means that the orientation of the anomaly plays an important role.

Before we look at this in more detail, let's define what we mean by a "magnetic anomaly" in the context of marine surveys. At sea one uses proton precession magnetometers (see, for instance, Garland, 1979) that are towed behind the ship. These instruments make use of the precession of hydrogen atoms after a strong (artificial) ambient field generated by a current in a coil surrounding a contained with many hydrogen atoms (e.g., water!) is switched off. When the initial field that aligns the hydrogen dipole moments is switched off the hydrogen atoms precess with a frequency that depends on the strength of the ambient field. The precession induces a current in the coil, which can be measured. Precession magnetometers measure the magnitude of the total magnetic field, not its direction (which means that they do not have to be orientated themselves!) and have a sensitivity of about 1 nT. The magnetic anomalies are determined by subtracting the IGRF from the observed values.

In vector notation, the total field $\mathbf{B}$ is given by the contributions of the main field (the core field) $\mathbf{B}_E$ and the magnetized body $\delta\mathbf{B}$:

$$\mathbf{B} = \mathbf{B}_E + \delta\mathbf{B} \tag{3.87}$$

But the scalar magnetometer measures

$$B = |\mathbf{B}| = B_E + A \tag{3.88}$$

with A the magnetic anomaly.

$$\begin{aligned} B &= |\mathbf{B}| = (\mathbf{B}\cdot\mathbf{B})^{\frac{1}{2}} = \{\mathbf{B}_E\cdot\mathbf{B}_E + 2\mathbf{B}_E\cdot\delta\mathbf{B} + \delta\mathbf{B}\cdot\delta\mathbf{B}\}^{\frac{1}{2}} \qquad (3.89)\\ \Rightarrow B &= \{B_E^2 + 2\mathbf{B}_E\cdot\delta\mathbf{B} + \delta B^2\}^{\frac{1}{2}} \end{aligned}$$

Typically, δB is of the order of several hundreds of nT whereas B_E is of the order of 20,000 to 60,000 nT. Therefore we can safely neglect δB^2 so that

$$B = \{B_E^2 + 2\mathbf{B}_E.\delta\mathbf{B}\}^{\frac{1}{2}} = B_E\{1 + 2\hat{\mathbf{B}}_E\delta\mathbf{B}|\mathbf{B}_E|^{-1}\}^{\frac{1}{2}} \tag{3.90}$$

with the unit vector $\hat{\mathbf{B}}_E$ in the direction of $\mathbf{B}_E$. The second term is small so that this expression can be expanded in a binomial series

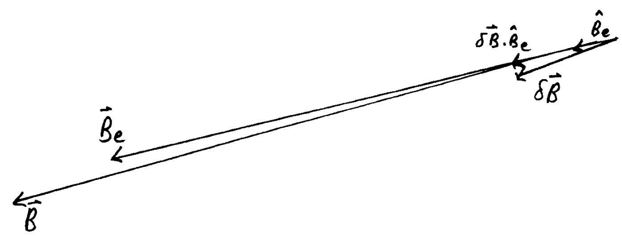
$$B \approx B_E\{1 + \mathbf{B}_E\delta\mathbf{B}|\mathbf{B}_E|^{-1}\} = B_E + \hat{\mathbf{B}}_E\cdot\delta\mathbf{B} \tag{3.91}$$

and we define the magnetic anomaly A, see (3.88) as

$$\boxed{A = B - B_E = \hat{\mathbf{B}}_E\cdot\delta\mathbf{B}} \tag{3.92}$$

In other words, the only components of the local field $\delta\mathbf{B}$ that we measure is the one parallel to the direction of the ambient (core field). A vector diagram shows that this is a good approximation since $\delta B \ll B_E$. (For a dipole, the field is given by the gradient of the potential $\delta\mathbf{B} = -\nabla V = -\nabla(\mu_0/(4\pi)^{-1}\mathbf{m}\cdot\hat{\mathbf{r}}r^{-3}\}$.)

Let's consider some examples in which $\mathbf{B}_E$ is approximated by an axial dipole. Assume an infinitely long block (lineation) that is magnetized in the



direction of the ambient Earth field. The magnetization direction can be decomposed into components parallel and perpendicular to the strike of the block. It can be shown that the component $\|$ to the strike does not contribute to a magnetic anomaly since the infinitely small dipoles that make up the anomalous structure all lie 'head-to-toe' and cancel out except for the ones near the end (which infinitely far away (in other words, the field lines never leave the body). In this geometry, the magnetized body only has magnetization in the directions $\perp$ to the strike (say, in the x and z direction). Consequently, a N-S striking anomaly magnetized at the equator will always have $A = \hat{\mathbf{B}}_E \cdot \delta\mathbf{B} = 0$ since $\delta\mathbf{B}$ is orthogonal to $\mathbf{B}_E$ (or, more precisely, there is no non-zero component of $\delta\mathbf{B}$ in the direction of $\hat{\mathbf{B}}_E$. In contrast, lineations magnetized at the pole and measured at the pole have $\hat{\mathbf{B}}_E \| \delta\mathbf{B}$ so that there will be a strong signal centered over the anomaly.

The effect of the orientation (which theoretically speaking gives rise to a phase shift) presents itself as a skewness of the anomalies (see diagram) which can be corrected for (for instance by reducing the anomaly to the pole, see diagram).

In addition to these distortions due to the phase shift there's the effect of **amplitude modulation**. Amplitude modulation consists, in fact, of two contributions:

1. usually, the magnetized body (at the ocean floor) is several km beneath the vessel. The observed magnetic field thus represents the upward continuation of the field at the source. As a result of the $1/r^3$ decay of the field there is significant spatial attenuation of the high frequency components. Sharp contrasts will be observed as more gradual transitions.

2. Since the magnetized body can be thought to consist of dipoles a homogeneously magnetized lineation will only display a magnetic anomaly near its ends; in between, the plus and minus poles cancel out (same argument as above) and no anomalous field is measured. This results in an effect known as **sagging**. See diagram on next page.

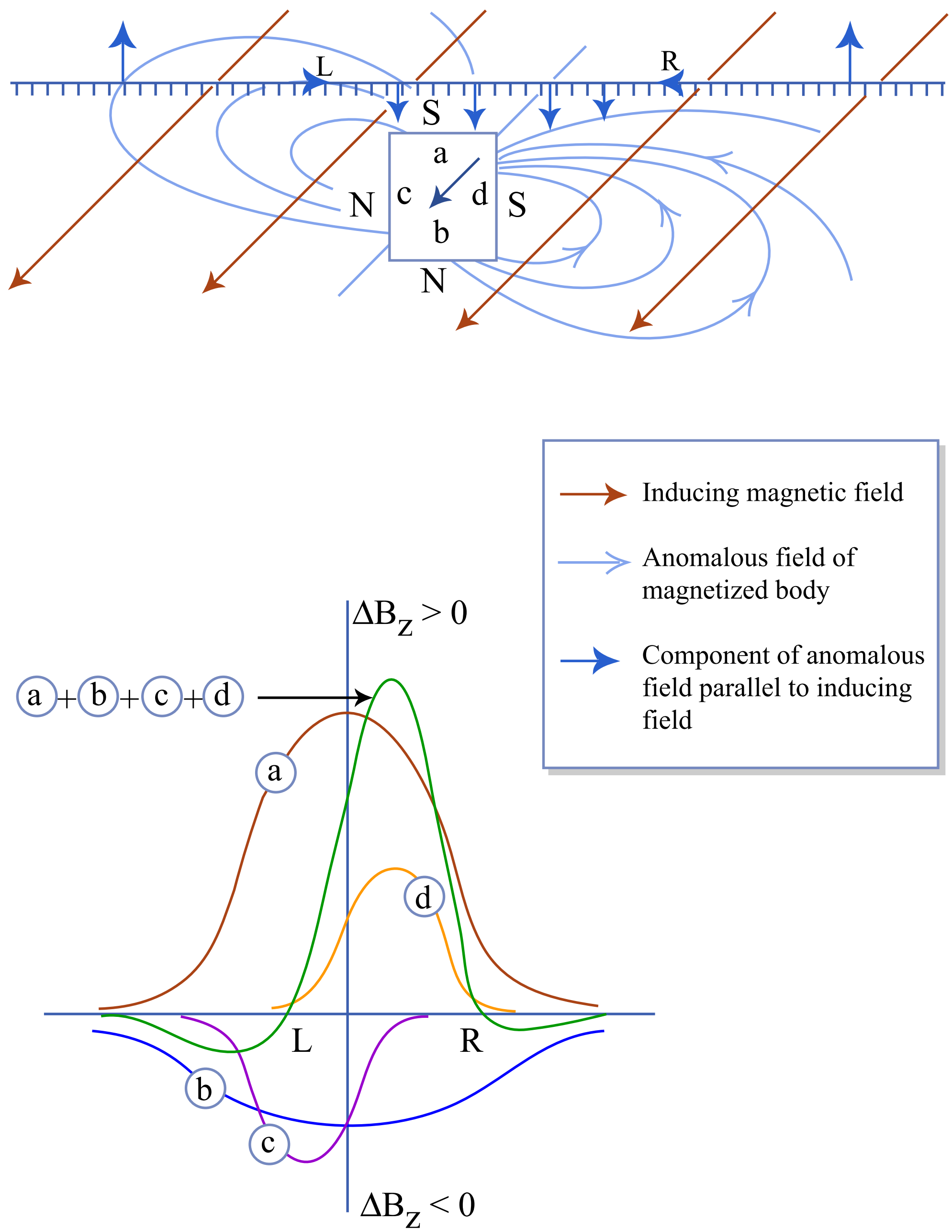


Fig. 3.24. Explanation of the origin of the magnetic anomaly of an infinitely long vertical prism in terms of the pole distributions on top, bottom and side surfaces, when the magnetic field (or magnetization) is inclined.

Figure by MIT OCW.

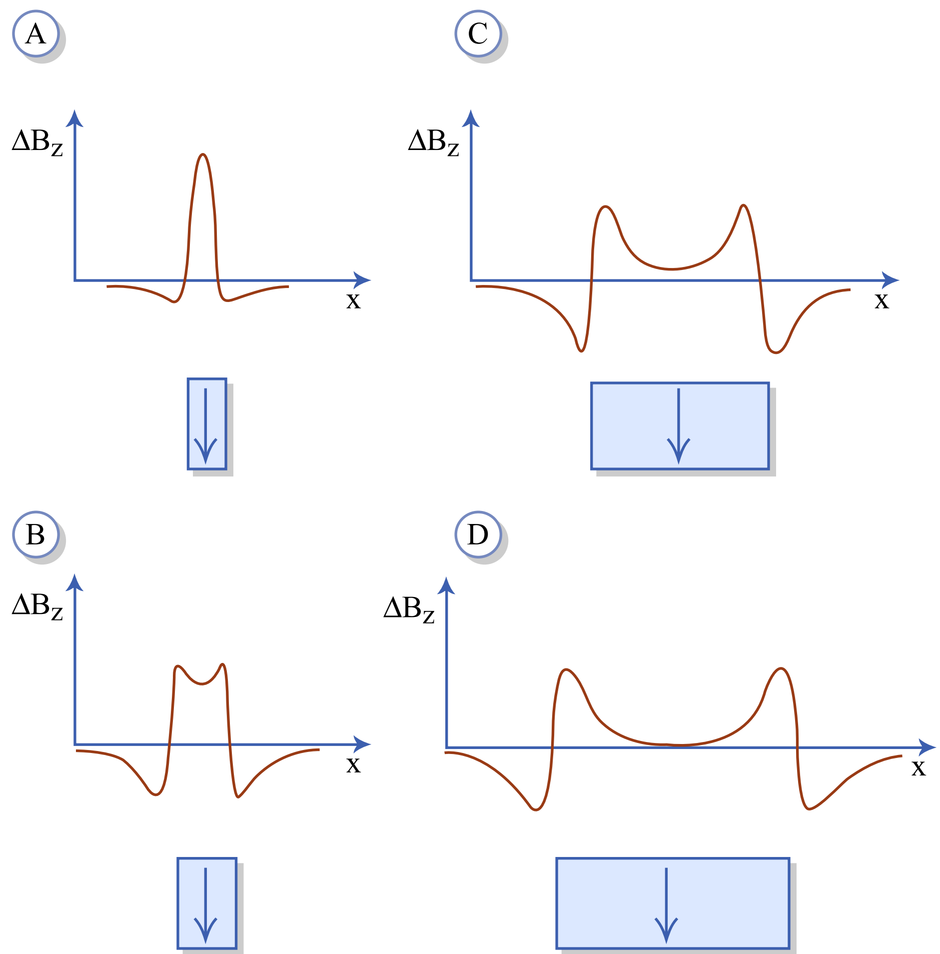


Figure 3.25. The effect of block width on the shape of the magnetic anomaly over a vertically magnetized crustal block.

Figure by MIT OCW.

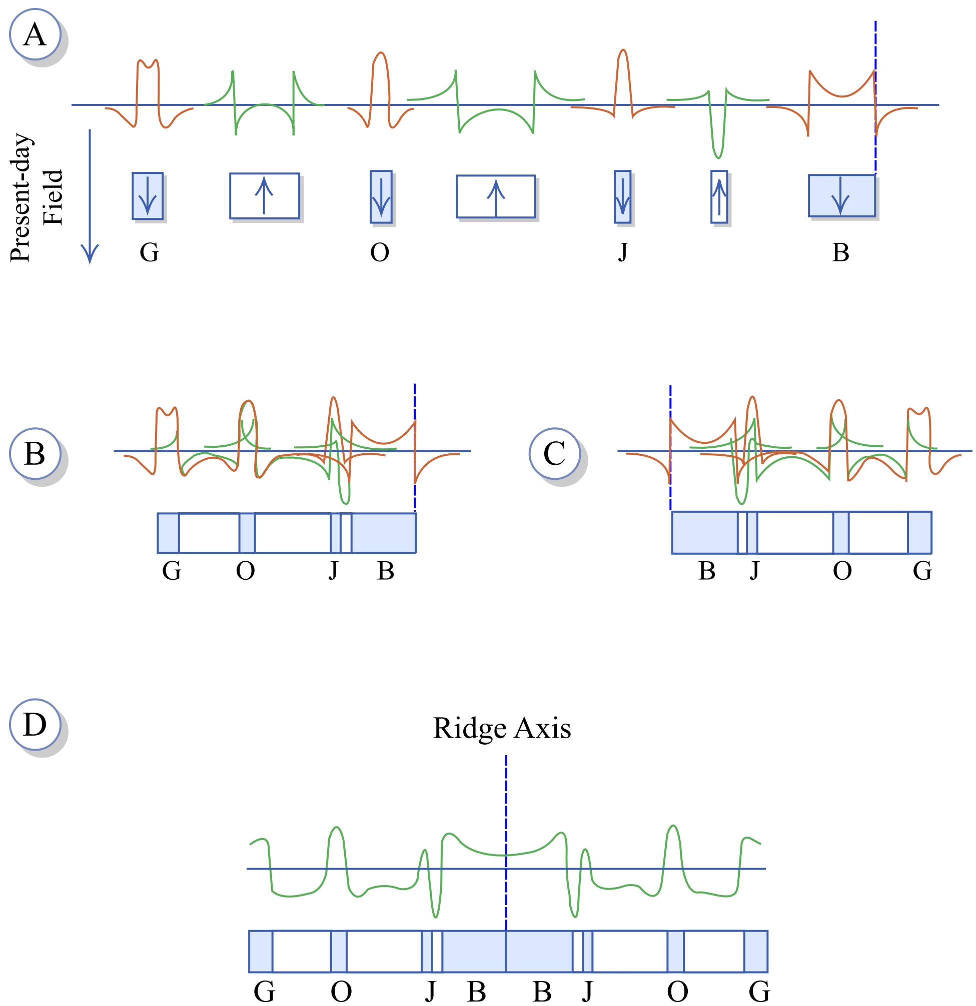


Figure. 3.26. Explanation of the shape of a magnetic profile across an oceanic spreading center: (A) the anomalies of individual oppositely magnetized crustal blocks on one side of the ridge, (B) overlap of the individual anomalies, (C) the effect for the opposite sequence of blocks on the other side of the ridge, and (D) the complete anomaly profile.

Figure by MIT OCW.